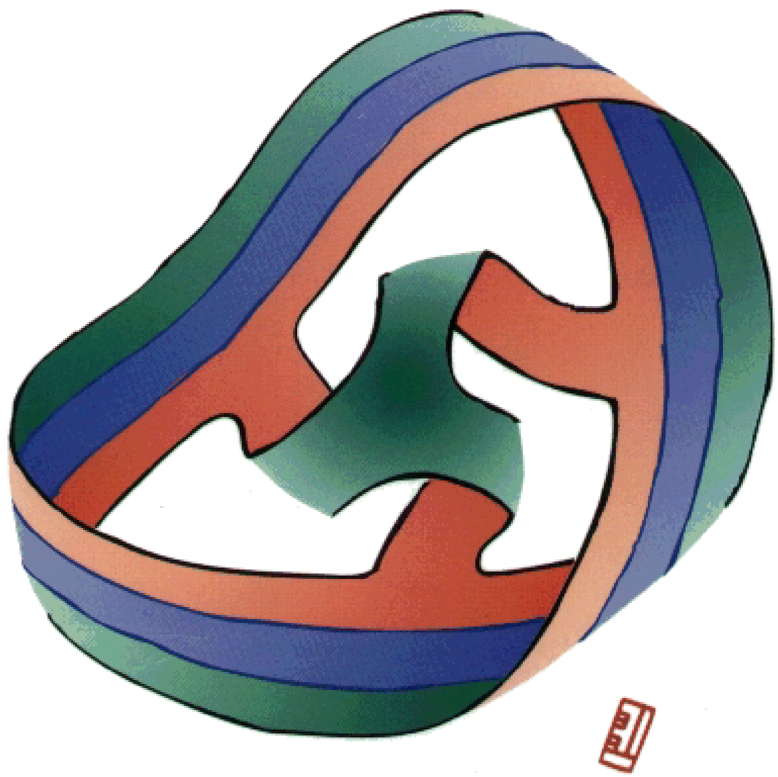
KNOT

The knot theory outlined by J. Lacan

Jean-Michel Vappereau



Translation by Quinn
Foerch

Presentation of the series of results booklets

1. In Freud's field, our results booklets take things seriously; they form a series. The series of our results in topology extension is aimed at those who want to make their way in this field, without being petrified with fear or steeped in indifference.

We borrow the expression "results booklets" from the Bourbaki team. The mathematicians in this group develop the construction of mathematics based on the terms of set theory. In the instructions for use of their treatise, which they have divided into books, they specify the function of these booklets:

"Certain books (either published or in preparation) are accompanied by supplementary booklets containing the results. These booklets contain the essential definitions and results of the books, but no demonstrations."

Their attempt differs from ours in a way that cannot be confused; our booklets are not appended to any treatise of comparable scope.

On the other hand, in our construction of the object of psychoanalysis, based on the foundation of set theory, we have Freud's work and Lacan's writings, the latter accompanied by his seminar teachings.

2. Psychoanalysis was invented by Freud when he discovered the unconscious. This invention was completed by Lacan through a critical commentary on Freud's text, which he tested against his own logic.

This practice is based on a method that produces a discourse.

The method is familiar to those who study texts. It was given its letters of nobility by Champollion, who had already used it successfully. The psychoanalytic method consists of comparing two versions of the same text, since analytical discourse is based on the hypothesis that our psychic apparatus is developed through a series of translations, transcriptions, and transliterations.

We group these different acts under the term translation, which is the subject of this study. To use this method, several versions of the text under study must be available. Psychoanalysis applies only to a speaking subject who himself provides, in a single discourse, the different versions of the same text (É, pp. 747-748) ([1] What is foolishly called applied psychoanalysis is nothing more than the use of the psychoanalytic method in literary criticism, for example.

The study of the problem posed by translation culminates in a practice of writing that Dr. Lacan finds in the writing of the Japanese language. The reader may know that Japanese scholars write their own language (kun-yomi reading) using characters that were used to write an archaic form of Chinese (on-yomi reading) ([2] which they are familiar with.

Fig. 1

This is how the element that we write as "water" *in our countries is said and written. This practice of writing calls for numerous comments. To begin the discussion, we will limit ourselves here to a few of them.*

The use of this writing produces a permanent translation effect.

This translation takes place within the same culture for the literate Japanese. This is most evident in written Japanese because of the characters that introduce another dimension to the translation; this is produced by a fiction of three.

We find this instance of the letter in Lacan's Écrits *when he presents the structure of the signifier using the oppositional pair men/women,*

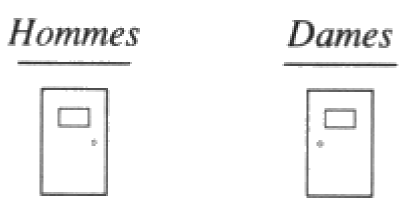


Fig. 2

which spans two identical doors, signaling in our country those isolated places subject to the laws of urinary segregation (É, p. 499).

There is something impertinent about illustrating the function of the letter in this way, but it is nevertheless present in our society, even in our public life, and its articulation seems to be erased in alphabetic writing.

It is in the same way that clinical elements can be understood in practice.

The words of the analysand must go so far as to encounter the structure of the Freudian field in order to reach the dimension of discourse. *This structure is topological because analytical discourse is part of an era of logical-mathematical science whose topology aims at the foundation. It is therefore through a series of translations that there is a transition from the particularity of the case to the universality of what is being founded. This gesture does not claim to take us out of this fantasy but aims to account for it.*

This is not an increasingly disembodied abstraction, as Husserl believed in his Foundations of Arithmetic*, with regard to the concept of a cat, but a materiality.*

literal, as Frege reminds him, where the foundation of concepts is based on the extension of specific cases to the dimension of the whole. The concept of a cat is not an abstract cat from which the fur, whiskers, eyes, etc. have been removed, but rather the collection of cats, when it gives rise to a whole according to specific conditions. We will therefore speak of a concept in relation to this collection, provided that a letter or a name can be assigned to it, and we will then say that it is a set. This assignment depends on textual constraints that are well known in set theory but less so in other fields ([3]*.This raises the delicate question of proper names.*

The practice of psychoanalysis interprets the fact of translation by relying on the drawings or mathèmes of topology and by using the topology used in mathematics, which does not lend itself to applied topology but, as in the reading of Japanese, achieves a bilingual speech.

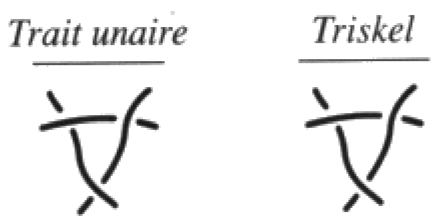


Fig. 3

We could multiply examples for each concept in psychoanalysis. Hence the necessity of our topological elements, for what can be said of a Japanese scholar who is unfamiliar with on-yomi reading (ancient Chinese) and claims to be able to write the Japanese language (kun-yomi reading) unambiguously without it?

Everyday conversation involves both the gaze and the voice, whereas Freud's practice *consists in principle of isolating the voice at the expense of the gaze in psychoanalytic treatment itself (a major stage in analytical* training, *[É p. 698]). This practice responds to what is discovered there, the torments of transference where, in the interplay of passions, ignorance hides behind love, sometimes behind hatred. But they must, as we have just said, be articulated with the whole of* the training *that makes this transference a formation of the unconscious.*

Lacan's practice *is part of this configuration. He undertakes to return to Freud by effecting, in the case of practice itself as well as in relation to each concept, a slow but radical reversal. His practice of structure then consists in setting aside the voice, which is what Lacan did at the end of his career, in order to focus attention on the gaze, especially with topology drawings. The turning point of this reversal, according to the structure of the Freudian field, finds its practical realization in short sessions. This stage, where this practice is reduced to a simple cut, is necessary.*

Our approach is not to remain stuck on one or other aspect of the structure, but rather to ensure that none of them are neglected. Our project is an approach to Lacan, an approach to Freud, in line with this dual movement that remains unsurpassable—the practice is thereby broadened.

The practice of psychoanalysis is certainly not aimed at producing mathematics, but it requires that we do not ignore it. The training ends, for the analysand, with the separation of the analyst from the analysand, which must be reported. Practice (clinical practice, structure, and action) does not hold if we avoid the dogmatic foundation of translation, that is, of reading the unconscious.

Analytical discourse *progresses from this practice, but it is already there for our generations. This was not the case for Freud, nor yet for Lacan; they did not have it at their disposal. Analytical discourse is the social bond that is formed as a result of this practice and its results. Here we see that it does not happen on its own. It begins with two, paired with the works of Freud and the writings of Lacan.*

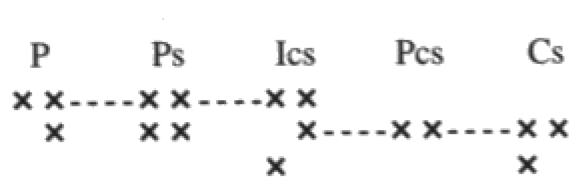
These reasons lead us to consider, in the preamble, topology as it will be discussed as the argument of the discourse. This discourse, which is currently being developed, appears in several versions in Freud's work and in several translations in Lacan's teaching. We must reason them through the use of the psychoanalytic method, the learning of which constitutes the other stage of training. This method cannot be neglected in the reports of the test, just as it cannot be neglected in the internal effects it produces.

3. *Freud's work is divided into two topics, separated by an intermediate moment (1914-1915) when the necessity of the transition from the first to the second is affirmed.*

There are three topological chapters in Lacan's teaching, related to three types of mathematical varieties: graphs (1953-1961), surfaces (1961-1971), and knots (1972-1981). We define and develop these concepts in our series of results.

Freud

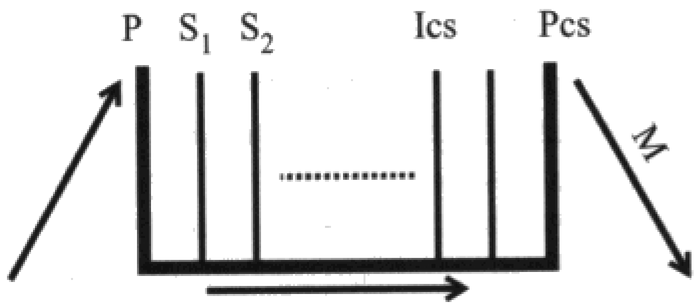
1st topic. *In letter 52 to Fliess, Freud draws a diagram* [4] *that corresponds to his hypothesis that the psychic apparatus is constituted by successive translations. The segments correspond to upheavals caused by translation.*



Les lettres du graphe de la lettre 52 se lisent ainsi :
P = Perception, Ps = Perception-signe, Ics = Inconscient,
Pcs = Préconscient, Cs = Conscience.

Fig. 4

Freud raises the question of the conjunction of the extremities of this graph, of the knotting together of perception and consciousness, where our "reflexive tradition" "has tested its standards of truth" [É, p. 69]. This question recurs in The Meaning of Dreams (*p. 460, note 1) when he gives a new optical version of his diagram, in which each stage of translation is rendered by a lens that produces a reversal of the object, as in a telescope.*



Shéma de la Traumdeutung

Fig. 5

This is the initial problem from which we will start the topology of the subject.

During this first period, Freud established the work of the unconscious in three major works: The Interpretation of Dreams, The Psychopathology of Everyday Life, *and* Jokes and Their Relation to the Unconscious.

2nd topic. *In his second topic, Freud questions the same structure that recurs in the pitfalls to be avoided in dualism in theory. To this end, in 1914, he introduced his theory of the ego with narcissism. The other who speaks through the stumbling blocks of my speech is not symmetrical to me, just as my unity does not depend on the unity of my organism. What makes one out of these two? Similarly, Freud distinguishes between the sexual drives that invest the object and the ego drives that are supposed to preserve it. He recognizes that they are the same thing (*Introduction to Psychoanalysis)*:*

> *"It seems that in a whole series of cases this could also be a conflict between different strictly sexual tendencies." (p. 330, retranslated.)*

and

> *"We have no reason to assert a difference in nature, which is not very graspable, between the two groups of drives." (pp. 389-390, retranslated)*

But they are not the same:

> *"But it is basically the same thing, because of the two conflicting sexual tendencies, one is always, so to speak, satisfying to the*

> *myself* (ichgerecht) *while the other challenges* (herausfordert) *the defense of the self. This remains close to the conflict between myself and sexuality." (p. 330, retranslated)*
>
> *"The two (groups of drives) are opposed to each other for us only as designations for the sources of energy of the individual, and the discussion, whether they are fundamentally one or different in nature, and even if (they are) of a (single group), when did they separate from each other?" (p. 390, retranslated)*

It is for this same reason that Freud introduced narcissism as early as 1914, since, in narcissistic neuroses, the ego is sexually invested as an object, and Freud believes he can elucidate this enigma by studying these neuroses.

> *"We certainly know much less about the development of the ego than we do about that of the libido, because only the study of narcissistic neuroses promises an examination of the structure of the ego." (pp. 330-331, retranslated.)*

This structural difficulty, which recurs from the presentation of the unconscious to the introduction of the death instinct into the doctrine, is also present in the 1914 article, in which Freud distinguishes, in the most certain manner, between the two terms "ideal ego" and "ego ideal," the fact that "we are nevertheless unable to distinguish their use in this text should be rather worrying" [É, p. 672]. Some believe this expression to be a reproach to poor readers of Freud who fail to distinguish between these two uses. This is not the case. If there is a reproach to be heard in this sentence, it is addressed only to those who are not concerned about it, as we assume they have read Freud's article carefully. In fact, if they have not read it, they cannot distinguish between these two uses in the text; if they have read it, have they realized that it is impossible to distinguish between them? Very few have read it and, consequently, very few are concerned about it.

In this intermediate period, in 1915, Freud attempted to write his Metapsychology *in twelve articles, of which only four remain, plus one that has just been rediscovered (1985). It was this failure that prompted him to construct his second topic, in which he radicalized his position by going so far as to address the outcome of his initial hypothesis, since it necessarily led him to subvert our conceptions of causality, under the title of the automatism of repetition.*

Freud seeks a rational explanation for this enigmatic fact in phylogenesis (Introduction to Psychoanalysis, p. 334). To this end, he has already constructed the myth of the primitive horde (Totem and Taboo), in anthropology. In biology (Introduction to Psychoanalysis, p. 390), he seeks support to the point of evoking Weissmann (Beyond the Pleasure Principle). As this structure appears in the material of analysis, he seeks the reason for it in philology in Abel (The Opposite Meanings of Primitive Words). The answer is logical; it is topological.

Lacan

Let us distinguish between historicity and structure. The temporal development of phenomena holds some surprises for us, such as feedback, reversals, interruptions, and resumptions that only structure can illuminate.

We must indicate by what rational process, in what reasonable context, Lacan was led to introduce his mirror stage [54] [5]. It is by realizing the fundamental dependence on the context, let us say the social or even familial context of the subject, that we must bear the repercussions of the radical inadequacy of the most precise account of this context. Better still, we arrive at an uncertain concept, or one that is simply paradoxical in appearance, of an acquired innateness. Here we see that our categories at the time are lacking something, that we lack the categories necessary in this situation. These are what we call structure, and we undertake to study them in this series of booklets.

Structure implies features or invariants, according to certain principles governing the actions to be performed.

What is invariant is what occurs on the occasion of each utterance. We will note this in the terms of the signifying condensation used above, as the product of this signifying involution is noted by Lacan with the letter a:

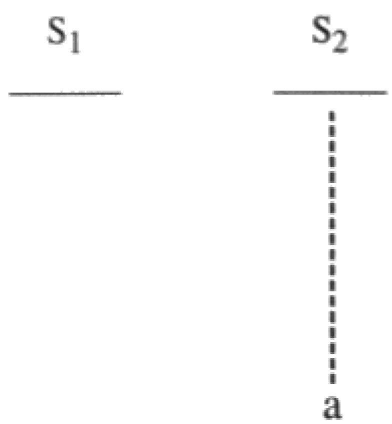


Fig. 6

This structure establishes, between structure and history, the reason discovered by Freud, which is not mentioned in the Rome speech and which Lacan begins to address in "The Instance of the Letter" *and takes up again in* "Position of the Unconscious" *[É, pp. 835-836].*

We will thus schematize the psychic event in Freud's sense, that is, the rupture of semblance which, according to Lacan, causes a trickle of small letters to erode the signified, which can only be resolved by counting the elements, provided that no letter is missed, in a rapture of having found the name. We present this in all of our work using the following T-shaped graph.

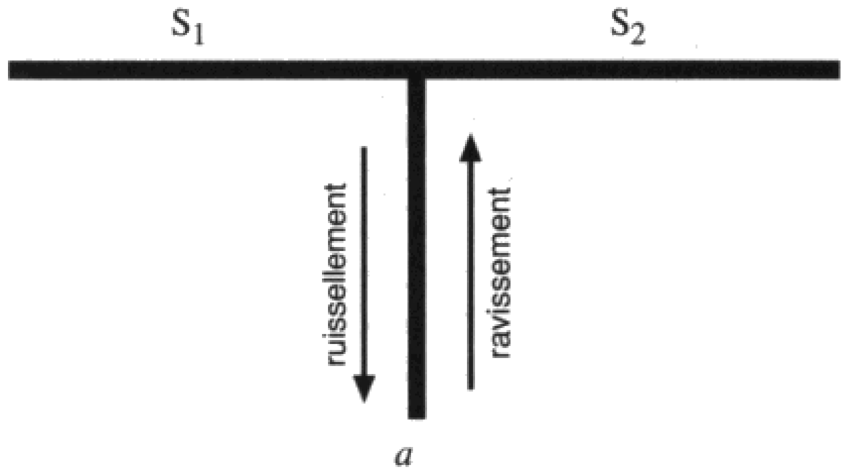


Fig. 7

We find this presentation at every stage of Lacan's teaching.

Chapter 1. *The first historical reference to topology in Lacan's teaching can be found in his first speech in Rome in 1953, concerning the structure of language. On this occasion, he uses the torus as an object to illustrate this structure [É, pp. 320-321].*

Dr. Lacan devoted the first period of his teaching (1953-1961) to a symbolization of the Imaginary through the alternation of the similar and the dissimilar [É, p. 821], in order to lift the Symbolic out of an imaginary quagmire into which psychoanalysis had fallen after Freud.

We must refer this Imaginary back to the instance of the Symbolic, that is, to the structure of language. From this period onwards, Lacan proposed a graphic solution to the conjunction of the extremities of Freud's graph.

Fig. 8

Let us take the graph of lines drawn by Freud in his letter 52. We replace the points with segments and the segments with points. The same terms are found there. By folding this graph of lines,

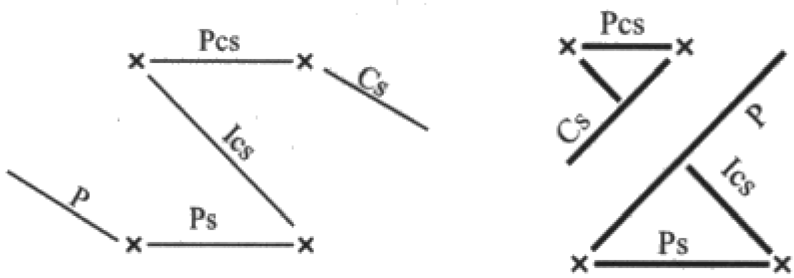


Fig. 9

we obtain the schema, referred to by us as schema F, which allows us to navigate the two Lacanian schemas contemporary with this early period of his teaching.

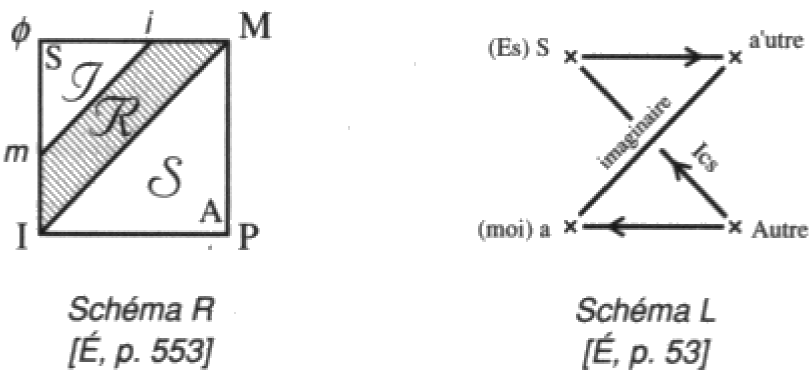


Fig. 10

We study the conjunction of these two diagrams on the surface of the projective plane in booklet no. 2. Thus, from the very beginning of his teaching, Dr. Lacan poses the enigma that we must relate to the structure of language, by asking whether these elements are one or two, thereby prolonging the subversion of our reflexive tradition.

Let us return to Freud's graph, transformed into his line graph. Thanks to our diagram F, we can transfer the letters from Lacan's diagram R (it should be noted that there are no points marked at the ends of the line graph; however, the letters m *and M correspond to these places) and the orientations of the edges from diagram L.*

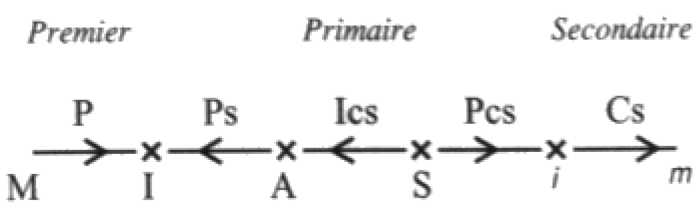


Fig. 11

The primary process, dominated by the pleasure principle, is a thought process that reigns in the unconscious [É, p. 650]. It would be a mistake to believe that what is primary is first. For us, therefore, there is a primary process that dominates the separate perceptions of *the unconscious through perception-signs. The secondary process, governed by the reality principle, dominates the conscious separate from the unconscious through the preconscious.*

We report these results in the structure diagram.

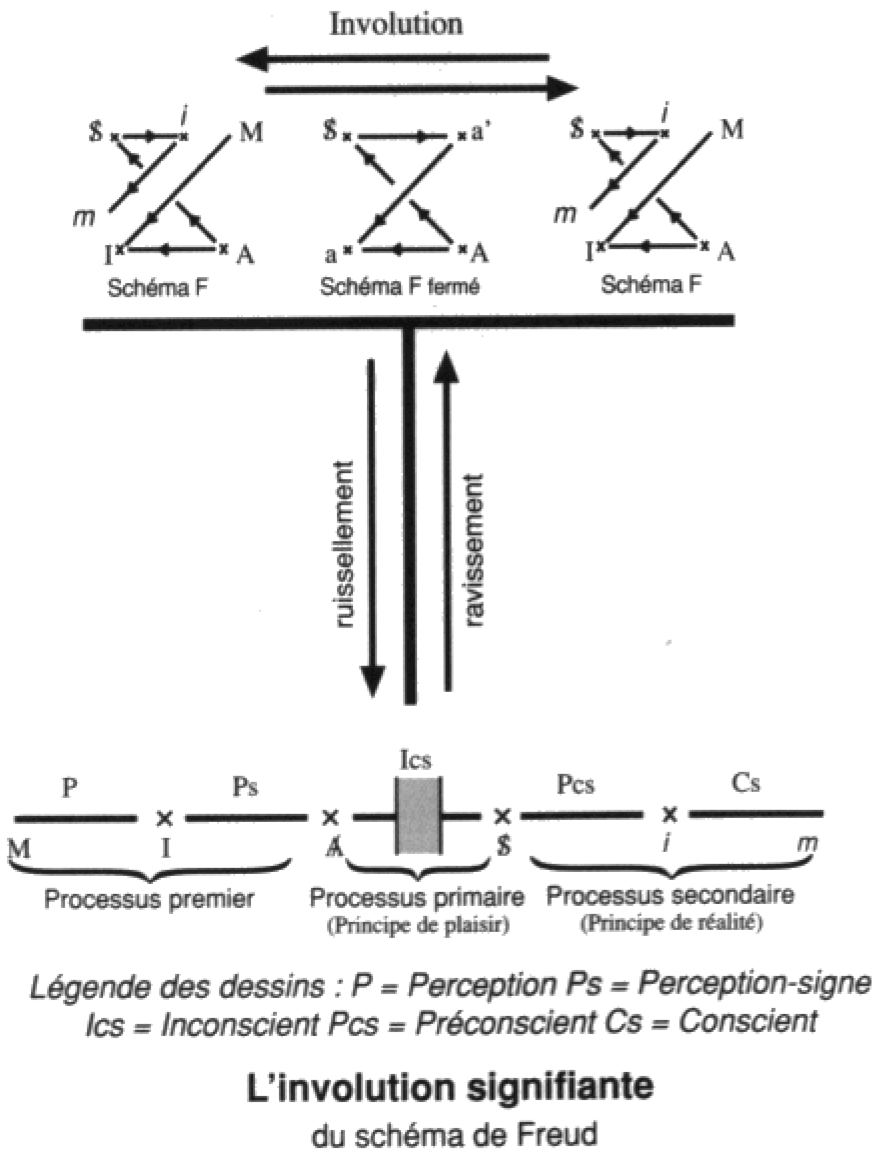


Fig. 12

We also transfer the letters from Freud's diagram to our diagram F, along with those from diagram R and the orientations from diagram L.

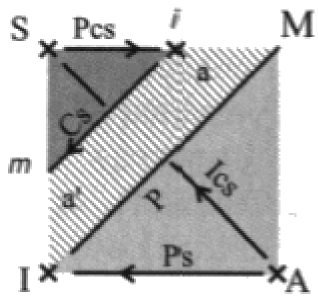


Fig. 13

In our diagram F, the orientation of the edges by arrows comes from diagram L and the grid of areas comes from diagram R. We replace the letters I, R, S with three different colors
— S on the side of the primary process, R on the side of the imaginary grid of reality, and I on the side of the secondary process, which divide the primary process Ics into three parts in our diagram.

This observation allows us to read Lacan's diagrams R and L placed in our graphic presentation of the structure.

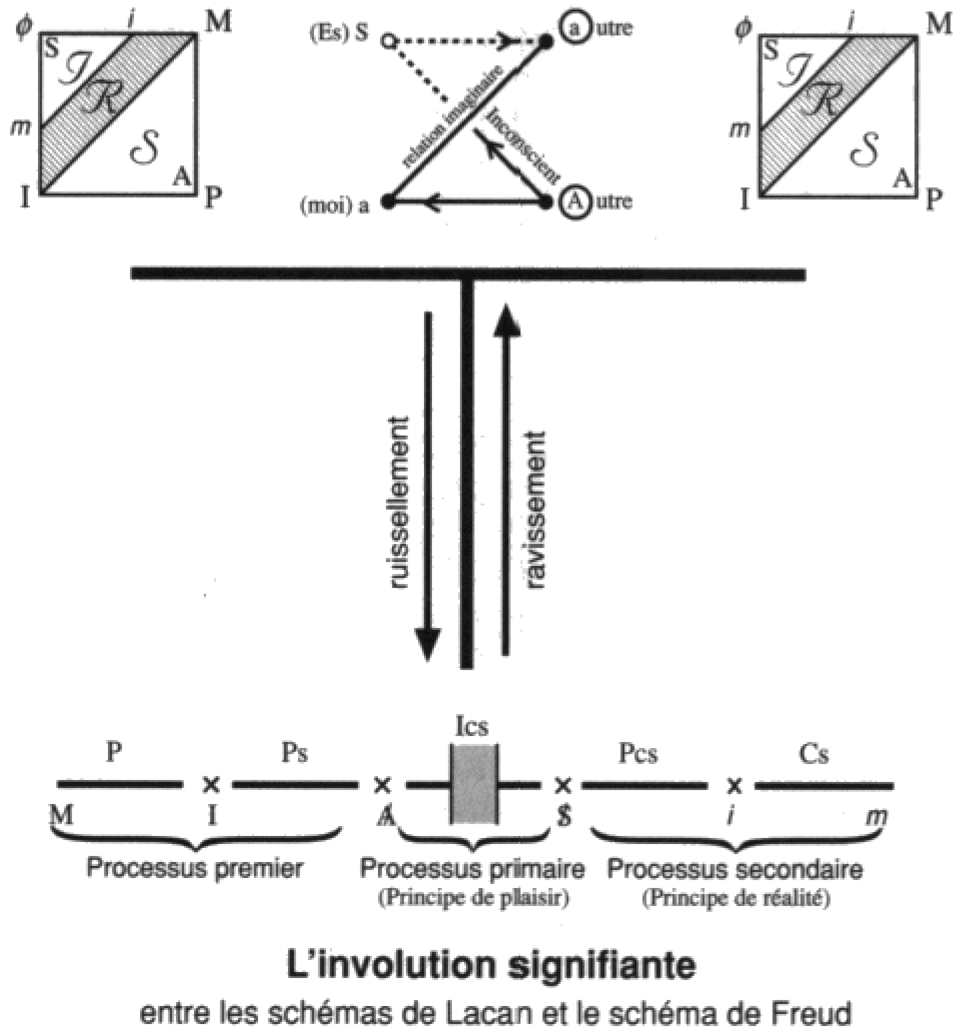
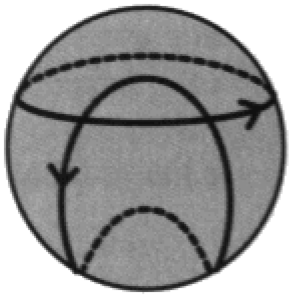


Fig. 14

Dr. Lacan extends this questioning in order to present to his audience the articulation of his schemas by constructing the graph of desire, which is the most developed version of our T-shaped graph of the structure that accounts for the links between structure and history, but where we see the problem of the disjunction and intersection of two logical sets in Euler-Venn diagrams placed on the sphere. He will develop this logical presentation in the next stage of his teaching.



Le point de capiton sur la sphère

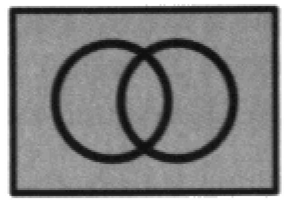


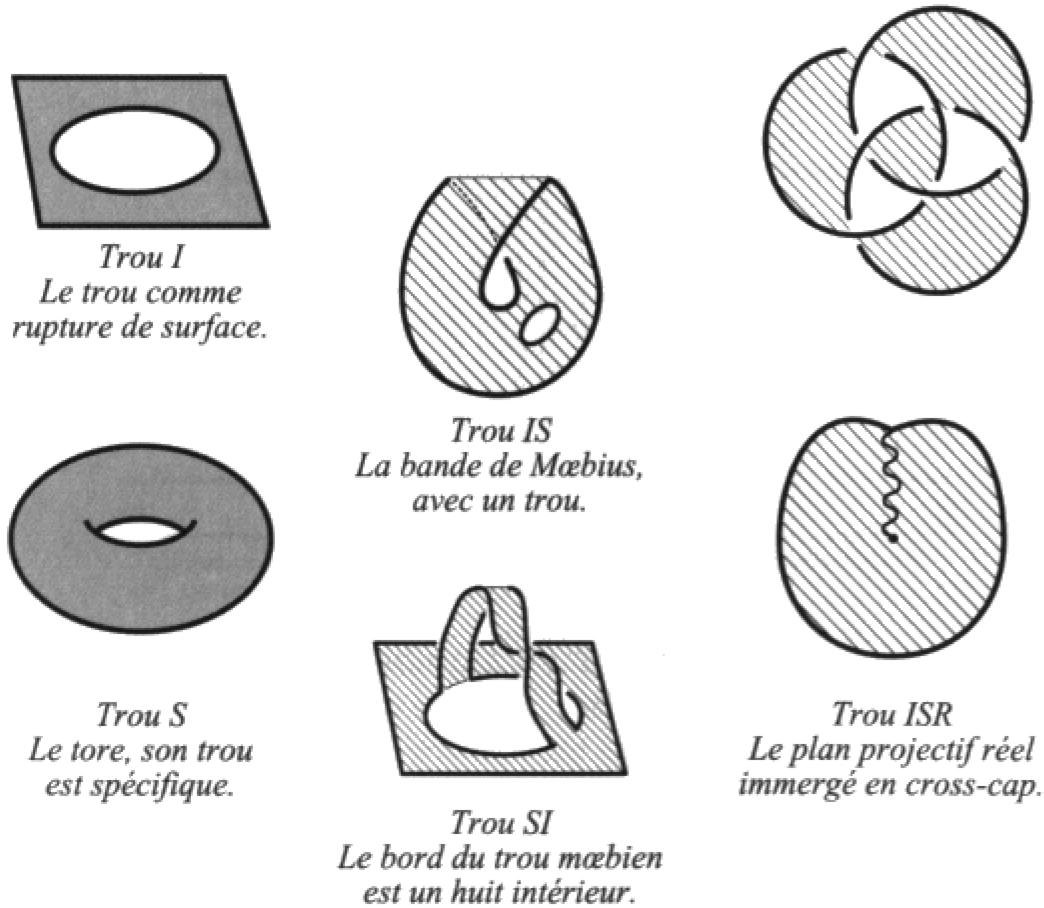
Diagramme d'Euler-Venn [6]

[6]Fig. 15

From the mirror stage onwards, Dr. Lacan deals with the ideals of the person, relating them to the structural schema that we read in the optical schema [É, pp. 673, 674, and 680]. This is what we do with topology, starting from the Imaginary in mathematics. But where the Imaginary retains a function.

Chapter 2. *During the second period of his teaching (1961-1971), Dr. Lacan practiced an imaginarization of the Symbolic by resorting to the theory of topological surfaces.*

It was during this period that he established a correspondence between the four objects of the drive and the four elementary topological surfaces.



Éléments remarquables de la topologie des surfaces

Fig. 16

*Freud's moment—which we must indicate each time as a double question—is formulated as follows: "Is it one? Is it two?" This moment returns to this stage in the form of the articulation of non-orientable surfaces to orientable surfaces in Lacan's construction, which he calls signifying involution, through which he summarizes Freudian repetition [*Logique du fantasme, *1967;* L'Étourdit, *pp. 26-27, 1971] (*[7].

We can place these elements of surface theory on our simplified presentation of the graph.

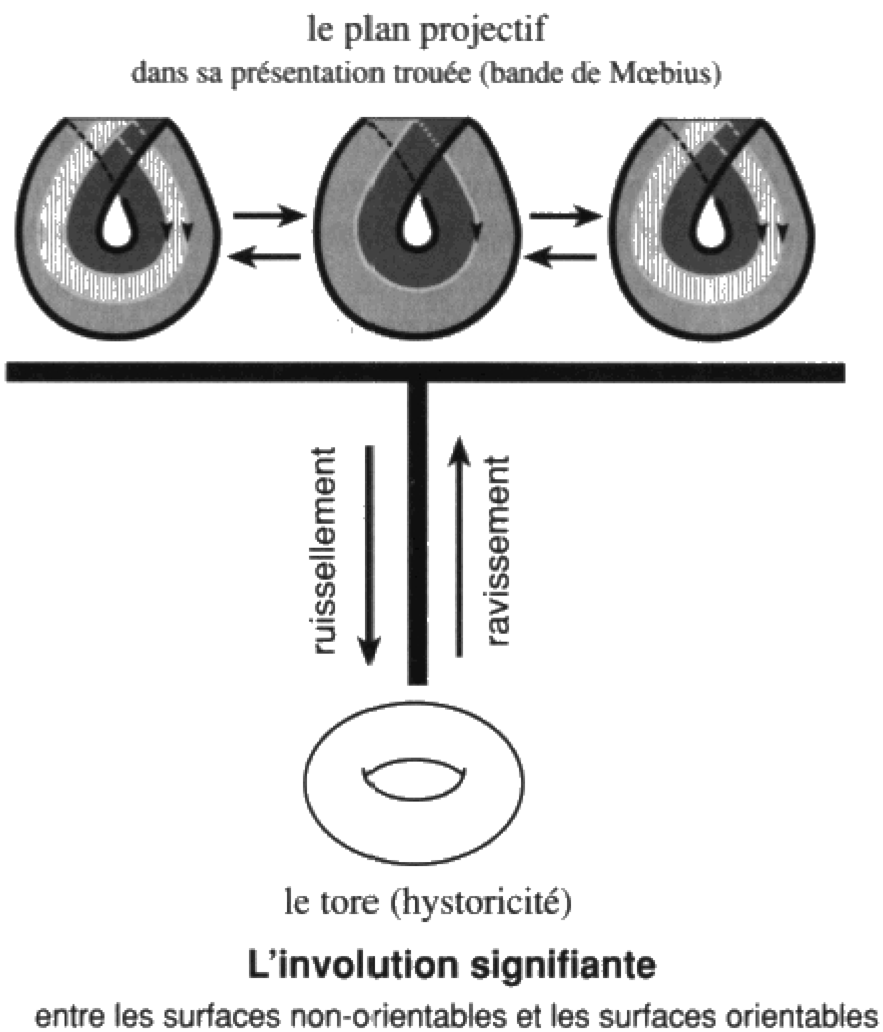


Fig. 17

Our work coordinates this structure with the logical version, replacing the Euler-Venn diagrams with the flattening of the nodes that come to us from the third stage of Lacan's teaching.

In fact, in the second period, he reconsiders step by step the entire construction of mathematical logic, respecting the three stages of propositional calculus, predicate logic with the Kanteurs, and set theory [8] .

Chapter 3. *In the third topological chapter (1972-1981) of his elaboration of analytical discourse, Dr. Lacan reformulates all of these questions in the field of existence of the knot. It is true that the previous formulations succeed in showing the framework of the structure while failing to write it down.*

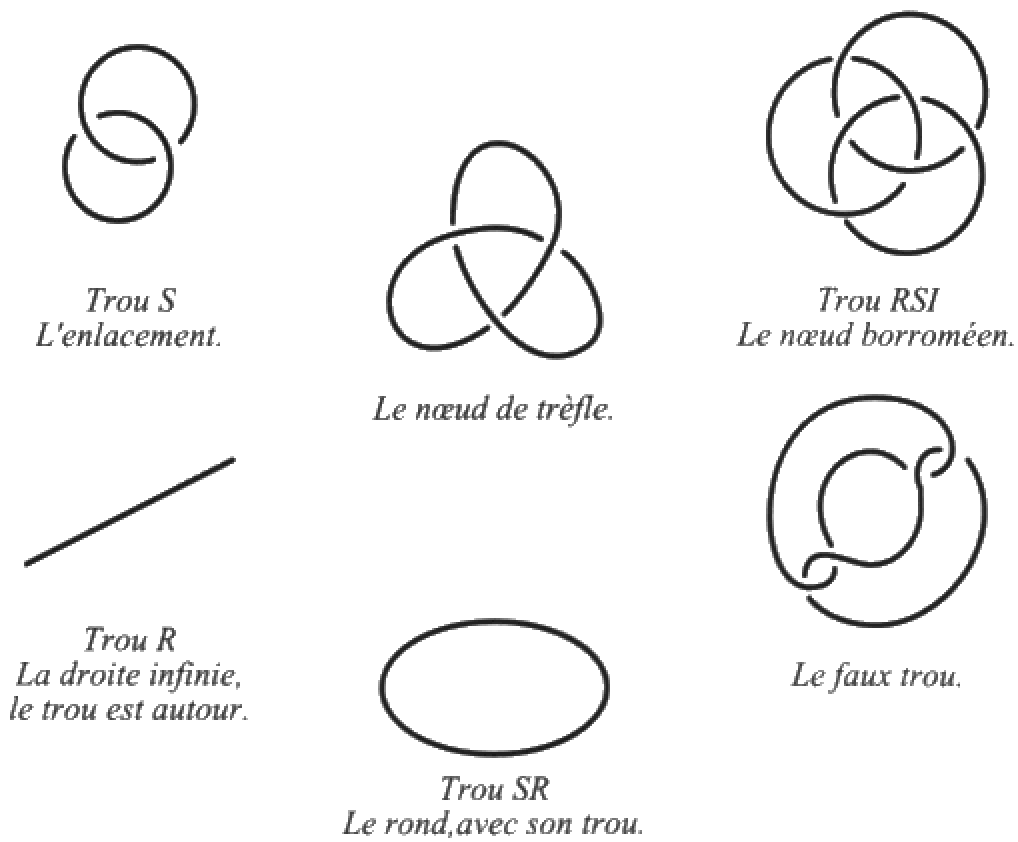
To avoid the pitfall of representation, previously avoided thanks to the projective plane, it cannot be constructed in three dimensions; the question of another writing is formulated in terms of knots.

It is, of course, a question of situating the Real, in addition to the previous instances of the Imaginary and the Symbolic, and no longer the psychic reality that is said to be implicit in the three-way knot (Seminar RSI*, 1974-1975).*

But topology is not a phantasmagoria of the Real, as some would have us believe in order to suggest that it is impossible, when in fact it is simply difficult for them. Topology does not claim, as we have already said, to take us out of fantasy but to account for it, in the manner of writing, with knots.

Admittedly, this topology starts from the Imaginary, as its detractors accuse it of doing, this time to disqualify it, Kant to this so-called Symbolic, whereas it produces an upheaval of the whole because of the Symbolic.

For our part, in order to read this state of completion, we consider in advance the edge nodes of the perforated surfaces, defined by the immersions of the surfaces; they have only an extrinsic existence.



Éléments remarquables de la topologie du nœud [9]

[9]Fig. 18

It is remarkable that the knot disappears in the intrinsic but that this presentation retains the trace of the knot (knot type). The theory of surfaces, argument of the chapter

Lacan's previous teaching is, for us, a means of investigating the space around the knot.

In our first booklets, we provide the necessary information for those who wish to verify by calculation what we do by drawing in the study of knots (starting with booklet no. 3).

Let us place these elements in our graph:

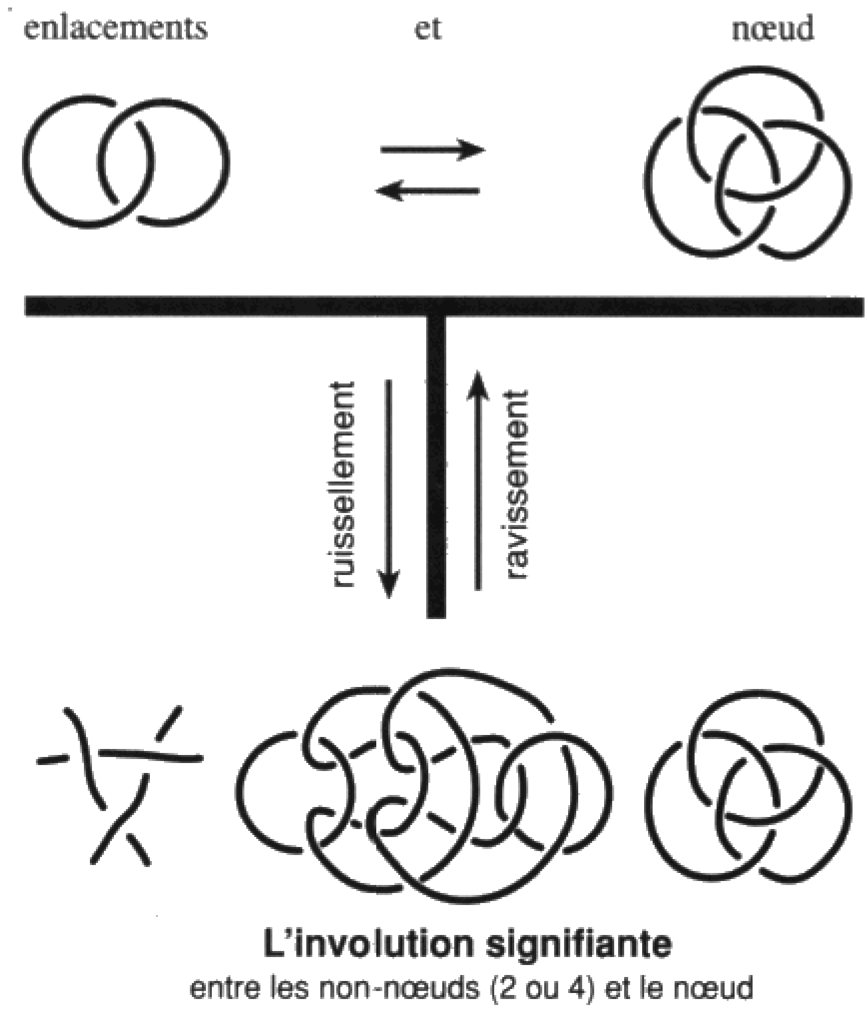


Fig. 19

For Dr. Lacan, it is a matter of starting from an impossibility encountered in the previous stages, such as a hole, which for him is the Real, in order to account for the imaginary function of the phallus that veils this hole. But in order not to fall back into this damned phallus in a philosophical manner, he must not succeed too easily. In his commentary on the structure, these categories always recur, as we have said, the Real: its impossibility becomes existence; and the Symbolic: hole (insistence), in an increasingly precise tightening, based on the imaginary consistency, of the failure to account for the sexual relationship.

He must gather his thoughts from another writing, in which this failure consists. That is to say, how this structure cannot be written [Introduction to the publication of the RSI seminar, *p. 88, and* Reading of December 17]. *A structure whose failure to be written accounts for its very impossibility. All the concepts of psychoanalysis are modified as a result, since each one, like any element of clinical material, bears the trace of this evanescent structure.*

To conclude, we have already achieved this other writing for the calculation of logical propositions. The rest of the construction is accomplished from there.

4. *From Freud to Lacan, a certain journey comes to an end. The term "completion" does not mean the cessation of practice, but its formalization from this turning point where the situation of psychoanalysis has become irreversible.*

It is now retroactively inscribed in this double turn produced by Freud's work and Lacan's commentary.

It remains to establish a series of readings that will allow us to move "towards Freud" in this return initiated by Lacan.

Cautious observers and those who have preferred to remain on the sidelines can rest assured that there is no risk of another phenomenon like Freud or Lacan. It is no longer necessary in this field. Besides, who would want to take on this now obsolete role, unless they were willing to slide down the slippery slope of mimicry, with no results to show for it? Today, the difficulties are of a different order.

5. *The stitching of the subject's place is now complete. It closes the gap that Freud and then Lacan kept open, and there is no reason to keep it open any longer. The double loop described by Freud and Lacan is over, and with the advent of Canrobert* (Introduction to Scilicet*..., p. 11), it can no longer be considered a label of use. Our results contribute to a new style of reading, whose mathematical significance goes beyond the interests of a single corporation. Our seriality is not one of filiation but of transmission and invention.*

A discrepancy between the rank of a term and its index always constitutes the major difficulty in the study of a mathematical series. The terms of a series are indexed by the set of numbers known as natural numbers. This set begins with the number 0.

Number 1 is not first; there is always an element before one. So we will give a booklet number 0 on logic, in order to situate ourselves in the sequence of this series.

There are six booklets in total:

No. 0: NONS (the topology of the subject)

No. 1: SWARM (the fundamental group of the knot)

No. 2: FABRIC (intrinsic topological surfaces)

No. 3: KNOT (a theory of the knot for psychoanalysis) No.

4: STEPS (chains of four or more circles)

No. 5: IF NARROW, LIKE THIRTEEN AND THREE... not wide

(the generalized Borromean knot).

Our booklets themselves respond to this structure of having been provoked by a rupture of pretense; they are based on a trickle of small letters in the first issues, a reading, a counting that follows the nodal structure and produces a delight oriented by names.

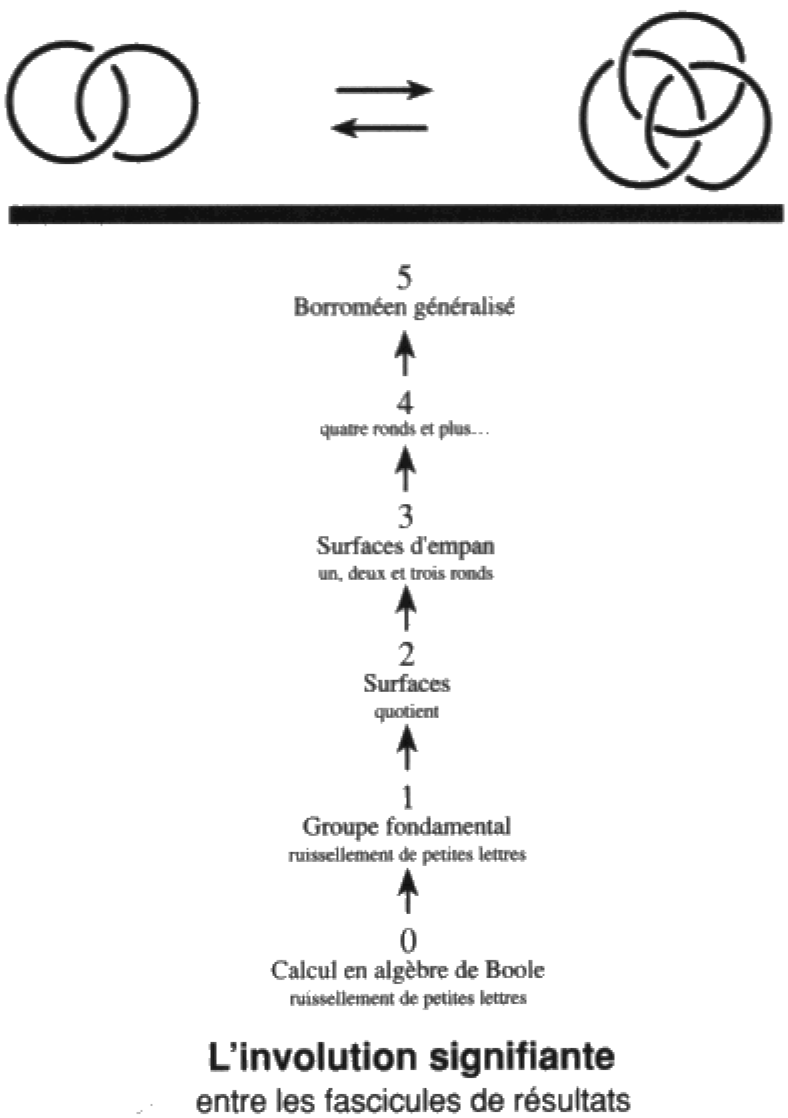


Fig. 20

6. *A major difficulty for our era can be summarized as follows: it is false that anyone, even in the Freudian field, does not want topology, and it is false that the same people accept it. This situation is described by Lacan when he compares psychoanalysis to architecture [É, p. 698]. He notes a discrepancy between a logical power that resembles discourse and the utilitarian ends that all power claims. Although useless, it is nonetheless central to our presentation of practice, as explained above. Utility is not a relevant concept; it is even dangerous in this field, where lost time produces its discovery.*

When we subvert conventional logic, those deprived of this imagination risk believing that they are dealing only with irrationality. It is as if we were removing the cork belt from a learner swimmer.

Some would like to substitute the natural for the artificial, without taking into account that there is nothing natural for a being who is subject to a double narcissism.

Thus, the early psychoanalysts were divided on these questions, and their contemporary scholars, most of whom were ignorant of the articulated logic of signifiers, of the very possibility of articulation, and even more so of the impossibilities that follow from it, fell into these traps at every turn. This was true of post-Freudian psychoanalysts, as well as neo-Lacanians.

For them, abandoning the categories derived from logic would be tantamount to losing their footing. A first step toward the truth consists in modifying them, and studying the effects of this modification itself provides the help we seek. Lacan understood this necessity by taking up, on behalf of psychoanalysis, research initiated by others (linguists, logicians, mathematicians, ethnologists). He endowed psychoanalysis with a topology of the subject that frees it from classical categories and cannot be considered an auxiliary discipline.

Those of his students who adopted the "weak" thesis of the auxiliary nature of topology did not use it for long and all admitted that they could not find any use for it in their practice or in their reports. Today, few of us use and practice topology, which we hold dear because of a stronger thesis:

It is wrong to say that "topology is psychoanalysis" and it is wrong to say that "topology is not psychoanalysis."

Since 1983, we have had a mathematical logic construct that modifies classical logic using a topological operator known as the interior operator. This is the topology of the subject. Our work consists of following the consequences of this structure when we encounter it in discourse, which is bound to happen in any context. It is this structure that we find in the approach to topological surfaces and which alone necessarily organizes the topology of the node. Indeed, there are entanglements, which are there and which are not there. It starts at four.

It is in order to achieve this degree of simple structure that we make available to readers the details at our disposal, when they are necessary.

Dr. Lacan indicated the necessary references without developing them in full, leaving it to his audience to refer to them and clarify them. It is not that he did not do so himself, as many can attest. He was content to use them in multiple and relevant ways, with enough care that by following his instructions, one could find what is only announced and used in the translation. Much work of explication in the areas addressed is yet to come, and there are already some drafts. Our series aims to be more than a draft.

7. *It is a question of using these clarifications in practice for the work of constructing the psychoanalyst, that of the object a. This task is ongoing; it is none other than that of Canrobert. A psychoanalytic clinic will emerge from it, produced by the interested parties themselves.*

We will also provide guidance for those who are looking for reasons to study this topology but have not yet committed to doing so. We will limit ourselves to the ideas that should be most easily understood, saving the new discoveries for our reading. This reading cannot be understood without the practice of topological mathematics, to which we constantly refer.

Others may extract other results from this topology. Moreover, we have the testimony of those who devote themselves to it for a time, that their work cannot fail to return to it.

We construct this topology of the subject in a way that draws on the subject, insofar as "consciousness without science is nothing more than complicity in ignorance."

The value of our series of textbooks also lies in their connection with everyday mathematics, a constraint that we have imposed on ourselves. We provide the classic algebraic components, i.e., *the elementary ones (*Bourbaki), *of the topology of the subject and those that are in progress, i.e.*, *as it is currently being developed in our field (*P. Soury), *which are necessary for reading Freud and Lacan.*

We take the development of the topology of the subject to the point where it is ready to turn into a mathematical theory; unfortunately for those who disapprove, we do not produce exclusively mathematical work. It will be up to mathematicians to reformulate it in their discourse in order to discuss it and discover its consequences in their discipline.

8. *We define topology in extension as Lietzmann speaks of Explanatory Topology [W. Lietzmann*, Anschauliche, visual*, verlag R. Oldenbourg, Munich 1955], but we give greater weight to logic since it is an eminent part of our topology of the subject, and special attention to the drawings we establish as mathematical formulas.*

We commonly encounter three different attitudes toward topology.

Firstly, not everyone may know what it is. Ignorance remains the norm, and this state of affairs is the responsibility of specialists. To see this, one need only note that, in France, through the academies, teachers use the term topology to refer to some of the activities they offer their students starting in the second year of preschool. There is therefore nothing particularly inaccessible about the approach to topology.

Then there are two related situations, one of which we will describe as studious timidity, which is necessary but insufficient on its own, and the other as the effective practice of topology, which at some point requires the former. The fact that these two attitudes occur separately is due to a particular feature of mathematical style and to the structure of discourse, which is divided into general topology and topology (algebraic, differential, semi-linear, combinatorial, geometric, etc.). There is the same articulation between these two domains, general topology and topology proper, as there is between mathematical logic and mathematics. Let us describe it.

The consideration of topological structures, in any field, is done through an investigation that consists of constructing invariant features during continuous transformations.

Thus, in the practice of topology, we encounter the need to define continuity. This definition is the subject of general topology, known as set-theoretic topology [10]*.*

Of course, topology assumes and presupposes the correct definitions of general topology, but in practice, the development continues, making room for these definitions without returning to them in each case. There is a principle of abbreviation that we can locate in the use of language, the language of categories [11]*.*

Conversely, the fact that some beginners get bogged down in general topology detracts from the effective practice of structure in favor of work of a different order. If they do not overcome the barrier that separates these two aspects of topology, they are reduced to endlessly refining definitions without ever finding convincing results, as formalism becomes increasingly cumbersome in this dead-end path. They then find themselves studying families of open sets, closed sets, neighborhoods, and filters, the interest of which few see as anything other than anecdotal, given how rich in nuances this field is. Our apprentice topologists fall into a relativism that is ill-suited to psychoanalysis.

Furthermore, it should be noted that there are mathematicians, and not the least among them, who dispute the impracticality of these general definitions for those who question the structure of a particular field, given that general topology has historically remained focused on classical functional analysis (i.e., the analysis of real functions with real variables). There is idealism and transcendence in this classic and limiting questioning among mathematicians themselves, when they become fascinated by the structure of real numbers without a real strategy, having failed to integrate the results of mathematical logic due to K. Gödel and P.J. Cohen.

It should be noted that in this brief overview of attitudes towards topology, we are obviously not even referring to the fanciful, supposedly topological activity of some. We understand that, among our contemporaries, the proponents of this fantastic topology are called "Lacanians." We do not wish to abandon Lacan's teaching to such a sad fate before claiming to be his students. We have the deepest respect for everyone's intuitions; the main thing depends on the presentation of the work.

To resolve the difficulty encountered in learning topology, without evading its profile, we would like to draw the reader's attention to a particular feature of style in mathematics.

We call this condensation, which has nothing to do with transcendence, the principle of abbreviation. This principle dictates that a work entitled topology, to take the example that concerns us, suggests and assumes, from its title or in the title of the series in which it is published, that the functions (morphisms of the category, transformations) it deals with are continuous and that the objects it studies are related to well-known topological spaces, without it being necessary to redefine them each time.

This is similar to how in mathematics we do not redefine material implication in every work. Nevertheless, some of our idealistic mathematicians complain that their colleagues pursue their work with little knowledge of logic and set theory, when naive theory seems to suffice.

It is as if they were demanding that every driver have a knowledge of mechanics in order to be allowed to drive a car. In doing so, they are simply denying the characteristic feature of the industrial method, the feature that has made it so successful and led to its development. Indeed, in the industrial empire, as in language, the user can make proper and relevant use of the object without knowing how it works. That is to say, without having participated, or even being able to participate, in the design and manufacture of the object. This naturally raises the question of maintenance, which was much better resolved in the era of Neolithic techniques.

Admittedly, in psychoanalysis, things work differently since, from the outset and throughout, the psychoanalyst, the person who consults the psychoanalyst, is held responsible for the unpredictable consequences of what they say, and it is so that they can reasonably assume this responsibility that topology is necessary in their teaching. But this fact must not, under the pretext of mathematics, go so far as to foreclose (freeze, holophrase) the style of mathematicians, as is done by some simplistic minds. This means that even in mathematics, some condensation is used.

Of course, in topology, general or renewed set-theoretic topology is assumed, but the strategy is different in the mathematical method, because it relates to the structure of language, that is, to a practice of the absence of metalanguage. It is this structure that is sealed in the industrial method.

We neither mock nor disregard these premises, and we encourage those who still parrot them. For we wish to point out to them that they are quite right not to understand the practical use of our topology, nor Lacan's practice when he resorts to topology, since they themselves approach it in an inappropriate manner. We wish to show them why.

That is why we propose to take things from both ends at the same time, each in its proper place.

In essays that immediately put topology into practice through varieties.

In a return to set-theoretic topology, not in general but in the specific and main question of the structures of propositional, predicative, and set-theoretic logic. This is in order to deal with each of these three chapters of mathematical logic in a topology in the general sense of the term. We thus obtain the premises of the topology of the subject by modifying negation in the manner of modal logic into a topology.

Our reference work in mathematics is that of E.E. Moïse [28] for the practice of topology. Some may find that there are too many results in this collection. This is because there is a resistance that detracts from any certainty. The results are distorted simply because they are recorded by the interested parties themselves. There are two ways to break down this obstruction.

One consists of establishing a protocol for experimentation and recording that leaves room for operation. The latter is increasingly supported by those who have already produced results.

The other can be summed up as communicating results for discussion by anyone, even those outside the field concerned.

These two solutions are only opposed by the ignorance of those who support the resistance that sustains the malaise in civilization, wonders and surprises that are constantly renewed in each case of transfer. For the rest, they can be undertaken jointly.

Plaisance, July 1996.

Chapter One

Scientific literature on knot problems

In the following pages, I propose to show that there is a topological technique that allows knots to be read: if this technique is applied, every knot appears as a topological process that has a regularity and can be perfectly inserted into the sequence of logical activities related to ordinary language. I also want to try to explain the processes that give the knot its strange, unrecognizable appearance, and draw a conclusion about the nature of the topological tensions whose fusion or collision produces the knot. I will limit my presentation to this point: it will have reached the point where the problem of the knot leads to broader problems, the solution of which requires the use of other materials.

1. **Initial approaches**

I will begin with a historical overview, as I will have little opportunity to return to it in the main body of the work. I will take this opportunity to comment on the different styles of investigation and indicate the different levels of difficulty in approaching the knot.

a1 - ***J. B. Listing***

The scientific conception of the node did not develop significantly until very recently, as the first published study on this subject dates back to 1847. This was J.B. Listing's habilitation thesis [25], which he defended in Göttingen.

It is worth noting that he was also the first person in scientific discourse to use the term topology. The first occurrence of this term is found in his thesis, which covers a series of structural problems relating to the orientation of the subject in space.

Moving from the mirror symmetry of a game die to the distinction between left and right spirals found in botany, he arrives at the presentation of flattened knots, which he introduces through cylindrical spirals, today's toric knots. We note that he already distinguishes between two types of areas determined by the flattening of a knot. He notes them with two different letters, l and d, and already seeks to form polynomials with these variables.

a2 - ***The precursors***

We can single out Descartes, who mentions the art of lace-makers as worthy of interest in his *Rules for the Direction of the Mind* [9]. Then also, Taking it one step further, Gauss drew "a collection of knots" in 1794. He also left two studies among his manuscripts, in addition to his 1833 text in his notebooks devoted to electrodynamic circuits [13]. He planned to write a book on the subject, as evidenced by a letter from Möbius to Gauss in 1847 [34].

In fact, it was not until 1877 with P.G. Tait [37] and 1885 with C.N. Little [26. a, b and c] that a truly developed presentation of knots began.

As chemists, they were both involved in this investigation by Lord Kelvin and produced the first tables of knots. We should also mention the contribution of
T.P. Kirkman [20], who accompanied their development with his comments.

Since Tait's work, the notion of crossing sign has been defined in a presentation of a knot or oriented chain.

For translations and planar rotations, there is only one type of crossing in a non-oriented knot or chain presentation:

Fig. 1

When the presentation is oriented, there are two different types of crossings:

Fig. 2

They are indexed by two crossing signs, denoted respectively by +1 and –1.

We leave it to the reader to verify for planar transformations the identity of any non-oriented crossing and the non-identity of these two types of oriented crossings.

Note that we are dealing here with presentations of knots and chains laid flat on a table, for example. These projections must meet the simple condition of not presenting multiple points other than those where at most two string elements intersect, and never more. The flattened presentations determine string arcs, crossings where the passages above and below are noted, and areas delimited by the portions of arcs and crossings([1])

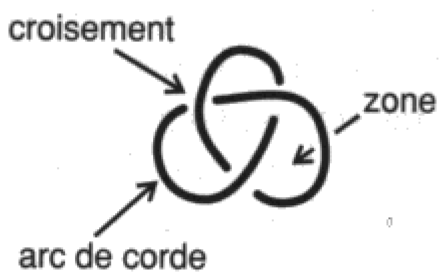


Fig. 3

a3 - ***Two other tables of knots***

For those seeking a material basis for their work, apart from the string constructions they may have made, we indicate here where a multiplicity of knots and chains can be found that offer themselves as a place of exploration. We will subsequently propose a renewed formulation of this field. Knots and chains are presented in mathematical literature in the form of a table enumeration.

Since the knot tables of P.G. Tait and C.N. Little, other tables have been established and successively expanded.

We are familiar with K. Reidemeister's table in his *Knötentheorie* published in 1932 [32] and the more recent one by D. Rolfsen in 1976 [33], which offers a drawn and expanded version of J.H. Conway's 1970 compilation [8], which we will discuss again in the history of knot polynomials.

There are other such tables in the available literature, for example in L.H. Kauffman [18. b].

2. **Tightening the knot at the turn of the century**

a1 - ***Entanglements and knots***

If we continue in chronological order, we must return after the English chemists to H. Brunn in 1892 [7. b] for the end of the 19th century. We note that in his study of the chain, he marvels at the presence

Borromean rings, which hold together and do not present any intertwining or consistent sub-chains. This study is an excellent reference for beginners who want to follow Lacan's attempt to isolate the knot through chains and knots.

This study is still conducted in the formal style of previous essays, before Poincaré came along and transformed *the* old *Analysis-situs*, named thus by Leibniz [31], into algebraic topology. Attention is then focused on the space around the knot, using more sophisticated algebraic techniques with the use of homotopy groups.

However, we will continue to pay close attention to the early period of topology in our work, as it is better to avoid uncontrolled use of group structure, the importance of which has been well known since F. Klein, and knowing that even these works, however archaic they may be, can be part of an algebraic structure formulated in the language of categories[2].

Let us now turn to works that are closer in style to today's mathematical publications.

a2 - ***First theorems***

M. Dehn in 1910 [10] and J.W. Alexander in 1927 [3. a] provided the first truly mathematical results at the beginning of the century, presenting them as theorems that required genuine proofs.

We owe Mr. Dehn the lemma that bears his name and which gave mathematicians a curious surprise. We will recount this anecdote shortly. But let us note that among other results, he formulated the method that we found by another route during our conversations with P. Soury, in the calculation of the fundamental group of the knot([3]) We will return to this at the end of this chapter.

We owe to J.W. Alexander [3. a] the construction of the first knot polynomial, whose search as a knot invariant is very characteristic of this moment in topology thanks to Poincaré.

We will comment further on this use of algebraic group structure and the use of polynomials in their current developments.

a3 - ***The adventure of Dehn's lemma***

Dehn's lemma is characteristic of the difficulty of our discipline, since in 1910 he believed he had proven the lemma that formulates the criterion for a knot to be trivial. However, in 1929, H. Kneser [21] showed that Dehn's proof was insufficient, and it was not until 1955 that

C.D. Papakyriakopoulos [30] finally provided an acceptable proof.

This would be nothing more than an anecdote were it not for the reputation of topology, and more specifically knot theory, as a risky discipline, a reputation that fortunately seems to be fading today.

3. **A knot theory**

We can now define what knot theory is, in the most current sense of the term. To do so, we will follow L.H. Kauffman [18. b], who himself adopts the spirit of the rapid overview of knot theory provided by R.H. Fox [12].

A knot theory is a problem of situation or placement. Given a space and an object (round), the problem is to know how this object can be arranged in this space, i.e., how can it be placed there or how can it be located there?

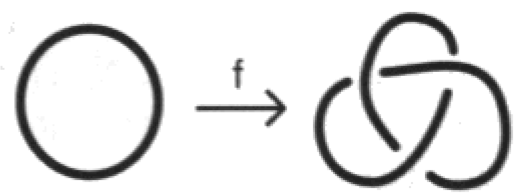


Fig. 4

In knot theory, the object is a simple circle placed in three-dimensional Euclidean space. The way in which the object is located in space usually corresponds to the notion of embedding, and we will give a definition of this notion later on, which the reader can intuitively grasp for now.

Classical knot theory attempts to classify these different circle embeddings. We therefore need a criterion of identity and difference in order to be able to say what is the same and what is not the same. This concept corresponds to an equivalence relation.

There are different approaches to such a theory.

We will adopt a formal knot theory in the sense of L.H. Kauffman [18. a], which involves implementing a combinatorial calculation of features (invariants) characterizing the presentations of knots thus identified by a relationship of equivalence.

For us, this will be an equivalence by elementary movements that change the appearance of the same knot or chain when read. These elementary movements have been known since K. Reidemeister [32]. A composition of these movements constitutes a change of presentation, which leaves the knot or chain identical.

The theory begins when we have the correct definition of the movements.

Another approach consists of studying the space around the knot (the knot variety) using algebraic topology techniques, or even differential geometry.

Before undertaking our study in the formal style we have adopted, let us explain how tempting this other approach can be, by commenting on the three steps required by the definition of knot theory we have just given.

4. **Commentary and definition of the elements necessary for a knot theory**

a1 - ***Situation or placement***

Let's take an example from our previous work on topological surfaces.

The projective plane[4] is a two-dimensional object; it cannot be placed as such in three-dimensional space[5]. There is an obstruction here, and this question is a good introduction to the problem of knots. It sheds light on the transition from the intrinsic classification of two-dimensional manifolds, surfaces, to the classification of knots, one-dimensional manifolds located in three dimensions.

Indeed, the reader may wonder why topology does not continue the classification of intrinsic three-dimensional, four-dimensional, and so on, manifolds.

It begins to dawn on the reader that there are several competing ways of placing an object in space, and this problem will increasingly take center stage instead of simply enumerating the different intrinsic objects of successive dimensions. The main purpose of this chapter will now be to explain this fact.

However, a more intuitive learning of the practice that does not use mathematics in the field of existence of the node can dispense with the definitions we are now giving. Readers who prefer this intuitive practice will return to our text (see § 7. *Movements* below) when we define the equivalence relation. We will define this equivalence in the space of circles using Reidemeister movements([6].

a2 - ***Embedding and immersion***

In this section, we provide more mathematical expressions that formulate this situation, for readers who are not satisfied with an intuitive approach.

We must therefore specify the order of difficulty of the methods used in topology. Topology studies are usually divided into three different categories:

— topological spaces (*TOP*),

— piecewise semi-linear spaces (*PL*),

— differential geometry (*DIFF*),

which are ordered here according to the greater or lesser fineness of the techniques used to divide up space.

The definitions we now give belong to differential geometry (*DIFF*), the most refined category. However, the study of knots more readily requires the category (*PL*) of piecewise semi-linear spaces.

An embedding of an object O (differential manifold) of dimension m in a space E (differential manifold) of dimension n, with n greater than m (n > m), is a mapping f: O → E.

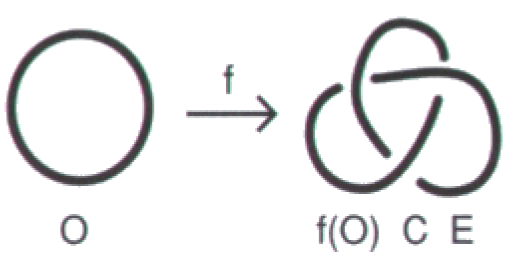


Fig. 5

This map f is injective and infinitely differentiable C, whose tangent map is injective everywhere.

An immersion is a non-injective embedding. The difference between these two types of embedding is that the threads may or may not cross themselves.

a3 - ***Identity and difference***

Let us now give the definition of equivalences, still in the context of differential geometry.

The equivalence relation corresponding to embedding is isotopy. Two objects O and O' are isotopic by a differential isotopy.

The equivalence relation corresponding to immersions is regular homotopy. This relation allows us to move from an object O to an object O' by crossing the string elements exclusively at the points of intersection.

A crossing is said to be clean when the two overlapping elements are from the same loop of string. In a clean knot, made from a single loop, there are only clean crossings. There is therefore only one clean knot relative to homotopy equivalence. For this relation, all knots are equivalent to the trivial knot, the simple loop.

a4 - ***Methodological considerations***

We have said that we are using differential geometry (diff) to give these definitions. But our aim is not to draw readers who do not do mathematics into this field. On the contrary, we want to give them the means to experience the knot by staying close to the realizations obtained with string or by practicing well-constructed drawings of these configurations. We invite them to experience topology, not to do mathematics, provided they remain logicians.

This position is consistent with Freudian theory if we remember the lesson given by Lacan in his seminar on Edgar Allan Poe's short story "The Purloined Letter" [2. É].

In this story, the very precise division of space does not allow the letter to be found. On the other hand, Dupin, by putting on green glasses and becoming more directly involved in the territory, by engaging with it, finds the only way to isolate the letter in the space of the minister's office.

It is a concept that no longer considers space *parte extra parte*, as was the case with Greek geometry. Thus, G. Ganguilhem, in a remarkable essay [50], finds that the circumstances of Antiquity may mitigate Aristotle's error; he knew no other mathematics and effectively declared, for a long time, that it was impossible to use this discipline in the study of organisms. But this great professor of the philosophy of science finds it unforgivable on Bergson's part to have peddled the same judgment, when he himself had been trained in mathematics at a time when topology already existed.

In our case, we are certainly not proposing to use it for the study of organic life in biology, but rather for the study of language and political ethics in psychoanalysis. It may therefore seem ironic and regrettable that the same judgment persists in this discipline, where the organic relationships of discourse, revealed by its practice, reveal such a structure.

As a result, our method of reading nodes, presented in the following chapter, is based on a simple but effective topological structure, refined by Lacan in the previous chapters of his seminar, in terms of graphs and then surfaces, for those who have a taste for simplicity.

You can therefore follow us in our exploration without resorting to particularly sophisticated techniques, provided you stick to the robust categories we propose and apply the elementary logic that can be found there.

Of course, we are not unaware of the results obtained in knot theory since the beginning of the century, which would undoubtedly not have been demonstrated without the techniques that produced them. We always try to present them to the reader through topology in extension, in a simple and accessible way, as we propose to do in the construction sketch that follows.

This construction shows how the knot can disappear under certain conditions, and warns us against the fact that the mode of investigation should not unexpectedly trivialize the structure[7].

5. **Let us clarify the main function of the node**

a1 - ***Antoine's construction***

We must mention here L. Antoine's memoir "Sur l'homéomorphie de deux figures et de leurs voisinages" [5] and [28], published in 1921, to illustrate the importance of knot theory in mathematics and its logical resonance, which explains why we would resort to its topology in the Freudian field.

L. Antoine studies the case of two curves C_1 and C_2, each located in space E, which we abbreviate as follows:

$$f_1 : O \to f_1(O) = C_1 \subset E$$

$$f_2 : O \to f_2(O) = C_2 \subset E$$

to indicate that these two curves are nothing more than two different immersions of the same object, the circle O. And he examines the extent to which the homeomorphism f, i.e., the topological equivalence that always exists between these two curves, can be extended to their neighborhood in space, i.e., a part of space that is more or less close to the space in which they are immersed; This can be represented graphically as follows:

$$\begin{array}{cccccccc} O & \to & C_1 & \subset & V_1 & \subset & E \\ i\downarrow & & \downarrow f & & \downarrow \varphi & & \downarrow \Phi \\ O & \to & C_2 & \subset & V_2 & \subset & E \end{array}$$

The notion of more or less close extension is conveyed by the position of φ between f and Φ.

The existence of the extension ensures that there is nothing more singular between C_1 and C_2 than between V_1 and V_2. The impossibility of extension, on the contrary, indicates that there is something between C1 and C2 that does not go beyond a certain neighborhood; we will then say that there is a knot between C1 and C2 in space E.

There is indeed a homeomorphism f between C_1 and C_2, since they are two embeddings of the same object O. This means that a circle embedded in different ways remains, intrinsically, the same. This notion is itself crucial to our problem.

Three cases are then possible:

— 1st case: where the homeomorphism of the curves extends to the entire space from f to Φ.

— Case 2: where the homeomorphism extends to a neighborhood that exceeds the curves without encompassing the entire space from f to φ but not to Φ.

— Case 3: the homeomorphism of the curves cannot be extended to any region outside the curves; f does not extend to φ or Φ.

To say that f extends to φ or Φ is to say, conversely, that there exist homeomorphisms φ or Φ of which f is the restriction.

1 In the case of plane curves, we are always in the first case, i.e., there are no knots in two dimensions. The structure of the curve completely determines the structure of the space that contains it.

We say that in this situation the knot is trivial. There is no knot.

2 - On the other hand, for curves in three-dimensional Euclidean space, all three cases can occur.

In this case, there will be a knot.

This knot can be erased.

Particularly when our curves are embeddable in a toric neighborhood, itself embedded in three-dimensional Euclidean space, f can extend to this toric neighborhood itself, without extending to the entire space.

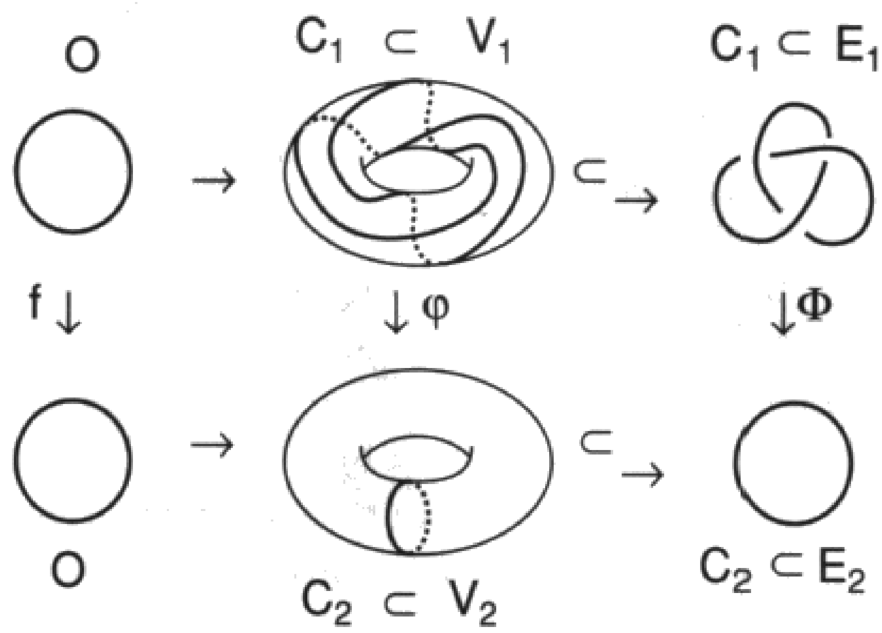


Fig. 6

The homeomorphism f from C_1 to C_2 extends to a homeomorphism φ between two toric neighborhoods, as we will show, but does not extend to the entire space, i.e., to Φ.

Since the homeomorphism f does not extend to the entire space, i.e., Φ, there is a knot. We will not show this.

But the extension of f to φ, a homeomorphism of the torus of which f is the restriction, shows that the knot disappears locally in a space of lower dimension. We will say that it trivializes or disappears in this case.

Demonstration of the homeomorphism φ, extension to the toric neighborhood of the homeomorphism f

Let us show the identity, for the topology, of the two situations where the circle is differently embedded in the surface of the torus.

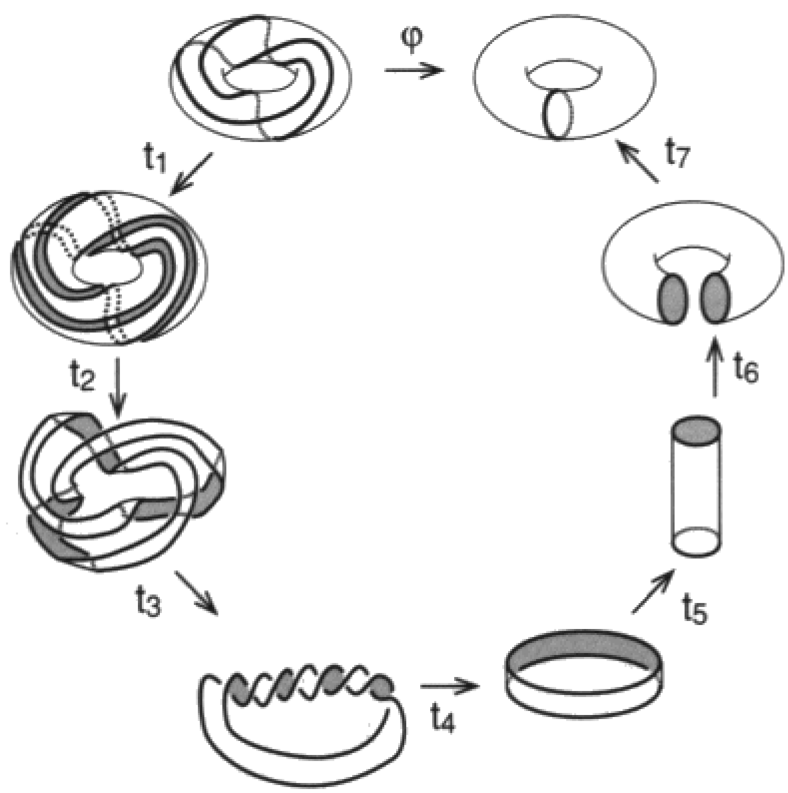


Fig. 7

This transformation applies intrinsically to the surface of the torus[8].

Let us provide a frame-by-frame commentary on this argument in favor of topological equivalence relative to space.

t_1 - We cut the torus along the trefoil knot. There is only extrinsic discontinuity.

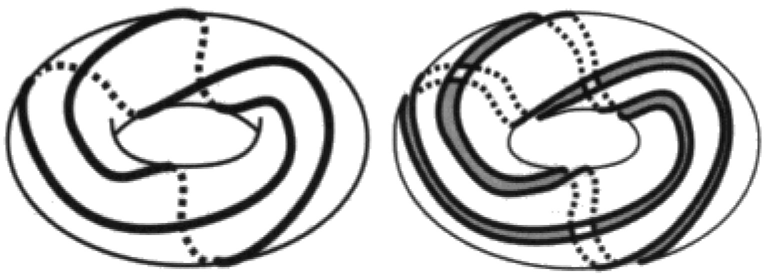


Fig. 71

t_2 - We continuously reduce the object to show that it is indeed a knotted and twisted ribbon.

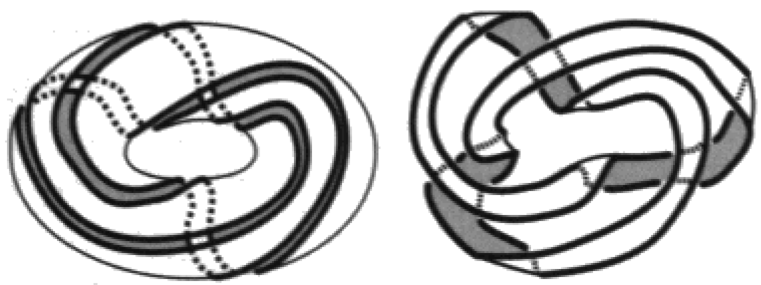


Fig. 72

t_3 - We undo the knot in the ribbon. There is only extrinsic discontinuity.

Fig. 73

t4 - We reduce the half-twists in pairs to obtain a belt without twists, which still preserves the intrinsic continuity.

Fig. 74

t5 - We deform the belt into a cylinder:

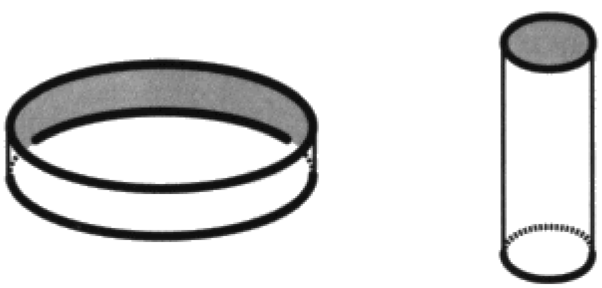


Fig. 75

t6 - We are only bending the cylinder:

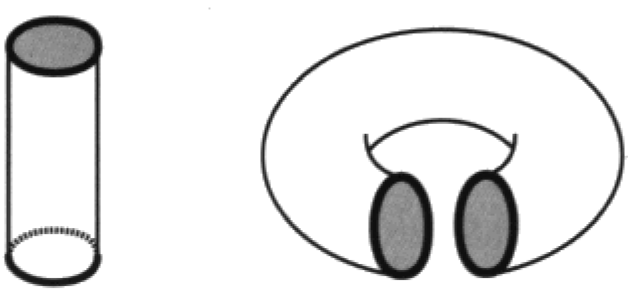


Fig. 76

t7 - We close the torus. This extrinsic counter-continuity still does not contradict intrinsic continuity.

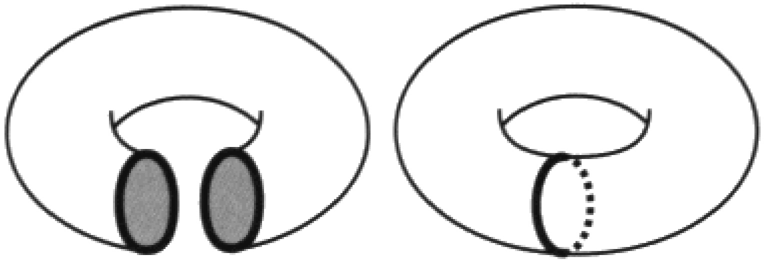


Fig. 77

It is important to understand the equivalence of the knotted and twisted ribbon with the belt, which explains the homeomorphism j.

There are, of course, different and sometimes more subtle criteria that may require further clarification in this game of topological equivalences, but the register we have chosen here, while still necessary, is sufficient to show what the knot between different extrinsic positions consists of and how it disappears in certain intrinsic situations.

3 - L. Antoine effectively constructs a curved arc on a torus whose correspondence with a straight line segment does not extend to any neighborhood. We therefore know that there are cases where the obstruction is extreme, and the knot is at its maximum. We are not concerned with this strictly mathematical situation. This means that Antoine's construction, which is much more complicated, is of less interest to us, because in this construction the knot does not disappear.

a2 - ***Let's return to the distinction between intrinsic and extrinsic***

We already discussed this distinction at length in our previous work[9] with reference to A. Lautman [22], in order to present the structure of narcissism between the space of which we are the subject, intrinsic, like our own body, the torus of the previous demonstration, and the three-dimensional Euclidean space in which our drawn constructions are immersed, where the torus is taken as an object, extrinsic.

Narcissism, as introduced by Freud [1. c], consists of a subject taking their own body as an object, i.e., moving from an intrinsic position to an extrinsic position. This distinction extends Lacan's optical schemas, which are a generalization of the mirror stage, where an instrument as artificial as the mirror is no longer necessary for the gaze. This formulation opens up, among other things, our position with regard to the voice in the invoking drive. This generalization involves formulating the structure that is at the root of narcissism, namely the structure of language, using different topologies.

This question is also very commonplace and rooted in history; it is still of particular interest today. If we focus our attention on the space we believe ourselves to be the subject of, three-dimensional Euclidean space, it is easy to extend it to give it an astronomical size. It then seems necessary, for many contemporary subjects, to situate it in another space from which it can be taken as an object, and thereby find themselves involved in a headlong rush, from contents to containers, in a series of infinite nesting. However, this extrinsic observation no longer seems necessary to astrophysicists since the debates on non-Euclidean geometries and the work of H. Poincaré and H. Weyl. The question of a knight who would plant his sword at the edge of the universe([10])no longer arises in a space without boundaries, considered from an intrinsic point of view. The difference between an intrinsic analysis and an extrinsic analysis is not necessarily obvious.

However, we note in the constructions studied by Antoine that, since the two curves C_1 and C_2 are homeomorphic, their intrinsic invariants determine, by virtue of Alexander's theorem [3. b], identical extrinsic invariants for their respective complementary spaces $E - C_1$ and $E - C_2$. But the identity of these invariants is not sufficient for the spaces $E - C1$ and $E - C2$ to be homeomorphic. They are not uniquely determined by the curves that can be inserted into them, and their extrinsic differences are irreducible. Here there is a kind of irreducible relationship between an object language and a metalanguage that disappears in the object.

If we note that the three-dimensional Euclidean space E is identical to itself and that it is the introduction of homeomorphic, i.e., identical, curves C_1 and C_2 in each of the two cases that makes it profoundly dissimilar, we realize how much the intrinsic invariants of curves reveal about the relationship between the curve and space in non-trivial cases.

a3 - ***Myth or structure of the paternal function***

The situational properties, which in the two-dimensional case of our torus example can be reduced to intrinsic properties, cease to be so in the three-dimensional case. In this register of reality, the distinction between aesthetics and analytics remains([11]) and this distinction gives rise to a variation in the case of our demonstration, because conversely we can say that extrinsic characteristics fade or disappear in intrinsic analysis. This is what is new here.

This pulsation is at the heart of our argument, as it is of the structure of language, and establishes the relevance of our use of node topology.

There is therefore more if the structure of language, as opposed to the code of communication, is defined by the necessity of metalanguage, an extrinsic position, but one that turns out not to be a departure from language, as R. Jakobson explains [51], i.e., an absence of metalanguage, a necessary return to the intrinsic.

This is the moment when, in Lacan's optical diagrams, the mirror tilts.

This erasure of the knot, its disappearance through trivialization in the intrinsic, requires a subject of reading in this topology, and the introduction to this pulsation constitutes the so-called paternal function in culture. It introduces the subject to the assumption of the structure of the signifier through the mediation of a metaphor that best illustrates it, as the use of personal pronouns [45], deixis, or performatives [41] are the best examples of this in grammar. It covers an irreducible original condensation.

In Freudian myth, it has its correlate in the death of the father, establishing the symbolic father, for reasons of structure rather than imaginary rivalry, which remains an effect and not a cause of this function, the extrinsic position immediately reestablishing itself, the erasure lasted only a moment, before the knot was restored in the institution of the ever-evanescent superego.

Our readers will thus understand how Freud made only a slight error in proposing a mythological origin that we no longer need, since, according to Lacan, the response to unconscious guilt is now only a matter of the subject assuming responsibility for it.

The lack of responsibility that characterizes madness is neurotic, if we define neurosis as a disease of the superego that consists of the subject harming themselves. Or we could say that neurosis is a form of madness whose entry into psychoanalysis marks the analyzing subject's decision to break with it. This is necessary because madness is opposed, even antithetical, to the decision to undertake the study, by putting oneself to the test through a questioning, such as analysis, of the mental causality to which the psychoanalytic object belongs.

There is nothing more futile and perpetual than masking this response to the impossibility of sexual intercourse with the war between the sexes, as proven by common experience, which thus echoes the myth of the Danaids.

The optical model borrowed by Freud to account for the spatially unlocatable place of the unconscious, with its pulsating structure of appearance and disappearance, is rigorously argued when we replace it with the topological construction that deals with forgetting, going from the dream to the time of analysis, a forgetting intrinsic to the act itself in its difficulty for the subject.

a4 - ***Knots***

But such erasure also occurs in higher dimensions.

For the immersions of an object Sm, a sphere of dimension m, in a space Rn, there are no knotted m-spheres if the dimension n of the space is greater than half of three times the dimension m of the object increased by one:

$n > 3/2\ (m + 1)$

Under these conditions, all embeddings of Sm in Rn are regularly isotopic [11].

Otherwise, if $n \leq 3/2\ (m + 1)$, the theory of knotted spheres has barely begun.

This formula justifies our placing ourselves in lower dimensions, as suggested by the title of Moïse's book [28], since it is mainly a difference in dimension.

Classical knot theory is a special case of the classification of immersions of $M = S_1$ in $N = R^3$; it is far from complete.

Indeed, $3/2\ (1 + 1) = 3$, and according to the previous formula for there to be a knot, in the case of a circle where m = 1, the immersed object is the circle S_1, so the dimension of space n must be less than or equal to three.

The previous theory has analogues in the PL and TOP categories, but there are some differences.

Conversely, for the case $M = S_{n-1}$, $N = R^{(n)}$, a problem dating back to Jordan and Schœnflies is whether an embedding of S_{n-1} in R_n extends, as in the problem studied by Antoine, to an embedding of the ball D_n in $R^{(n)}$, which is equivalent to the absence of a knot. This is true in the *DIFF* category.

In the *TOP* category, this extension is found for n = 2, but there is a counterexample for n = 3, due to Alexander, and well known as the horned sphere.

With regard to the erasure of the knot that we are structuring, let us retain the result that ensures the existence of an embedding extension, i.e., the absence of a knot, in *TOP* when n = 2.

Indeed, if n = 2, then n – 1 = 1. Thus, there are no knots made of 1-dimensional circles in 2-dimensional surfaces, as we have already pointed out.

Knot theory studies, by isotopy, the embeddings of 1-dimensional circles in a 3-dimensional space.

The difference between the dimensions of the object and the space in which it is embedded is more important in these questions than the dimensions of the objects themselves. It is time for the reader to realize that a space can always be taken as an object for a higher-dimensional space, just as it can be taken in itself and have its boundary canceled by compactification in an appropriate topology, which can avoid the idealism of higher astrophysical dimensions, since lower dimensions are sufficient to deal with this difference.

We will call the difference between the dimension of an object and the dimension of the space in which it is embedded the co-dimension of the embedding. Knot theory is a theory of co-dimension, which explains mathematicians' interest in the space around the knot (knot manifold) because of this subtraction.

This subtraction of space, from which we subtract the knot in which it is immersed, highlights the importance of this complement to the knot, known as the knot manifold, which is revealed by the fact that the knot is rather incomplete from the perspective of classification. This incompleteness justifies our use of the term "supplementarity" instead.

We recognize here a situation specified by Lacan on the occasion of speech.

When Lacan discusses desire, emphasizing that it should not be confused with demand, he is making a distinction. Asking for something should not be confused with the need that motivates that request. Desire is obtained precisely when the need is subtracted from the demand. Desire is to be found in this difference, just as the knot exists in the subtraction of the object, which consists of a circle in the space in which it is immersed. We will talk about the space of demand and the consistency of need. It is desire that gives the demand for love its unconditional character, which cannot in any way be reduced to need. The latter, because it is caught up in this difference, becomes the drive. We will start from this insight to develop the notion of the existence of the knot of desire, provided that we specify that this knot does not exist in the consistency but exists in the hole produced by the circle. It is the hole that exists in consistency.

And it is indeed the function of the father, emphasized by the discourse of analysis, to learn to detach the absolute condition of desire—absolute meaning separate—from the unconditional demand for love. Erasure tempers but weaves anxiety into this passage, which is collected by the object that is said to be the condition or, better yet, the cause of desire.

Our aim is to identify this object in the field of existence of the knot.

6. **Algebraic and graphical results in the space around the knot**

a1 - ***Groups***

Before L. Antoine proposed his construction in 1921, we had already encountered the fundamental group in 1910 in the work of M. Dehn. The first volume[12]of our series of results booklets is mainly devoted to it. In it, we adopted a method of calculation based on the drawing, very similar to the construction proposed at the time by Dehn. This calculation establishes the necessary link between our formal style of investigation through the topology of the knot and the mathematical techniques we discuss in this chapter.

Another way of calculating this group, for each node and each chain, is more widely used today because it is more suited to mechanization. This method was developed by Wirttinger, and is well known by that name. However, we prefer Dehn's method, as it allows us to work directly on the figures and because our process forms words in the areas, which we then use([13]).

We refer to *Essaim* for the question concerning algebraic topology, of a correspondence between topological objects, such as knots and chains, and algebraic groups. At the time of writing, it had not been proven that the correspondence between prime knots and groups was bijective. This situation justified A. Gramain [15. a] giving only the first part of his report to the Bourbaki group on classical knot theory in 1976.

Since 1989, this has been done for proper knots.

We already knew that two proper knots (or two chains) that are equivalent by isotopy have identical fundamental groups, but the converse had not been proven.

The proof of the theorem that ensures the homeomorphism of the complements of two proper knots whose groups are isomorphic was established by
W. Whitten [39] in 1987.

C. Mac A. Gordon and J. Luecke [27] proved, in 1989, the isotopy of two proper knots whose complements are isomorphic.

Thus, today we know that two prime proper knots with isomorphic fundamental groups are isotopic [15. b].

There is no equivalent theorem for chains made up of several circles, and this state of affairs explains why some mathematicians today are so interested in the polynomials corresponding to knots and chains.

Our own results fall at this juncture, between proper knots and chains made up of several circles. We will show below how a correlation is established between proper knots and certain chains.

We must also take into account, with regard to the fundamental group of knots and chains, the two articles by J. Milnor [29. a and b], in which P. Soury [35] attempted to draw a link between this great mathematician, who was passionate about topology, and J. Lacan's work on knots. He highlighted the correlation between Borromean chains and centralizer series in group algebra.

a2 - ***Seifert surfaces***

There is also a process for determining graphically, directly from the drawing of a knot or chain, a surface that can always be oriented. This surface, known as a Seifert surface [36], is constructed using defined cycles, called Seifert circles, on the oriented representation of the knot or chain.

Let's illustrate this method with an example that is easy to generalize. If we start with an oriented presentation of the knot or link:

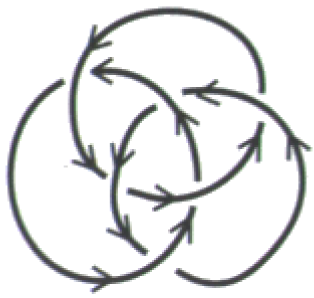


Fig. 8

We remove the crossings from this figure in order to retain only the oriented arcs.

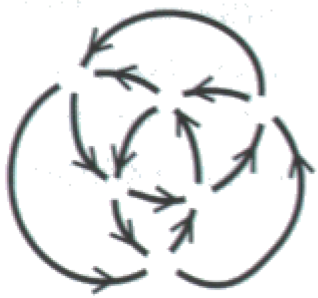


Fig. 9

We must then find a way to join these oriented arcs in order to form Seifert circles, which are disjoint oriented cycles that respect the orientation of the given arcs.

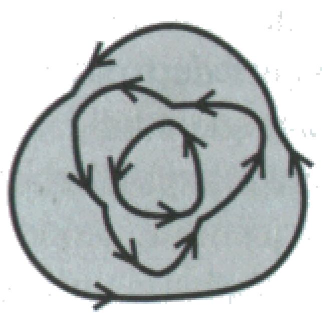


Fig. 10

At this point, we can label the circles with different orientations using the numbers +1 and -1. We will call the sum of these values the Seifert number. Here, for example, $\delta = +3$.

By restoring the initial intersections in this figure, we obtain a montage of superimposed discs, joined by twisted straps, the Seifert surface in a presentation of stacked layers, seen from above.

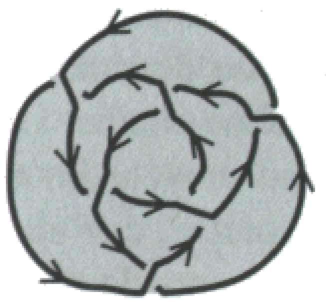


Fig. 11

This surface, which is always orientable in Seifert's case, allows the knot or chain as its edge. We can always transform it into an orientable empan surface[14] by changing the presentation appropriately.

The span surfaces that we define in the following chapter, and which we already discussed in the previous work, are a generalization of these Seifert surfaces. We will refer to them as orientable span surfaces. They will play a prominent role for us. In the case of alternating chains and knots, they will specify what we thus isolate as non-knots.

7. **Movements**

We can finally approach the beginning of the theory by giving the elementary movements due to Reidemeister [8], whose composition ensures equivalence between two presentations of flattened knots or chains.

There are three Reidemeister moves. The first makes or undoes a loop, the second makes a non-alternated stitch, and the third move modifies a non-alternated triskel.

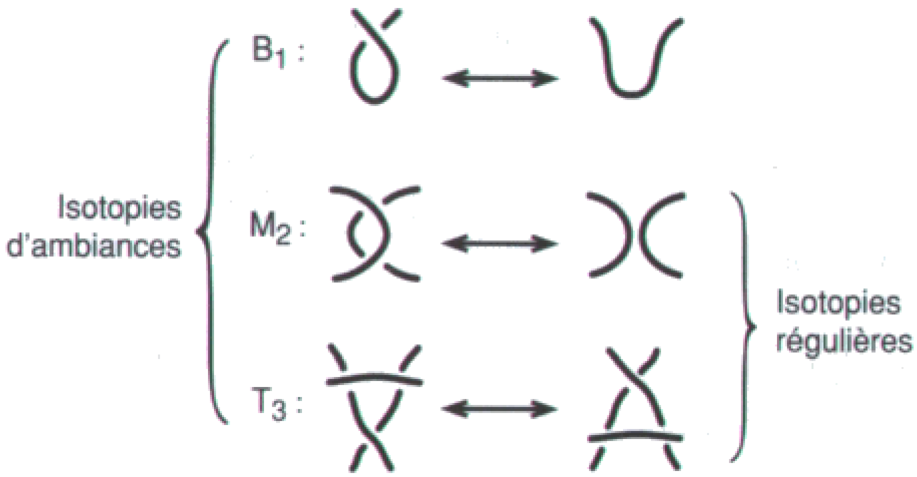


Fig. 12

Using these three generating moves, we can account for any change in the presentation of a given diagram of a knot or link to another presentation of the same knot or link. Describing these changes in presentation using elementary moves ensures identity by isotopy, also known as ambient isotopy.

In the case of changes in presentation that only use movements M2 and T3, we refer to identity through regular isotopies.

These modes of equivalence, in terms of movements, being well defined by mathematics, make it possible to dispense with more detailed descriptions, although these are present in an underlying manner, and therefore lend themselves to an abbreviated calculation that can be performed by drawing.

The most recent polynomials, which are invariant or regular for these objects, are respected by these movements, depending on whether we favor ambient isotopy or regular isotopy. The same applies to the orientation or non-orientation of circles.

These distinctions give rise to different polynomials.

8. **Polynomials of knots and chains**

But let us return to the chronological sequence of works from the beginning of the century that we mentioned earlier, to now encounter results that lead us, through an algebraic approach, to the most recent state of the theory.

Apart from the construction of the fundamental group and other homotopy groups, which have been among the main techniques of topology since Poincaré, we owe Alexander his famous polynomial in 1923, which inaugurates, more certainly than Listing, the series of this type of invariant that will give the most current and precise knot invariants.

Let us now enumerate the progression that has continued to the present day.

After Alexander, we can place J. Levine [23] in 1965-1966. Then mainly Conway [8] in 1970, building on Alexander and developing Tait and Little.

We owe Conway the construction of a relationship between knot polynomials. This relationship is written between three polynomials, corresponding to three chains or knots, which differ from each other only at the height of the same crossing. If we start with a flattened knot, the knots or chains obtained by inverting and smoothing a crossing correspond to polynomials related to the polynomial of the starting knot.

This relationship can be written as:

It gives rise to a polynomial calculation process, Conway's *skein calculus*, which we will discuss in the appendix to this book.

In 1985, W. Jones [17] produced a new polynomial that differentiates knots more precisely, in particular the two trefoil knots. This advance represents a very important result.

In 1987, five different teams of mathematicians simultaneously constructed a generalization of the Jones polynomial.

The journal that received their contributions suggested that they write a single article [16], signed with their respective names, with a comment specific to each of them. This polynomial, named after their initials, Homfly, is a version of the Jones polynomial with two variables.

These different algebraic expressions are invariant up to ambient isotopy for oriented knots and links. Other polynomials invariant up to regular isotopy can be associated with them.

More recently, L.H. Kauffman [18. d] constructed various polynomials for unoriented knots. His calculation is based on the relation:

Kauffman's polynomial provides a very elegant way of recovering Jones' polynomial. (See Appendix Chapter II). Following Jones' discovery in 1984, it appears that we can move towards a vast family of chain invariants indexed by a Lie algebra *g* and an integer (level) *k* [19].

9. **To situate our work**

L.H. Kauffman showed [18. c] that, in the algebra of solutions to Yang-Baxter equations, a calculation plays the same role for knots and chains as linear algebra does for Euclidean geometry.

It is remarkable, at this point in the theory, how Reidemeister's third move, T3, which exchanges non-alternating triskels, reveals a principal function, which we will use later. We will propose a different approach to this.

In the next two chapters, we propose another orientation in chains and knots. This is in order to clearly define the knot. We see it appear in the relationships between entanglements and what is a knot. The knot is a curious organism that disappears under certain conditions, which can be quantified in our topology of dimensions.

This orientation and the terminology derived from it are particularly relevant for alternate objects made of one, two, and three circles, thus offering a new approach to dimension.

Using a method of coloring presentations (*diagrams*) of knots and chains, we isolate characteristic cuts in each alternating presentation.

Non-alternation is homologous to this cut. Non-alternability and cutting can even disappear in the case of two circles.

The study of the variation in the cut, in the case of chains made up of several circles, leads us to formulate a relationship:

$$Cp - 2\Sigma p = vi - 2\Sigma i$$

which is the subject of the main theorem of this first part and remains valid regardless of the number of rounds, whether the object is alternable or not.

This relationship is verified for any presentation and establishes a link between two types of orientations, torsion and characteristic.

A tension arises in what follows between the graphical description we arrive at for alternable objects and the nodal plasticity of the entanglement and the knot. This tension is established around a small arithmetic of the knot and the entanglement.

In the three middle chapters, we will tackle the study of the graphical description of alternable objects and the study of nodal plasticity head-on, dividing each chapter according to this criterion.

We will undertake the study of the graphical description of alternating chains and knots, which is deduced from our colorings and provides a new enumeration of these objects. The eminently nodal question of non-alternating objects remains. This is why we accompany each of these descriptive parts with arithmetic and plastic concerns that will lead us to the final problem of this study.

Initially, the average cut number Σ_p, which stands out from these results, can be interpreted through Reidemeister moves and proper (homotopy) and improper Gordian moves
, which we define here for the first time.

The number Σ_p is the sum of the number of knots and the average cut number Σ_0 of the non-knot contained.

To this end, we propose a theory of non-knots for objects made of one to three circles. This theory will lead us in the last chapter to the definition of the knot number.

Another type of movement, knot movement, which is also defined in this last chapter for the first time, gives another interpretation of Σ_p and leads us to the knot number.

This number is an invariant of ambient isotopies. It is defined in terms of orientation by torsion. This number is added to *the* well-known *linking number*, which depends on orientation by characteristic in the study of knots and chains.

But in order to better compare these two remarkable numbers, we propose a new calculation of the number of entanglements for objects made of one to three circles, in terms of torsion, using the concept of contained non-knots, thereby calibrating the number of links and the number of knots.

This new way of counting entanglements reveals a structural break between chains made of three circles and chains made of four circles. This break is relevant to the definition of dimension and allows us to understand the stakes of Lacan's last seminars. It is a matter of homogenizing the impossibility of representation with the structure of language at play in analysis.

This structural flaw will be the subject of our next book, the title of which can already be announced: ***Pas****, les chaînes de quatre ronds et plus* (fascicule de résultats n° 4).

In the last chapter, we will return to the nodal aspect that interests psychoanalysis in order to situate P. Soury's work in relation to the constructions sketched out by Lacan in the last years of his seminar. This will allow us to situate our own contribution to this debate, before returning to it in the last volume of this series of works. This final volume, ***Si étroit comme treize et trois… pas large,*** *le borroméen généralisé (*results booklet no. 5), will be devoted to the study of the generalized Borromean knot as Lacan designated it in 1978 and to the theory that can be deduced from it. This will be the theory of generalized knots, which is the theory of knots that we propose for psychoanalysis.

We will therefore be led to define several theories with different objects in order to filter the trickle-down effect of the multiplicity of chains and knots.

Before formulating, for the proper knots and chain knots of classical theory, a relationship of equivalence that subverts the number of loops in order to enumerate them in a more relevant, more topological than descriptive way, and to deal on this occasion with the distinction between proper (one) and improper (several).

So, after this first chapter of pure scholarship, which can serve as a working tool for those who want to get involved in this field, let's begin to approach the knot in our own way, which is more topological than mathematical.

Chapter II

The method for reading a knot

1. **Analysis of an example of a knot**

The title I have chosen for this chapter indicates which approach to the problem of the knot I am inclined to follow, among the less traditional ones. I propose to show what can be read from the knot, and if I contribute to clarifying some of the questions dealt with in the scientific literature concerning knots, this will be only an incidental benefit, a side issue to the essential problem I have to solve. My assumption that knots lend themselves to being read places me elsewhere, between numbers, letters, graphics, and plastic dimensions, despite the prevailing theories, and in fact despite all knot theories [18 a and b].

Changing topology means changing object, as Quine [57, p. 119] says in another context. But this does not mean forgetting classical theory.

Claiming to be readable requires some explanation. We are careful not to say that this practice of knotting is a form of writing, not to write that the knot is a letter.

Saying this is another issue that requires some preliminary clarification so that anyone can claim to accept the consequences of the answer we intend to give. We will not present our theory of writing here.

For now, we will limit ourselves to showing that these knots and chains are legible, just as there is legibility in the notches carved into the bone found at Mas-d'Azil and preserved at the museum in Saint Germain-en-Laye.

This phase of legibility is necessary for writing as such to come into being, even before we can speak of a constituted writing, before we can claim a specific writing in psychoanalysis. Thus reversing the naive order of precession between writing and reading [53] and [2 Sém IX, lectures of December 20, 1961, and January 10, 1962], we will speak of the reading of objects that our modern minds would be wrong to equate with an imaginary, even animistic, projection. This term explains nothing, just as the term suggestion did nothing to explain hypnosis before Freud's discovery of the libido.

Writing will therefore be another step, the act of those who, linked together by discourse, by a social bond, use, in their actual practice, material that is already there, or other material, but always recycled material, the remnants of another discourse that has fallen into disuse.

First, we would like to clarify that our use of the term "reading" is not an analogy, as is often the case. Reading a presentation of such objects is not like reading coffee grounds. Here, there remains the distinction between calculation and language, where the metaphor comes into play, whose principle is meaningful condensation that stems from an involution.

Reading is such an involution between the gaze and the voice. The most assured presentation of this structure is given in our first booklet, in terms of truth first, to extend it to speech, the insoluble utterance to communicationists who cannot help themselves. Speech leads us to the knot [59 c and d].

As we have seen, scientific theories of the knot do not place the problem of reading at the forefront of their concerns, as it is already fully addressed by the algebraic component of their approach. For them, it is less apparent that the knot is an opportunity for an act to be performed by the subject who practices the object and who loses himself in a condensation of the numbers that flow around him. They aim to substitute a known writing for a topological body and do not distinguish between the two stages, graphic and plastic, after that of empirical observation.

Thus, these two aspects, graphic and plastic, are very little differentiated with regard to the identity of the nodes recognized thanks to the algebraic invariants of standard mathematics.

The latter seeks to replace, in the name of algebraic topology, the plastic object with an algebraic group [1]or a polynomial [2], this object of algebra being a special case in a vast family of more sophisticated and already known invariants [18 c, and 19]. This is the first point.

In our approach, the formalization of the object is not to be confused with mathematization. Like the formation of statements, it differs from the demonstration of theses in a formal language in mathematical logic. This confusion is caused, rather than resulting from, the forgetfulness in which our significant alienation occurs.

Our formalization, on the contrary, takes condensation into account since it is a graphic formalization of the presentations of these topological objects and a mathematics of their plasticity. This is the main point, examples of which can be found here in coloring, cutting, the duality of presentations, Terrasson's graph, regular montages, Gordian movements, and knot movement.

Encryption has a history, and the lack of distinction between calculation and language plays a prominent role in the inertia that exists in recognizing the actual actions performed in these practices. There is a subject at this stage, even if it is doomed to be dismissed once the process is complete.

Then comes the stage of mathematization, if it takes place, when a structure is found, the prototype of which remains the example of algebraic structures and their role in number theory. Here there is a conversion, in the psychoanalytic sense of the term, of a series of indices into symbols because of this structure, which acts as a text, as a context for these elements.

This interpretation assumes that drawing is an opportunity for an involution between place (*topos*) and discourse (*logos*) correlated with sight and voice. We will consider it as accomplishing a break, which is only justified once it has been flattened and written, in algebra, in lowercase letters, with the numbers that we can assign to it or attribute to its singularities, which thus flow down onto it. This may be erased in practice but we cannot forget or ignore it if we intend to use it. In fact, there are theorems relating to the graphics and plasticity of the node.

Let us define the significant involution that is the subject of our topology as "the copula that unites the identical with the different" [2 Sém XIV, leç 15.02.67].

We will show, at the same time, on the side of numbers and algebra, what escapes this graphic presentation, the problem of non-alternable objects, but which is dealt with by the finally isolated plastic aspect. That is, what is forgotten but insists through its plastic presence, thus showing the main topological difficulty of all future theories of the knot.

Having emphasized the difference between formalization and mathematization, we must highlight, in approaching this involution, the existence of "the charter of structure" emphasized by Lacan [É a' 21 p. 75] in the context of the historically significant example of Newton's formula of gravitation.

This formula cannot be understood, but it is explanatory, illuminating, and above all, resolutive. Lacan thus introduces the notion of the littoral function of the letter to designate its retroactive effect of upheaval [É a' 23 p. 5]. Here we can see that, at the extremes, it is neither the trace nor the impression that supports the metaphor of the letter he uses at this stage and the practice of reading in psychoanalysis. This practice is situated, as in mathematics, between that of the Oracle of Delphi and that of J.F. Champollion.

To link this question of modes of expression to what we are concerned with here, we will focus on the most accessible aspect, but one which, once again, should not be reduced to crude similarities. In terms of material, let us show how chains and knots can be used as a practice covering the entire spectrum of writing.

This spectrum ranges from the mathème to the poem. They are linked if we do not forget the signifying involution that is at the root of this range. From the use of a single letter in logic to the practice of calligraphy. Glimpsed by Wittgenstein, it immediately returns to the sidewalk. We thus make our contribution, with these few clarifications, to the developments required by Lacan's indications.

— On the side of the mathème for chains and knots, it is a question of referring to the structure of a statement, a text, a piece of writing, as is done in symbolic logic with the grammatical notion of well-formed statements, but which risks, as we have already emphasized, masking the reflection of signification if we are not careful. This consistency can be taken very far in their strict use, as shown by the notion of assemblages in set theory. Here, it is the consistency that is usually masked by meaning, as evidenced by the authors of the books signed N. Bourbaki. These assemblages do not designate sets, they are the sets themselves [2 Sém XX, Encore, pp. 46-47]. We will discuss the strict use of letters in mathematics on this occasion.

For example, in Bourbaki's Book I, entitled *Set Theory* [40 p.E II.6], the character called the empty set:

This strict nature is rarely noticed and rarely studied for a reason that has to do with the prohibition of the very existence of the structure itself. We could then return to the link between intuition, not only mathematical but also philosophical, and the validity of statements whose writing is yet to come.

The quality of a knot, and among these, a Borromean knot, will have this function of holding, unlike other chains, but this is not enough. We must not forget that between statement and utterance, between object language and metalanguage, this holding is subject-dependent but can always be formalized to the point of destitution of this subject.

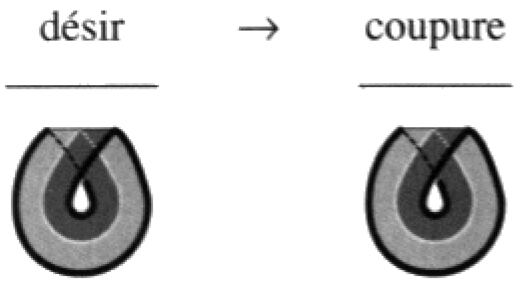
It is true that this strict usage quickly exhausts itself in supporting itself in practice, to the extent that certain function symbols are introduced. This is particularly true in classical mathematics with the introduction of the mathème (f: a → b), which writes the application in set theory [52, p. 21]. This exhaustion requires further efforts at formalization, but does not
rejects them all.

— In terms of poetry, writing goes so far as to suit the art practiced with ink and brush, Chinese poetic writing [42].

There is a function of writing that is eminently metaphorical, provided that it no longer owes anything to analogy here either, whose doctrinal reference can be found in a text by J. Lacan (pp. 493-528) when he discusses the instance of the letter in the unconscious.

As we explain in the introduction to this series of works, this function extends to the example of the writing of their language, borrowed from China by the Japanese, as it is commonly practiced in the discourse of analysis and provides a pole of materiality.

Let us give another example of such writing, which is only valid in the context of the results gathered in this work.



We maintain that the knot can be inserted into a topological writing of holes, the place of existence of the structure of the subject, as an important link, similar to others and of equal value. This writing of the *drive* (*Trieb*) realizes what Freud says about it [1 d], what Lacan specifies [É a 30, pp. 846-850], it depends on the edge, the knot, provided that a surface, the libido, is placed there, which turns out to have a structure, desire, our cut. We began to elaborate this topology of holes through the theory of intrinsic surfaces([3) and] we continue it here.

This reading of the knot implies, as we have said, assigning it a topological structure, a component provided by the various theories whose definitions are set out in this work. What would we say about a Japanese scholar who, when reading a text written in Japanese, claimed to be unaware of the archaic Chinese reading of the letters he uses to write contemporary Japanese? Our contemporaries may claim to accord it only a purely scholarly importance, supposedly outdated, foreclosed since Lacan's death, but horror remains linked to the prohibition in action, particularly in psychosis, since this prohibition returns in the real.

We will return to this reading practice in our last chapter in order to give a nodal presentation of the clinic of the sinthome with the Freudian structures of neurosis, perversion, psychosis, and analysis, and their mutual articulation, which causes so many difficulties in reading for the analysands of Freud and Lacan due to the lack of their topological component constructed here.

If we take into account the entire spectrum of variation, we see the effective exercise of a pulsation between the graphic dimension and the plastic aspect of the object, whose invisible presence we have already[4] emphasized in relation to masks and tattoos, as being at the principle of identification in the Freudian sense [59 a].

This is something other than the use of images [46] in explaining the function of symbols [49, p. 652] [53]. This question must be revisited starting with Gide's character, as Lacan [É a 27] points out in paying tribute to J. Delay, who effectively addresses it at the beginning of his essay on Gide's youth, but this aspect must be extended to the point where we are taking it here.

This was our second point.

Thirdly, with a view to this practice of drawing, we construct an algorithm that was previously lacking and apply it until we extract a formula for nodal gravity that corresponds to it.

This algorithm, extended to several circles, is required by this topology, as Lacan pointed out in a lesson from his seminar [2 Sém XXI, leç 12.03.74].

A more rigorous algorithm, which he wanted to construct at the time, for the knot insofar as it involves more than one circle of string, as he puts it, and extending, as he later says, Dehn's well-known lemma in the case of proper knots, made of a single circle.

We propose, at the same time, to undertake to address the articulation of the one and the multiple [55]. For we must be mindful here of the fact that Lacan evokes, in addition to this algorithm, in this same lesson, at a specific moment in his teaching, a passage that he says he has already made from the Borromean knot (several circles) to the trefoil knot (a single circle). The previous year, in his seminar [2 Sém XX, p. 111], he had briefly touched upon the idea that it is necessary to refer to the Borromean knot in order to study the first primary knot, the trefoil knot([5] .

This remark will occupy us for a long time to come, if we note that Lacan used this procedure only in the last seminar lesson of 1979, in December [2 Sem XXVII], before dissolving his School in January 1980. We do not know whether he ever explicitly defined this passage, which we will nevertheless construct, with the knot movement([6] thanks to the means we now wish to provide.

It is also remarkable, and worth noting, that this seminar lesson [2 Sem XX, lesson 15.05.73, pp. 111-113] is constructed according to the same plan and presents the same objects as the chapter dealing with knots in a book on popular mathematics [58, pp. 261-268].

Let us clarify the terminology we are adopting to begin discussing our subject.

When we study an immersion of several circles, we will refer to it as a chain (*link*). When we study an immersion of a single circle, we will refer to it as *a* proper *knot*, in accordance with Conway's terminology (*proper knot*) [8].

This distinction is important here, because our analysis will reveal the existence of chains with constant cuts. We will refer to this type of chain as improper knots or *link* knots.

In all cases, we refer to objects when talking about chains or knots interchangeably.

We will begin by formulating, in this chapter, the algorithm predicted by Lacan. a1 -

Preliminary account

We will work from *presentations* (*diagrams*) of knots or chains laid flat *in a general position*[(7)] which we will refer to as *flat diagrams* S.

Fig. 1

In the general case, a presentation is non-alternating.

Alternation of a presentation

We say that a presentation is alternating if, when going through each component in succession, each strand of string alternately passes over when it has passed under and under when it has passed over the string loops it encounters in succession.

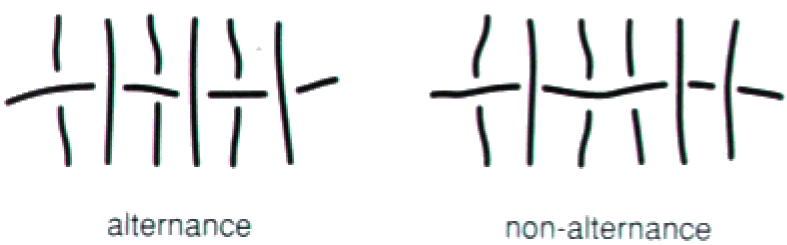


Fig. 2

We will refer to a non-alternating presentation in the opposite case.

For any object in a given presentation, if it is not itself alternating, we cannot be sure that there is an alternating presentation. There are therefore alternating objects and non-alternating objects.

Starting from the flat diagram S, it may be tempting, in order to quantify the alternation, to naively mark with a plus sign (+) the crossings where a component passes over and with a minus sign (–
) at the crossings where it passes below the string elements it encounters.

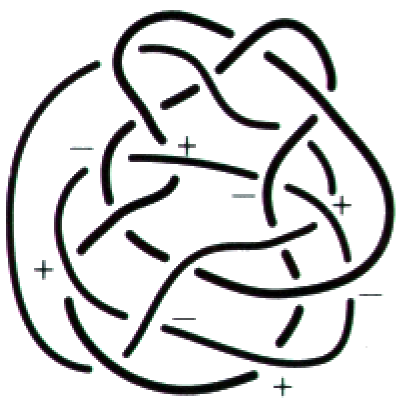


Fig. 3

But the irony of this structure remains that when we perform this calculation on all the components of the object, which is done from the very first in the case of a clean node, we find that all the crossings are marked in the same way by the presence of both signs, + and –, at each of them.

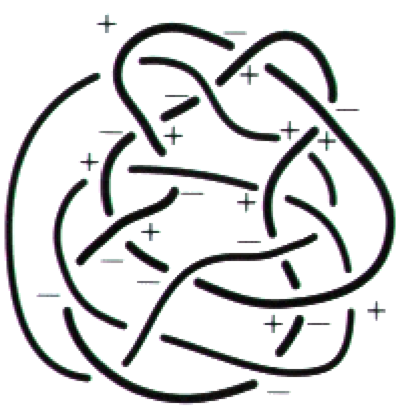


Fig. 4

Let us propose a coding of the alternation that shows the consistency of this particularity and will enable us to clarify, using our method, what this distinction is based on.

Freudian quantification

We therefore proceed differently, and we can even say in a manner contrary to this initial intuitive tendency.

Here we discover a particularly Freudian calculation, in the sense that it must have been Freud who discovered the Ics in order to calculate intuitively in this way—remember his interpretation of the dream of "the beautiful spiritual butcher" that contradicted his theory of dreams. We are not even trying to find out how it came to him; we simply acknowledge it in order to situate what psychoanalysis depends on. The psychoanalyst's desire includes this unknown.

Starting from the flat S diagram, in order to quantify the alternation, we propose marking the first intersection with an indifferent sign, for example the plus sign (+).

Then, starting from the intersections already marked, we go through each component:

— to place *the same sign* as the preceding one at the next intersection if this component *alternates* between one intersection and the other,

— to write *the opposite sign* to the previous one at the next intersection if this component *passes* from one intersection to the other *in a non-alternating manner*.

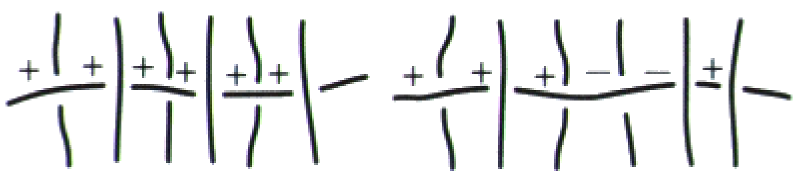


Fig. 5

This means that we apply a numbering principle that we formulate as follows:

When the string elements are alternated, we do not alternate the signs; when the string elements are not alternated, we alternate the signs.

This can be seen even more clearly in the following fragment.

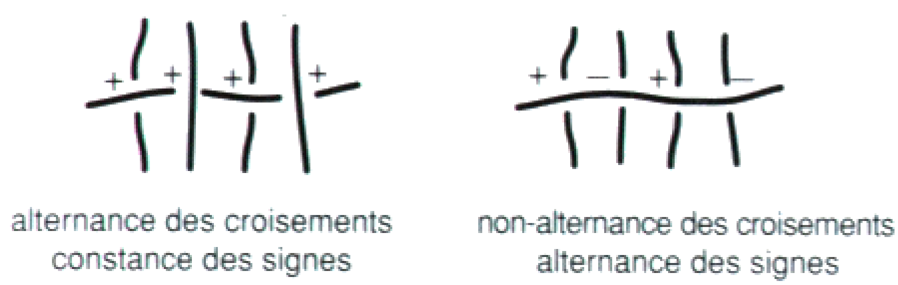


Fig. 6

We refer to this type of notation as Freudian.

Let us perform this coding on the same example, starting as follows:

Fig. 7

Once completed, it gives the following result:

Fig. 8

where we can see that, among the crossings, there are two halves that alternate within themselves but are not necessarily connected, or, if you prefer, two types of crossings: crossings marked with a plus (+) and crossings marked with a minus (–).

We will introduce a new orientation[8] into these presentations in order to reflect this fact and give meaning to these figures, which currently only exist in graphic form.

a2 - ***Node of July 23, 1993***

Présentation d'une chaîne non alternée, mise à plat[9]

Fig. 9

a3 - ***Analysis***

Let us illustrate, in three drawings, the main stages of the analysis that we will carry out using coloring thanks to our algorithm, for each node and each chain.

Premier temps. La surface d'empan de la présentation donnée.

Deuxième temps. La surface d'empan est non orientable.

Troisième temps. La coupure qui oriente la surface.

Fig. 10

The colors we use here are rendered by patterns that are used consistently. Their respective functions will become apparent over the course of the different stages.

2. **The three algorithmic stages**

We now present a very simple initial coloring method for studying each flattened node or chain.

2.1. **First step: the span area**

This defines the span area of the given presentation.

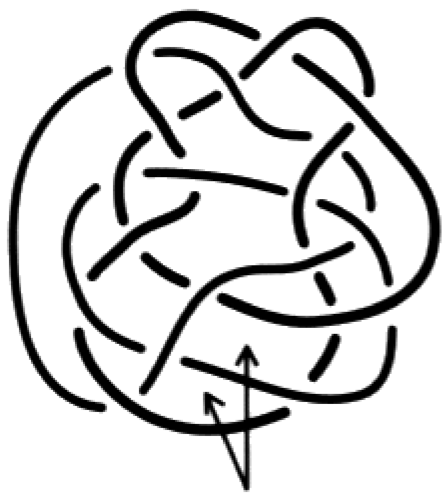


Fig. 11

a1 - ***Purpose of this step***

It aims to reveal a surface in the drawing of the flattened object; this is a true surface drawn without its folds.

The folds appear as half-twists of straps. We obtain a damage of

the plane.

a2 - ***Process implemented***

Equipped with a binary set of signs, we scan and mark all the areas by passing through the arc portions[10] and changing the sign from one to the other. This crossing is done in the middle of each arc portion, avoiding intersections and their proximity.

All adjacent areas of the flattening are then marked with opposite signs, bearing in mind that two adjacent areas are separated by an arc portion.

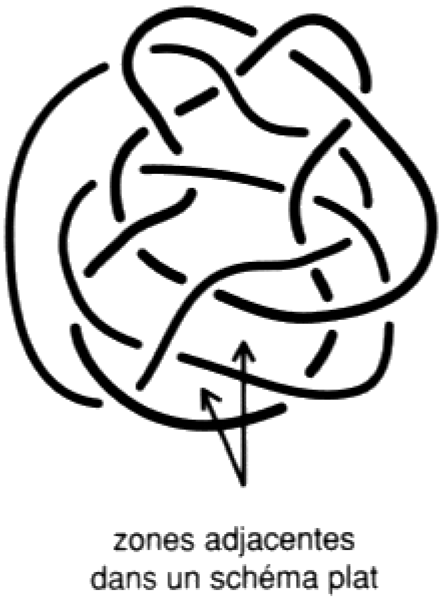


Fig. 12

To define the span area on the example of our object, let's give ourselves a pair of signs, such as (+, –) or (0, 1), or (white, gray) or any other pair of distinct and opposite signs that we use as primary differential elements.

Let's start by placing one of the signs in any area, using the binary (0, 1) here.

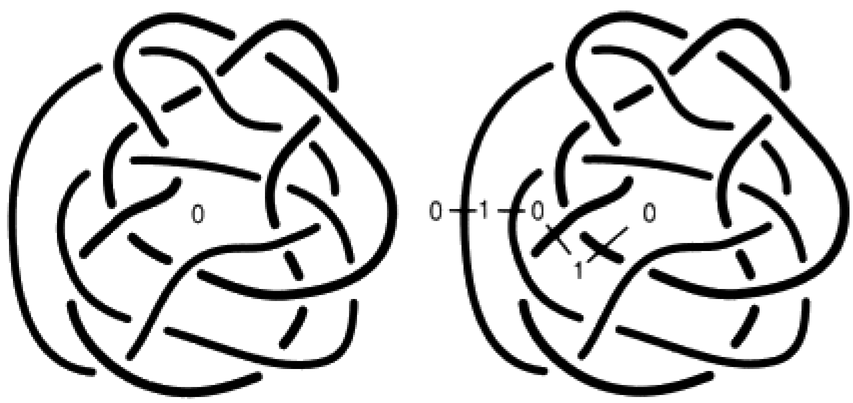


Fig. 13

The sign 0 is written in a first area. Let's move straight across a portion of the arc to an adjacent area. We will write 1 in the area we have reached. Then, from this area marked 1, let's move to another area by crossing another portion of the arc where we write 0.

We continue in this way from zone to zone, always crossing the arc segments in the same way, avoiding intersections, until all zones have a sign (0 or 1).

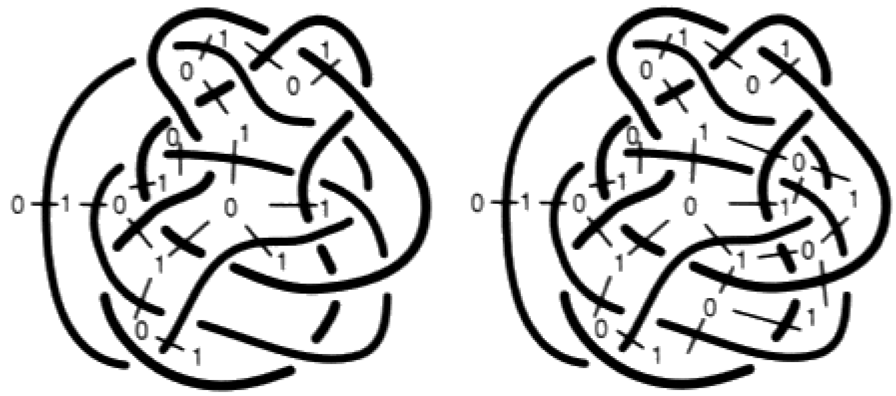


Fig. 14

Note that this algorithm never leads to a contradictory situation: the same area will never have two opposite signs; on either side of an arc segment, there will never be the same sign, as confirmed by Jordan's theorem of plane curve theory.

We therefore obtain two distinct sets of areas: those with the sign 0 and those with the sign 1.

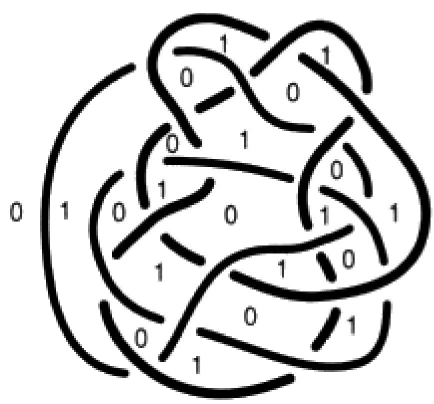


Fig. 15

End of the algorithmic process.

a3 - ***Evaluation of the result***

We then adopt a terminological principle that will allow us to define the span of a presentation.

The scope of a presentation

We agree on the following.

The set of areas marked with the peripheral area sign is the set of empty areas in the given presentation.

Therefore, we define the set of solids in this presentation as the set of areas bearing the opposite sign to that of the peripheral area.

Thus, the set of solids, respecting this convention, connected by half-twists, defines *the span surface of the presentation.*

Let's color this surface to highlight it. The knot or chain then appears as a distorted checkerboard.

The span area of the given presentation. Fig.

16

Let P be the number of solid areas (here, P = 11) and V the number of empty areas (V = 10), not forgetting the outer area.

The first step of the algorithm is complete.

Now, if we denote C as the number of crossings, we have the formula derived from the Euler-Poincaré indicator[11] of the sphere, when paving this sphere with the graph of the solids, or its dual, the graph of the voids[12].

This formula tells us that the number of solids (vertices of the graph of solids) minus the number of crossings (edges of the graph of solids) plus the number of voids (faces of the graph of solids) is always equal to two on the sphere. This can be written as:

$P – C + V = 2$

which we can transform using a small calculation similar to arithmetic, which is legitimate since these letters are supposed to refer to numbers. Thus:

$P + V = C + 2$

which gives us what we will call the *elementary formula of the node*: $C = P + V – 2$ or,

in our general case: $C = 11 + 10 – 2 = 19$.

a4 - ***The case of alternating presentations***

In alternating cases, in their alternating presentation, the minimum number of crossings allows the search for the presentation of the minimum span area.

In these cases, we designate the most numerous areas as full of the minimum span area and the less numerous areas as empty.

Minimum number of crossings

We are certain that for each object there are presentations with a minimum number of crossings; we will call these minimum presentations, but we do not know how to find them in all cases.

When an object is alternable, its alternating presentation is minimal, and L. Kauffman has demonstrated, using his polynomial, in the context of the first step of our algorithm that determines the span area, that this minimum number of crossings is a topological invariant of alternable knots.

In the case where the alternating presentation is found, we are assured that the object studied is alternable and therefore in its minimal presentation.

In their alternating presentation, if it exists, thanks to the colorings produced by the algorithm, we will be able to determine the graphic type of the objects. This typology will only be a starting point for terminology, given the nodal and plastic structure that can be discovered from there.

These colorings also remain feasible in any non-alternating presentation, and provide us with valuable information for counting interconnections, for example, or for the use of transformations.

The minimum span

In the alternating presentation of an alternable object, the set of the most numerous areas, chosen as full, connected by half-twists, defines the minimum span area. The empty areas must be the least numerous areas.

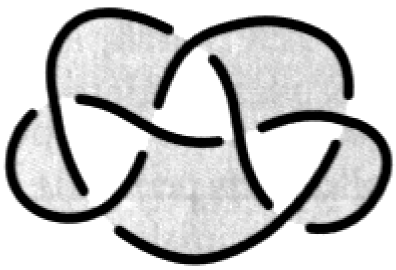


Fig. 17

Here, the span area of this presentation is the minimum area, because V = 4 is smaller than P = 5.

However, the minimum area, defined by the solid areas, does not always correspond to the span area of the given presentation as we have defined it.

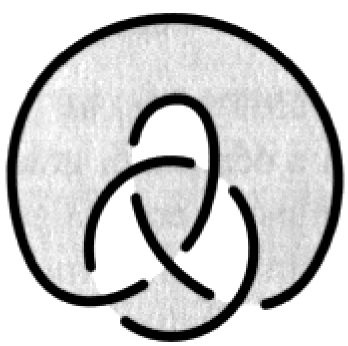


Fig. 18

For example, here the area of this presentation is not the minimum area required, because P = 3 and V = 4.

There is a need to exchange the quality of full and the quality of empty between the two sets of areas defined by the algorithm in order to reverse this ratio and obtain P = 4 and V = 3.

However, the area obtained is no longer the span area of the given presentation; it no longer meets the condition that we imposed on this area in its definition in order for it to be considered as such. It is therefore the span area that meets this definition to be that of the dual presentation.

Let's explain this by giving precise definitions.

Duality

We will refer to the exchange of solids and voids[13]in a given presentation *as duality*.

Dual surface

We will refer to surfaces that are dual to each other, the two surfaces obtained from each other through duality.

In the case under consideration, where we are looking for the minimum span surface of an alternating presentation, in the presence of this minimum surface, dual to the surface of the given presentation, we must remain attentive to the definitions.

However, the previous convention, which defines the span area of a presentation, requires us to change the presentation if we want this dual area to be the span area of a presentation, so that the empty areas have the same sign as the peripheral area, as required by this definition.

We thus move on to the dual presentation.

Dual presentation

Simply flip a peripheral arc, making it travel around the other side of the figure, to obtain the dual presentation of a given presentation.

Alternatively, we can simply surround the figure with a circle and then connect this circle to a peripheral arc.

This planar trick, which involves using an additional circle, is in fact a change of presentation. It is indeed a continuous deformation of the peripheral arc in question.

Let's illustrate this process using the example we chose at the beginning.

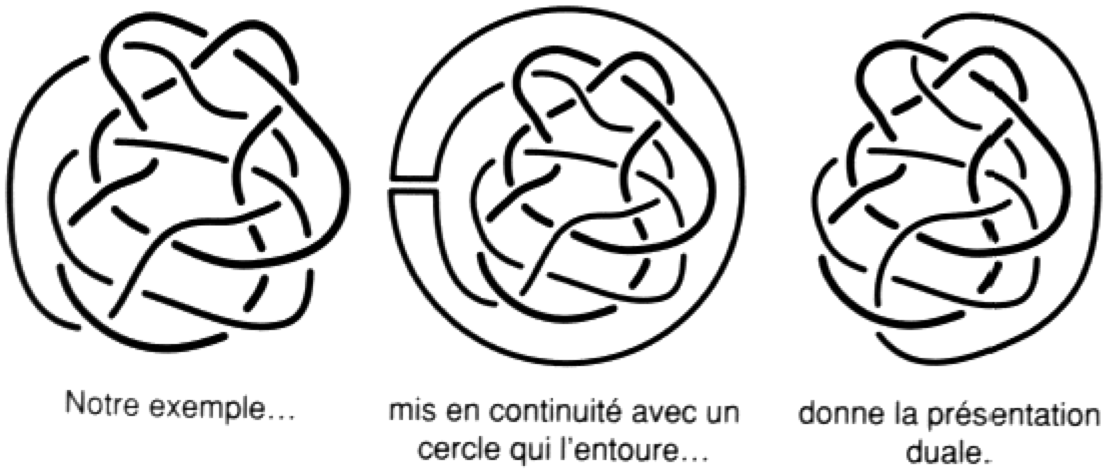


Fig. 19

This change in presentation which involves all the intersections of the presentation if we pass the deformed arc above or below the figure [14], is even more regular on the sphere without holes because, in this case, the deformed arc travels along the hidden side of the sphere and does not involve any intersections.

This change in presentation can be repeated several times.

On the sphere with a hole, our sheet of paper, we will then refer to *presentations* that *are dual* to each other, depending on whether the peripheral area, the area bearing the hole in the sphere when it has a hole, is part of one or the other half of the areas determined by our first algorithmic process.

We thus solve the problem posed by Listing at the end of his habilitation thesis when he discusses the different presentations of the same object laid flat. He had identified this binary of areas, which he had labeled λ and θ.

We will devote a more detailed study to this very important concept in our drawings later on.

Now that these definitions have been clarified, let us return to the example of the alternating case for which we are seeking the minimum span area and show how it is the span area of the dual presentation of the one we had, moving from one presentation to the other using the seemingly artificial process which may appear artificial, of the additional peripheral circle, but which we will take as a practical and graphical definition of the duality of the presentations.

Fig. 20

The search for the minimum span area in alternate cases when they are in their alternate presentation led us to change presentations by means of this still enigmatic movement, the duality of presentations, which will be explained further on.

With the span of this presentation, we obtain the minimum span, V < P, of this object:

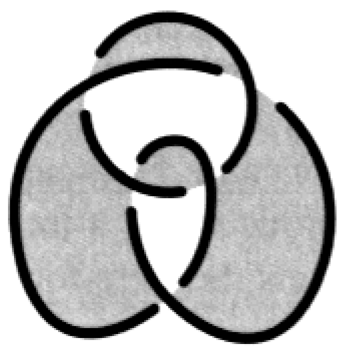


Fig. 21

since V = 3 and P = 4 in this case. This is indeed the same object, as the change in presentation proves.

We also encounter the case of balanced presentations.

Balanced presentations

We say that a presentation is balanced when P = V.

In such cases, the two dual span areas can be said to be minimal.

a5 - ***Crumpled surfaces***

Readers of Freud may recall what little Hans says about the crumpled giraffe. As Lacan points out [2 Sém IV], if the large giraffe represents the mother, it is easier to sit on a small giraffe drawn on a piece of paper, thus marking a key feature of this observation, since it is then something other than the real giraffe. This is a matter of symbolism, which indicates the register of nonsense for the little boy at that moment. Freud insists on this at one point in his commentary when he states that Hans has not yet entered into analysis until he has developed the register of fiction to which his nonsense belongs.

This dimension of fiction, this dimension of truth that we have deemed necessary to establish in a calculation, is the subject of the first volume, devoted to logic[15], in this series of introductory and review works on topology and mathematics in the Freudian field.

If we return to the origins of this discipline, the significance of dreams, we can highlight how Freud's optical apparatus is necessary to free his readers from the prejudice, still prevalent today, that the subject must be located within the mental structure. Lacan took this idea further with his slightly more elaborate optical model. But from this point, the analysis of a painting, and not just any painting, but Velázquez's *Las Meninas*, can be used to bring into play the real lines of construction of linear perspective. They are not located in space, if they can be reproduced at any time. It is therefore sufficient to move on to the virtual objects of our topology, of which no material gives more than a local glimpse, in order to grasp the location of the structure. Today, electronic animations can realize nodal space insofar as it can be calculated by recursive processes; all that remains is to read them, and for that we need a reader.

In Freud's desire to explain the rhetoric of dreams and their location, he moves towards the necessity of this topology. We will not say, in a crude approximation, that dreams are written on a crumpled sheet of paper, because they are as if knotted by the work of dreams, desire; they are written on a libidinal substance, the fabric of which the text reveals to us.

2.2. **Second stage: orientable character**

It determines whether the span surface is orientable or non-orientable.

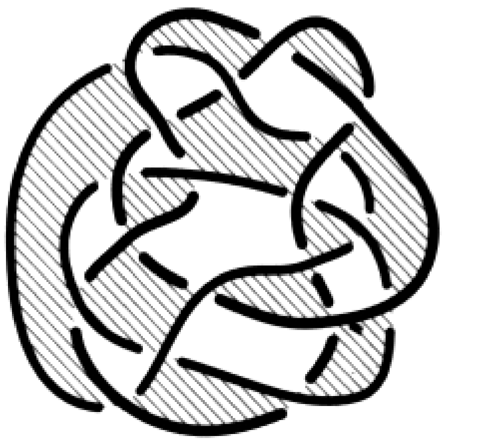


Fig. 22

a1 - ***Purpose of this stage***

We seek to decide whether the surface produced by the previous step is unilateral or bilateral[16]. Let us recall the definitions of the character of a topological surface, when taken in its orientable (bilateral) or non-orientable (unilateral) aspect.

Bilaterally: means that the surface has two sides (like a disc) and can be oriented.

Unilateral: means that the surface has only one side (like a Möbius strip) and cannot be rotated.

A principle results from the second algorithmic step, which determines the character of the span surface. We will use it to decide on the answer or to verify the result obtained after using the algorithm.

a2 - ***Principle resulting from the second step***

If there is at least one void with odd valence, the surface is unilateral. Otherwise, the surface is bilateral, with all voids having even valence.

Definition of the valence of the zones

Each zone is bordered by a certain number of intersections; this number defines the valence of the zone. We will call zones with valence one loops, zones with valence two meshes, and zones with valence three triskels.

Note that the valence of an area also gives the number of arc segments adjacent to that area.

The use of this principle is immediate.

If all the voids have even valence, the surface is *bilateral*. We color it with two contrasting patterns, one for each side.

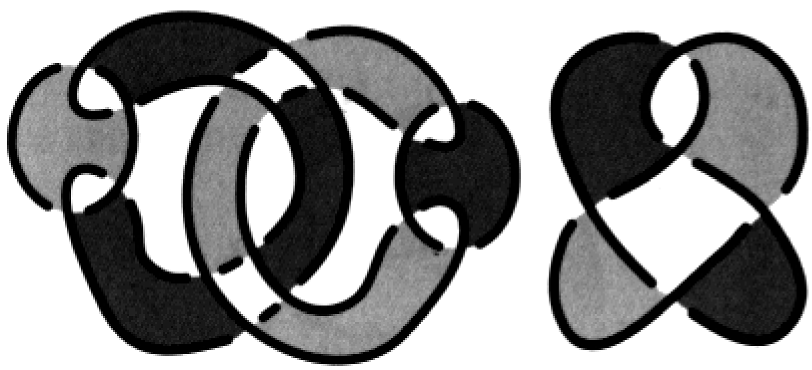


Fig. 23

Otherwise, there is at least one void with odd valence, and the surface is *unilateral*. We color it with hatching.

Fig. 24

The parity of the valence of the empty areas is the relevant feature retained by this principle in determining whether or not the span surface is orientable.

Before deducing this principle, let us formulate the second step of our algorithm. a3

- ***Method used***

To this end, we traverse the solid areas, marking them with distinct signs. They are connected to each other by half-twists. This time, we move from one solid area to another solid area using the half-twists.

To determine whether the span surface is bilateral or unilateral, we need a new binary system; let's take (+, –).

We use this new binary to mark the solid areas that make up the span surface.

Let's start by writing + in the first solid area:

Fig. 25

Let's do a half twist: let's write – in this second solid area.

From there, let's go through another half-twist and write + in this new solid area,

Fig. 26

And so on, trying to use all the half-twists.

— Either we find two opposite signs in the same area.

Fig. 27

We may be forced to borrow the same filled area several times, led there by different half-twists, and as a result, the same filled area may carry several signs. Furthermore, these signs are not always identical within the same area; they may be opposite, in which case the process can be interrupted.

— Either we have used all the half-twists at least once and found no pairs of opposite signs in the same zone.

End of the algorithmic

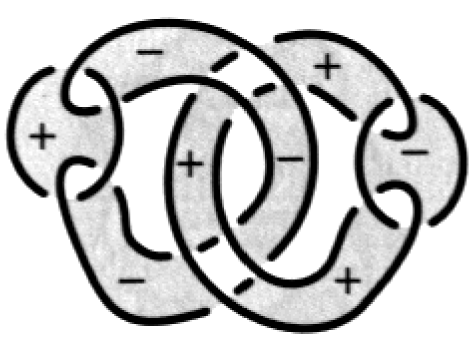
process. a4 - ***Evaluation of***

the result

Two cases may then arise.

First case

There is no contradiction. Each full zone bears only identical signs. This is the case in the following example.



La surface d'empan est orientable.

Fig. 28

In this case, on either side of each half-twist, the filled areas have different signs. The span surface is bilateral, with a + side and a – side.

We will say that the object in question appears *to be a* non-node.

Second case

There is a contradiction. The path taken leads to writing + and – in the same filled area.

This is the case in the example we have chosen:

La surface d'empan est non orientable.

Fig. 29

In this case, all the filled areas are both + and –.

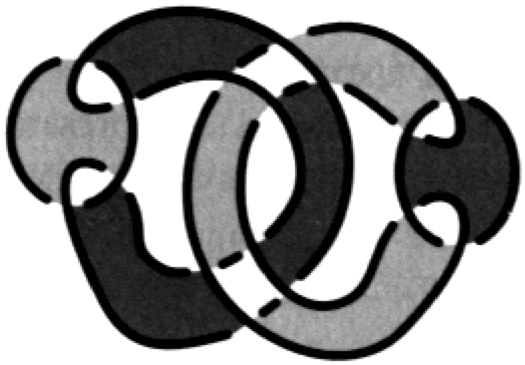
The surface area is unilateral; there is only one side. The object in question is presented *as a* node.

Definition of a presentation as a non-knot

The object in question appears *as a* non-knot, or its presentation is a non-knot presentation when the girdle surface is bilateral.

As we mentioned above, this characteristic cannot be determined with certainty until we have gone through all the half-twists. We can only be sure that a surface is bilateral under this condition.

Non-knot presentations have a bilaterally symmetrical surface; we color it with two contrasting patterns, one for each side.



La surface d'empan est orientable.

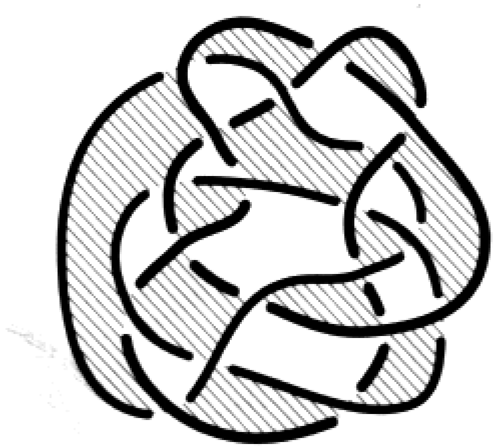
Fig. 30

Definition of a presentation as a knot

The object in question is presented *as a* knot, or its presentation is a knot presentation when the surface area is unilateral.

It may be that the non-orientable nature of the surface, revealed by this contradiction of signs in the same area, does not appear as quickly as in our example. As long as the signs marked in the same areas are homogeneous with each other, we cannot decide with certainty on the nature of the surface until we have taken all the half-twists.

Objects, when presented *as a* node, have a unilateral span; we color it with hatching.



La surface d'empan est non orientable.

Fig. 31

In the case where the surface is unilateral, we can reorient it to make it bilateral. All that is necessary is to make a cut. This cut can always be made connected and appear as a circle. This will be the subject of our third step.

The second stage is now complete.

a5 - ***Demonstration of the principle deduced from the second step***

From this second step of the algorithm, we deduce the more direct principle that we have formulated in order to determine the bilateral or unilateral nature of a surface of span, and thereby, in alternating cases, the node or non-node type of the presentation under consideration.

Let us recall the principle we now wish to deduce. If there is

an odd-valency void, the surface is unilateral.

We have defined the valence of zones as the number of intersections or the number of arc segments adjacent to that zone.

Considering only empty zones, we are concerned with the parity of their valence, as in our example.

Fig. 32

In fact, the parity of these numbers has an immediate consequence for our process, if we note that it is sufficient to make a circular path around an empty area of odd valence, writing alternately + or – at each half-twist until returning to the starting area.

Fig. 33

The last sign and the first sign, written in the same terminal and initial zone of the cycle, are different since the path describes an odd number of half-twist passages. There is therefore a contradiction between the signs at the height of a zone.

We therefore conclude that the surface area is unilateral if there is at least one odd valence void. This is the principle we have stated.

Otherwise, if there are only even-valued voids, this fact never occurs, and the surface is bilateral.

This second step of the algorithm determines the main feature of the classification of surfaces presented in our work on intrinsic topological surfaces[17].

a6 - ***The case of alternating presentations***

In the case of an alternating presentation, of an alternable object, both cases may occur.

If its minimum span area[18] presents it as a non-knot, we will say that it is a non-knot, as non-knots offer the purest presentation of the distributions of the number of crossings[19] .

If its minimum span area presents it *as a* knot, we will say that it is a knot in the sense that it contains a knot in the knotting that characterizes this presentation. This knot will be revealed by the cut necessary to reorient the surface. Our purpose, thereafter, will be to calculate the characteristic number of this knotting and the number of knots it contains.

Balanced presentations

If the presentation is balanced[20], i.e., where P = V, we must consider the two minimum span surfaces.

If one of the surfaces is bilateral, it is classified as a non-knot, and we can then refer to its minimum span surface.

If both surfaces are bilateral, it is classified as a non-knot, and the two dual span surfaces can be referred to interchangeably as minimal.

If both surfaces are unilateral, we will see later that they are characterized in the same way by the cut.

Knots and non-knots

Thus, in the set of knots and chains, which consist of tangles of one or more loops of string, we distinguish, among the alternating cases, knots and non-knots as two types of objects closer to the truth of the knot in its distinction from entanglement.

In a chain, that is to say, an entanglement, one of the loops passes through the hole of another loop. In a knot, no loop passes through the hole of another loop; when a loop enters the hole of another loop, it comes out again [2 Sém XXII, lecture of May 13, 1975].

This distinction is central to the first part of our discussion; it is the most obvious and is supported by our colorings and accompanying commentary. In the following two chapters, we will show how the link between intertwining and two-color non-knots on the one hand, and knots and monochrome surfaces on the other, is established through alternating presentations.

The smallest non-knot is a chain; it is an entanglement.

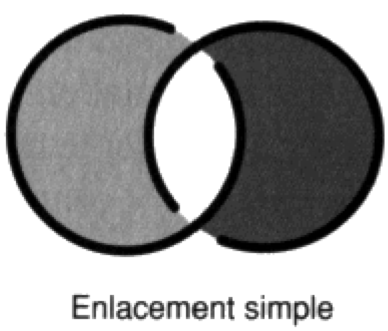


Fig. 34

There are non-knots made of a single circle. These are clean non-knots. The smallest known example is the knot that Jacques Lacan proposes to call the "Lacan knot" [2 Sém XXIII, lecture of 02.17.76].

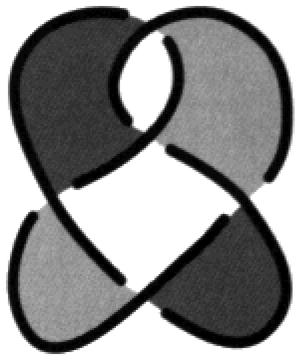


Fig. 35

a7 - ***Structure of the libido***

Let us note that the surface characteristics of the fabric depend on the interlacing and knotting of the edge, thus corresponding to the structure of the *drive (trieb*) described by Freud, where the constancy of the drive (invariance of the fundamental group[21]) is connected to the source via its edge (prevalence of bodily orifices, erogenousization through language).

It was necessary to introduce this surface (quotient of the fundamental group[22]), identified with libido as explained by Lacan [É a 30, p. 846], to show this crucial link in the structure of the Freudian drive. The knotting and intertwining fade away like the crumpling of fabric, in the transition to the intrinsic, leaving a trace, in the form of these characteristics, of what was once the knot and the chain.

The cut, which we now introduce in the case of monochrome fabrics, non-orientable crumpled surfaces, traces the path that reveals the structure of the libido. Thus, the reader can grasp Lacan's remark [2 Sem XIII] that these non-orientable surfaces, associated with the gaze and the voice, are necessary in order to correctly situate desire [É a 21, p. 601]. Indeed, orientable cases, such as the sphere and the torus, are insufficient to account for this articulation; Lacan associates them with the objects of pregenital, oral, and anal drives.

We identify this cut, which condenses the disorientation of the surface, with desire, as a metonymy [É a' 21, p. 70]. This should be read at a specific moment in J. Lacan's commentary, in the involution he practices between metonymy and metaphor. At the moment when he comments on Freud's explanations of double inscription, in his attempt in 1915 to write his metapsychology [1 e].

The interpretation of the dream therefore consists in determining the cut, thanks to associative materials, i.e., the main intrinsic characteristic of this unfindable surface fiction; Freud calls it libido, which is the substance of jouissance that is not there.

This approach will prove to be more rigorous, if not exact, with the number and invariance of the cuts, when there are several of them due to the number of circles.

2.3. **Third step: the cut**

This determines the path of a cut that reorients the span surface.

Troisième temps.
La coupure qui oriente la surface.

Fig. 36

a1 - ***Purpose of this step***

In the case where the surface is unilateral, we can reorient it, making it bilateral. It is necessary and sufficient to make a *cut*[23].

This cut can always be circular; if it has several components, they can be made connected.

a2 - ***Process implemented***

To determine the cut, a new color binary is required. Let's take two contrasting frames, light gray and dark gray:

code 1

Using this color binary, we color the portions of the arcs of each circle alternately, following the successive paths of these circles and applying color to the side of non-orientable surface produced in the first two steps, as in our example reproduced here.

Fig. 37

We begin by coloring a portion of the arc with one of the two colors, chosen at random.

Fig. 38

It should be noted that, in order to remain on the side of the span surface, it is necessary to change sides at each intersection. It is at this point that we change arc segment and, consequently, color.

Fig. 39

We continue coloring until each arc segment has a color for the circle that we are following in this way.

Fig. 40

In the case of a clean knot, with a single circle, the determination of the cut is completed at the end of the path.

When dealing with a chain, the procedure for coloring the arc segments of the same circle returns to its starting point without having colored the entire surface. We must repeat this as many times as necessary depending on the number of circles, choosing to start with any arc segment and one of the two colors for each circle.

Difference between one and several circles

When dealing with a chain, the algorithm experiences an initial halt and freezes. The procedure for coloring the portions of arcs of the same circle returns to its starting point without having colored the entire surface.

We must restart this period, choosing to begin with any arc portion and one of the two colors:

Fig. 41

The procedure continues along the second component until it has been covered in its entirety.

Fig. 42

The procedure stops again, so we move on to another circle, arbitrarily choosing a new arc segment and one of the two colors.

Fig. 43

We continue in this way until we reach the last circle.

Fig. 44

The coloring procedure is now complete. End

of the algorithmic process.

a3 - ***Evaluation of the result***

All that remains is to interpret it by drawing the cut.

Based on this coloring, some solids are monochromatic, because all their arc portions are the same color, while others are two-colored.

Fig. 45

There are two types of areas. Monochrome areas can be colored with the color of the arc segments that border them. Two-color areas are defined by intersections where two arc segments of different colors meet within the same area. We will refer to these intersections as *cut intersections*.

Fig. 46

We can sketch the cut by separating the two colors at each of these intersections with a fragment of edge that consists of solid areas.

And it is by joining these edge fragments that we obtain the components of the cut.

The cut passes through each of the two-color solid areas so as to separate the two colors. In fact, this is equivalent to saying that the cut will pass around the empty area or areas where all the arc segments are the same color.

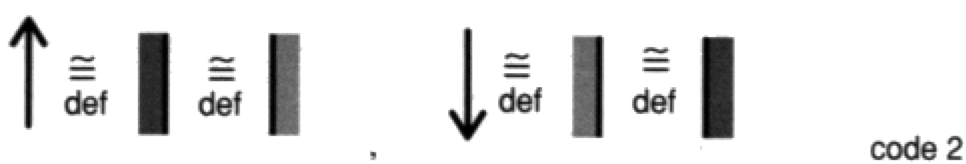
Fig. 47

The cut, running through the two-color fills, joins the cross cuts. The third step

is complete.

Coloring and orientations

This final coloring strictly corresponds to the orientation of the chain and knot loops [24], according to the following principle of correspondence.

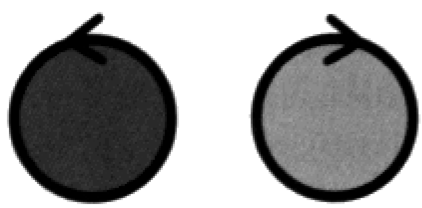


The orientation of a string element is indicated by a color applied to one side of that string element.

Given this chosen correspondence, an orientation of the loops that form the edge of the fabric can be associated with a coloring of the fabric.

Fig. 48

Another way of noting the adopted code can be extended to the drawings, where the colored areas of a given color have their borders oriented in a corresponding manner.



code 3

Knot part and non-knot part of a presentation

In a colorful presentation, we will discuss the node part (cut part) for the compound of solid areas crossed by the cut and intersections through which the cut passes. This part may have several components.

We will refer to the non-node part (non-cut part) for the compound of monochrome solids and crossings through which the cut does not pass. It may also have several components.

These parts are isolated in the drawings following the outline of a subgraph of Terrasson's graph[25]:

Partie nouage et partie non-nouage.

Fig. 49

The sources from which these different parts originate [26] and their mode of composition [27] are the subject of specific studies.

With the definition of the node part and the non-node part of a colored presentation, the algorithm is complete.

a4 - ***The case of chains made up of several circles***

In the case of a chain made up of several circles, we have seen that the procedure is interrupted and that we must resume it arbitrarily by choosing a new arc segment and a color. A different choice can be made between the two colors for the arc segment chosen at this point in the coloring process. These different colorings do not produce the same result: there are therefore several possible breaks in the case of a chain with several circles.

Here is an example based on the general case:

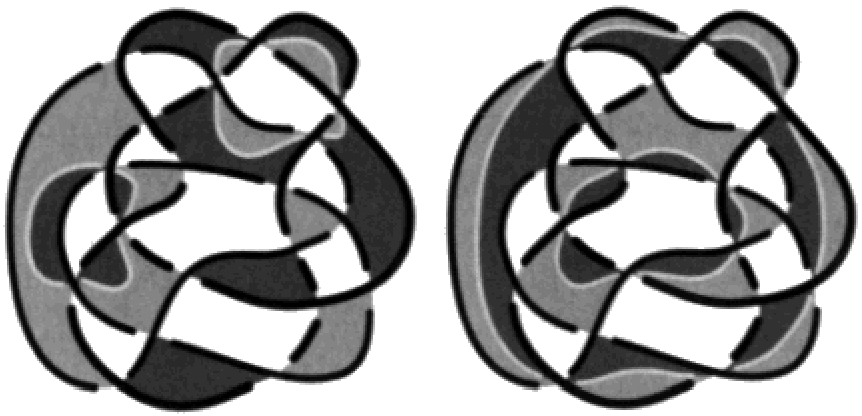


Fig. 50

In the case of chains, there are therefore several cuts. If the number of circles is denoted by r, the number of colorings is (2^r) and the number of cuts is (2^{r-1}). These different cuts have the same parity. The theory of intrinsic topological surfaces([28]) assures us of this fact, since it is always the same non-orientable surface and is equivalent to a projective plane (odd cases) or a Klein bottle (even cases) to which a certain number of tori are attached according to the main theorem of intrinsic surface theory.

a5 - ***The four interpretations of the dream of "the beautiful butcher"***

Lacan gives an example of dream interpretation [É a 21, pp. 620 to 627], saying that he does not do this often but that on this occasion he has elevated it to a paradigm. It is the dream of "the beautiful spiritual butcher" transcribed by Freud in his major work [1 a].

Freud's initial interpretation is quite surprising, given that this is a dream that the woman brings to the psychoanalyst, contradicting his theory of dreams, according to which a dream is the fulfillment of a desire. Freud presents it this way, and only Freud could respond to the beautiful hysteric by saying that she has the desire to have an unsatisfied desire.

He then completes his commentary with the first lines of his theory of identification, specifically hysterical identification, thus adding a second interpretation, which he does not share with the butcher's wife. Her desire is to identify with her hysterical friend who appears in the associations because, although thin, she pleases her husband, who prefers fat women.

Lacan extends this interpretation with a third one that follows on from the second, emphasizing that the dreamer also identifies with her husband in this dream, because she wants to answer the question that all hysterical women ask when they play the role of men: how can a man desire what he does not love?

Finally, Lacan adds that the lady identifies with the salmon when he evokes the gauze net that separates the slices of smoked fish, which he likens to the veil masking the phallus that we have just discovered in the frescoes depicting the demon of modesty on the walls of the Villa of the Mysteries in Pompeii. That makes four.

How, then, can we get closer to the fact that a dream is susceptible to four different interpretations, each of which is equally accurate and coordinated with the others, if not by grasping these cuts that condense the disorientation of the surface span of a chain made up of several rings?

Four breaks are worth three rings according to our algorithm and the little calculation we have just indicated.

The cut is what the interpretation of the knot must trace, which does not have to be exhaustive, passing through all areas; it suffices to summarize the disorientation by reorienting the entire surface, giving meaning to the areas of the non-knot part that it does not cross.

We will immediately return to the result, concerning the number of cuts, in the following chapter, in order to interpret the variation in the number of cuts in terms of intertwining.

a6 - ***The case of alternating presentations***

In the case of the minimum span surface of an alternating presentation, we are led to distinguish two families of knots as opposed to the isolated non-knots in the second step of our algorithm. These two families are defined according to the parity of the cut.

The cut passes through a certain number of half-twists. We will call this number the *cut number* and denote it **by k**.

Parity of the cut

We will refer to the parity of the cut as the even or odd nature of the number of the cut. If

the cut is odd, the alternate node is from the same family as the clover.

If the cut is even, the alternating node is from the same family as the Listing node.

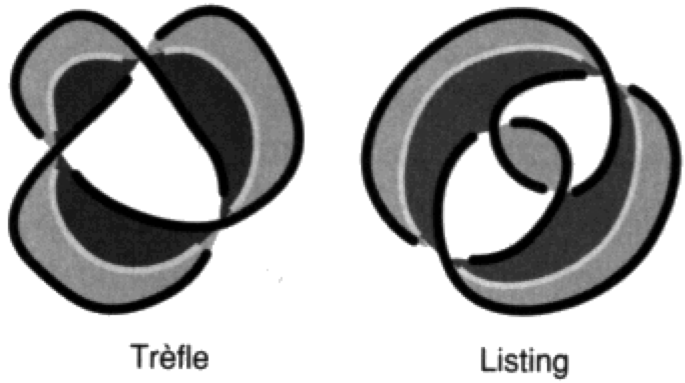


Fig. 51

The non-nodes already encountered have a zero cut, the same parity as the listings[29].

Balanced presentations

When balanced, the uniqueness of the family to which the node belongs, when it is alternable and in its alternate presentation, is also ensured. If the two dual empan surfaces are unilateral, it is easy to show that, in balanced cases, the cut of one and the cut of the other will be of the same parity.

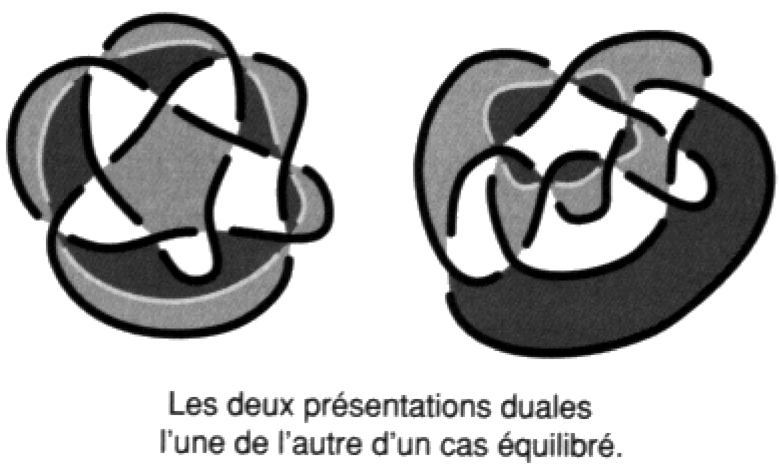


Fig. 52

Indeed, if we revisit the basic formula for the knot that we established following our first step[30]:

$P + V = C + 2$

And remember, as we defined at the same time, that balanced nodes are such that P = V.

Under these conditions, this formula becomes:

2P = C + 2 or 2V = C + 2

Thus, it is easy to see that, in the specific case of balanced nodes, the number of crossings is even:

C = 2 (V – 1)

The node part and the non-node part therefore have the same parity, since their sum is an even number.

This clarification ensures the definition of the parity of the cut of balanced nodes and chains. These nodes and chains clearly belong to the same family, without any possible ambiguity.

3. **Summary**

We have compiled a table of the vocabulary adopted, based on the distinction made in mathematics between nodes made up of a single circle and chains made up of several circles.

We replace this criterion relating to the uniqueness or multiplicity of circles with another distinctive feature relating to the necessity of cutting, depending on whether this necessity applies or not.

We call the alternating cases where cutting is necessary "knots," meaning "there is a knot." We refer to proper knots in the case of a single circle, and improper knots when there are several circles.

We call non-knots the alternating cases where the minimum span area is bicolorable, i.e., does not require cutting.

<table>
<tr><td colspan="3">knots (one)</td><td colspan="3">chains (several circles)</td></tr>
<tr><td colspan="2">Cut (knots)</td><td>Not of cut (non-knots)</td><td colspan="2">Cut (knots)</td><td>No of cut (non-nodes)</td></tr>
<tr><td colspan="2">clean nodes</td><td rowspan="3">nodes Lacan</td><td colspan="2">improper knots</td><td rowspan="3">Entanglements</td></tr>
<tr><td>pair</td><td>odd</td><td>pair</td><td>odd</td></tr>
<tr><td>Listing</td><td>Clover</td><td>Listing</td><td>Clover</td></tr>
</table>

Terminology for chains and alternating knots of 1, 2, and 3 rounds in their minimal alternating presentation.

Our terminology is particularly relevant in alternating cases consisting of one, two, or three circles. Below, we provide the reasons behind this designation of objects, and we will discuss the generalization to a larger number of circles.

In non-alternating cases, we adopt the distinction formulated by the phrase "presentation *as a* node" where cutting is necessary, and "presentation *as a* non-node" where cutting is not necessary, when there is a coloring that does not require cutting.

The main consequence of these three stages of the algorithm is that each proper node and each alternable chain belongs to a family with a unique name, which we will use in our description of the multiplicity of nodes and alternable chains.

This is because:

— the parity of the cut is fixed for chains made of several rings;

— the parity of the cut is fixed for clean knots and chains with a minimum span area and a single span area (unbalanced knots and chains: P > V);

— the parity of the cut is fixed for proper nodes and balanced chains (P = V) regardless of the minimum span area chosen from its two dual span areas.

Proper and improper knots are divided into two families known as Trèfle and Listing.

Non-knots are divided, according to the uniqueness or multiplicity of the number of loops, into Lacan knots and Enlacements.

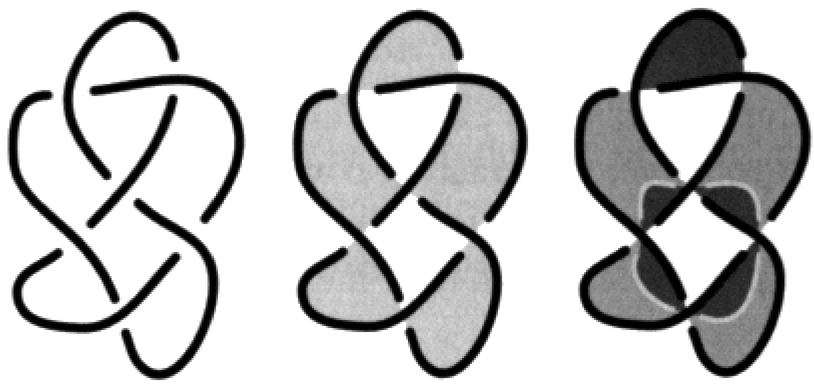
The existence of improper knots among the chains commonly referred to as such deserves some clarification, which we will now address by considering the question of variation in the cut in cases involving several loops.

4. **Exercises**

e1 - ***Coloring***

Find, in three steps and with few movements, a cut in a knot or chain, when necessary.

For example, the three steps in the case of knot $_{62}$:



la présentation alternée, la surface d'empan, et sa coupure.

Fig. a

Perform the same exercise for each of the following nodes and chains:

Fig. b

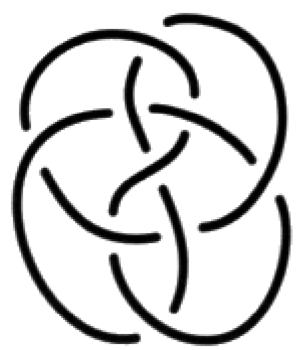


Fig. c

Fig. d

Pay attention to figures b and c. Refer to the following exercise if you need further explanation.

e2 - ***Passing the cut through the folds***

1. Transform the drawing of a given span surface in this exercise, creating folds at the height of each half-twist [31]. In this exercise, you can check how the cut passes through a fold and how the colors are distributed at the height of the intersections.

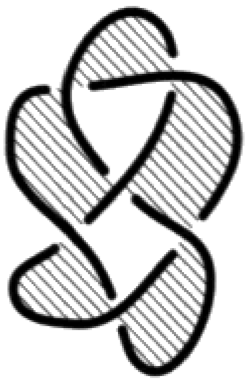


Fig. e

2. See how the cut passes through the folds and check that it can pass through twice in order to join the two components of the cut of knot 9_{40}, whose coloring you determined in the previous exercise, into a single circle.

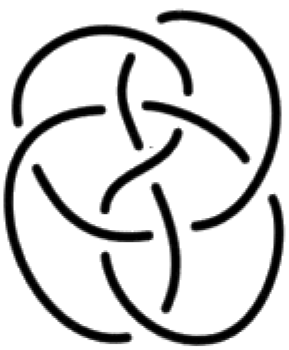


Fig. f

Chapter III

The knot is an accomplishment of cutting

When you have followed a narrow path and arrive at a high point from which you can see vast vistas in various directions, you stop and wonder which way to turn first. That is how we feel after having constructed the algorithm that was missing for chains involving more than one loop of string. We find ourselves in the full light of a sudden discovery.

Of course, our procedures do not generalize Dehn's lemma, since our algorithm does not directly answer the question of whether the knot or chain is trivial, i.e., made up of unknotted and scattered loops. We will subsequently deduce a graphical process that will assure us of the existence of a minimum characteristic number, but whose value remains undecided.

The first indication provided by this practice resolves the difficulty that can be encountered when attempting to orient oneself in the flat plan of the object. It indicates that, regardless of the layout of the flat diagram in the plan, the aim is to distinguish between different parts, rather than trying to orient oneself in terms of left and right, top and bottom, or in relation to some predetermined reference point, as in analytical geometry. The presentation of the object is not, however, a chaos of discordant lines resulting from a randomly drawn fantasy. We will specify what the elements are and how they are assembled in the following chapters([1]) thanks to the fact that a cut is made there.

We would like to clarify here what we mean by this break.

One thing should be noted about this distinction between parts for a given presentation and coloring. In chains made up of several circles, these parts become entangled according to the variation in coloring that can be achieved with them. We observe that the variation occurs according to a system of cutting, which contrasts with its completion, relating more to the non-knot contained in the chain than to its knot.

We will begin by accounting for the variation of the cut by means of a count, then the movement of the cut by refining this calculation depending on the elementary movements of the theory. The highlighting of non-knots, characterized by the absence of cuts, will reveal the relevant register of what we call its completion. This completion will appear with certainty through arithmetic formulas. In our next chapter, we will attempt to precipitate it in a legible manner in the drawings.

1. **Passage of the cut in a fold**

We have one additional clarification to make regarding the previous chapter. The layout of the cut in the flat diagrams (*diagrams*) may cause difficulty for the reader when transition to surface twists. There is a color distribution here that deserves comment.

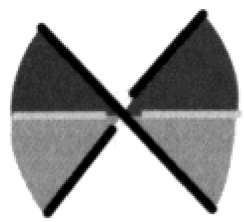


Fig. 1

To do this, we need to refer back to what we showed in the previous volume devoted to surfaces[2]. There, we explained that the drawing of a fabric twist is the outline of a fabric ribbon fold. The fold can be restored at the height of each twist. It is sufficient to perform this transformation, which corresponds to a calculation, in order to verify the relevance of our colorings.

Let us consider a crossing of rope elements, provided with a span surface. We add a fold line and show the edge line, which passes through the fold below the ribbon, as dotted lines, as we explained in our second booklet.

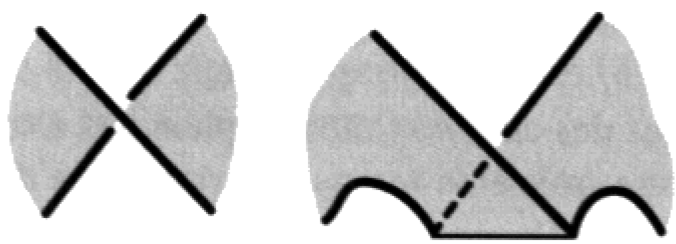


Fig. 2

In the case of an adjustable ribbon, there is a change in the colors of the visible side when passing through a fold.

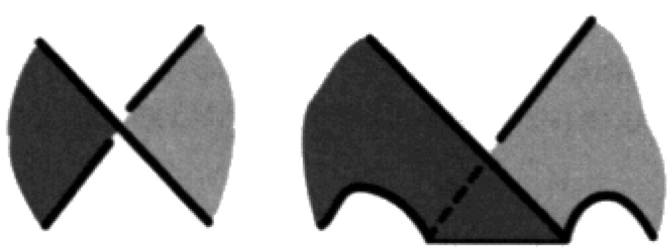


Fig. 3

In the case of a ribbon with a non-orientable surface, reoriented by the cut, we can check the path of the cut:

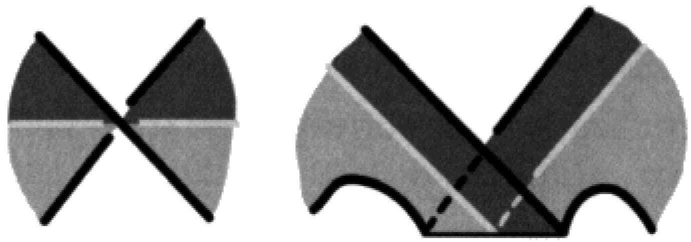


Fig. 4

and, in the drawing with a fold, explain much better the distribution of colors in the twist, since they are always back-to-back in a bilateral surface.

It is clear that we can adopt the same principle for cutting as we do for flattening nodes, namely that no more than two line consistencies should ever intersect at the same point.

Fig. 5

This principle is not respected in the presentation sketch in terms of torsion for reasons of convenience, since it will mainly be a question of counting the intersections through which the cut passes.

2. **Variation of the cut**

In the case of chains made up of several circles, repeating the process of our third algorithmic step gives rise to several possibilities. As a result, there are several different cuts, the number **k** of which, attached to each one, may vary. These are the different numbers of intersections through which these cuts pass. The parity of these different numbers of cuts is constant for the same presentation. We will distinguish this variation in the cut for the same presentation from the movement of the cut through changes in presentation.

Our example, consisting of three circles, has four cuts that reorient the surface. These numbers can be written in a distribution table:

	7	
11		11
	11	

This layout corresponds to the arrangement of the following drawings:

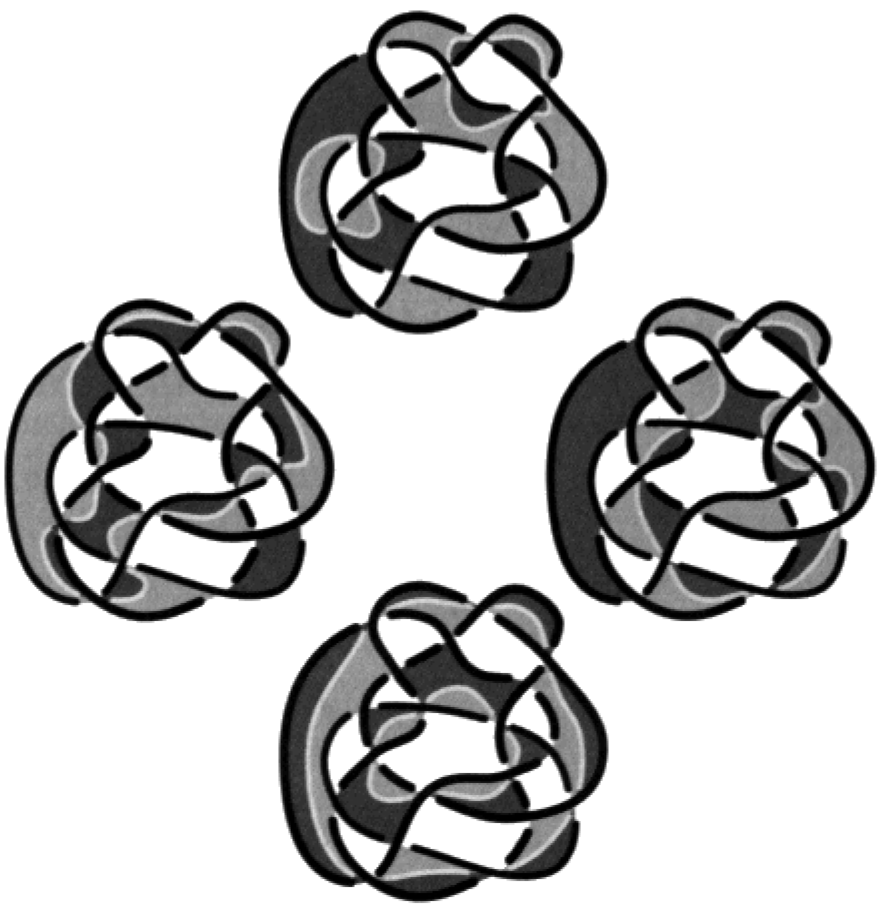


Fig. 6

However, we must bear in mind that the crossings may not be alternating, as in the case we are considering here.

If we formulate this fact, noting as in our preliminary account, using two opposite signs, plus (+1) and minus (–1), the non-alternating crossings between them.

Fig. 7

The numbers attached to each cut have been modified. In our example, the cut to which the number 7 is attached passes through four intersections with an alternation (+) that is different from the other three (–). In this same example, the cuts passing through 11 intersections curiously pass through four intersections with an alternation noted (+) that is different from the other seven (–). These numbers characteristic of the cuts are therefore reduced and oriented.

We thus obtain a new distribution, still corresponding to the layout of the drawings.

+4 –3

+4 –7 +4 –7

+4 –7

+1

–3 –3

–3

That is:

We can see then, knowing that the number of interlockings between the circles in a chain, well known in mathematics, is the first, simplest figure that lends itself to calculation, that the difference between the numbers corresponding to these cuts coincides with the number of interlockings of each circle with all the other circles[3].

This is the problem we want to study here in order to isolate it. For greater precision, we need to define the *linking number* between the loops. This number depends on the orientation of the direction of travel along the string loops. This orientation of the loops produces an orientation of the crossings. However, at the same time, it will become necessary to have another orientation of the objects. Therefore, we will simultaneously provide definitions for these two modes of intersection orientation.

3. **Another orientation in the knots and chains**

Since Tait's work, as in the definition of *writhe*, it has been common to note the crossings of oriented loop elements using two signs, +1 and -1.

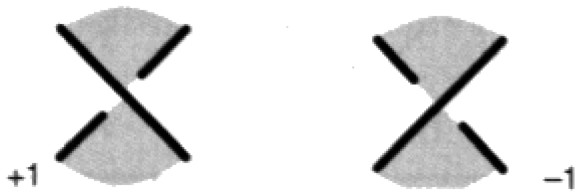
+1 –1

This defines the sign of each crossing (*crossing sign*), which we will call the characteristic (si) of each crossing, given the orientation of the circles, denoted *by i*.

There is another orientation of crossings in an object presentation, this time taking into account the distinction between solids and voids, which is independent of the orientation of the circles. Solids and voids are defined by the first step of the node reading algorithm we have proposed. To a presentation (*diagram)* of an object, we match the choice of solids and voids that assigns the value of empty to the outer area (infinite area of the plane), i.e., the area around the presentation.

a1 - ***Twisting***

Let us assign two signs, +1 and –1, to the two types of crossings that are not oriented by the direction of travel on the string circles, where only the difference between solids and voids is decided.



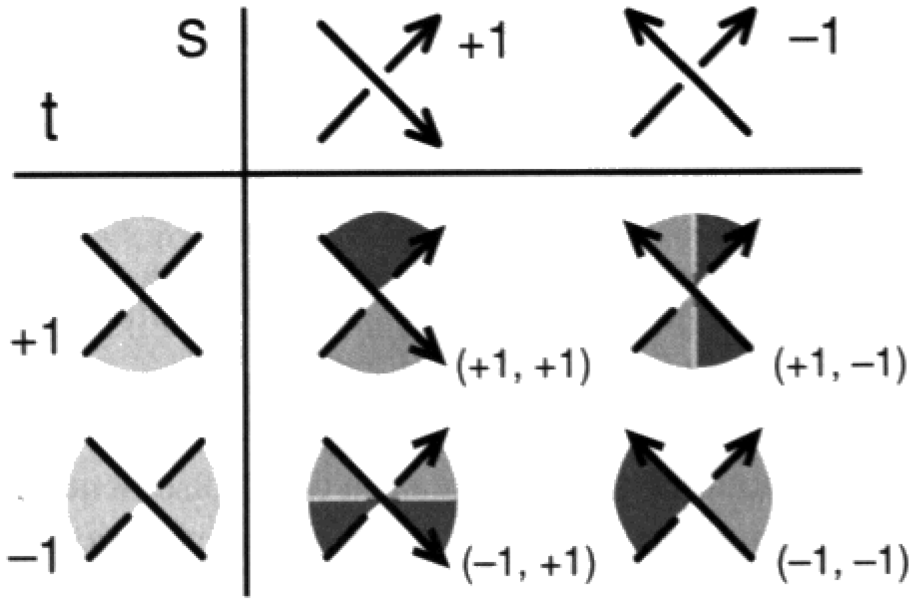
These two crossings, oriented by the solids and voids, can no longer be superimposed in the plane. We will call this incidence sign +1 or –1 the torsion ($_{tp}$) of each crossing[4].

This distinction between the orientation of intersections by torsion and the orientation of intersections by characteristic has been little developed by mathematicians until recently[5].

Calculation is less useful due to the global nature of this orientation by torsion, whereas orientation by characteristic can be localized at the height of each round. This question also concerns representation and calculation, and supports our approach with a notation specific to the knot, accurate in its drawing and rigorous in its articulation in discourse, which cannot ignore standard algebraic notation.

a2 - ***Four types of crossings***

Thus, each intersection can receive a double orientation in a presentation of an object where solids and voids have been distinguished, when it is oriented with respect to the circles. There are therefore four types of intersections.



Aesthetic principle

The surface, which is used to define the twist, thus arranged at the height of a crossing, allows us to show the coloring produced by our algorithm as suitable for accounting for the orientation of the circles by colors arranged on the side of each string element.

This correspondence between the orientation of the circles and the coloring of the areas will be the subject of a separate short study[6].

We index these four types of intersections with a pair (t, s), where the numbers t and s are taken from the pair {+1, –1}. The first notes the twist, the second the characteristic.

Let us also denote p, q, b, and d as the respective numbers of these four types of crossings (+1, +1), (+1, –1), (–1, +1), and (–1, –1) in an oriented or colored presentation. a3

- ***Three remarkable numbers***

Given this double orientation, for a presentation p, where solids and voids are distinguished, and orientation in terms of circles or coloring *i*, let us set:

— the number of crossings oriented by the twist:

$_{cp} = p + q - b - d$

where p indicates the choice of presentation.

— the number of crossings oriented by their characteristic, i.e., the writhe:

$v_i = p - q + b - d$

where *i* indicates the choice of orientation of the circles or the coloring for the given presentation.

If X_i and Y_i are two circles in the presentation oriented by *i*, we refer to the set of mutual crossings of X_i and Y_i, which is written **as crois** (X_i, Y_i), and we define the *linking number* of these two circles as

$$\mathbf{enl}\,(X_i, Y_i) = 1/2 \sum_{x \in \mathbf{crois}\,(Xi,\, Yi)} s_i\,(x)$$

obtained as half the sum of the characteristics of their mutual crossings, where $_{if}(x)$ is the characteristic of the crossing x.

— We thus define the third remarkable number in a presentation, the chain number Σ_i, as the sum of the linking numbers (**enl**) of round to round oriented by the characteristic.

Let P_i be the set of distinct pairs of circles $\{X_i, Y_i\}$ in the presentation oriented by *i*, then:

$$\Sigma_i = \sum_{\{Xi,\, Yi\} \in \mathcal{P}_i} \mathbf{enl}\,(X_i, Y_i)$$

Other definitions

In a presentation, we will call the mutual crossings of two circles improper crossings and the crossings of a circle with itself proper crossings.

Thus, clean knots made up of a single loop only have clean crossings, and there is no question of the number of entanglements. We say that their chain number Σ_i is zero.

There are chains that only have improper crossings.

In the most general case of a chain, both proper and improper crossings may be present.

The chain number Σ_i is in fact half the sum of the characteristics of the improper crossings of the entire colored presentation.

We distinguish this from the respective entanglement numbers **enl** (X_i, Y_i) of the pairs of circles, which give a more detailed overview of the entanglement state of the object.

In this regard, we will have to consider two types of distributions:

For a colored presentation, we will have to consider the distribution of the respective entanglement numbers of the pairs of circles.

For a non-colored presentation, we will also discuss the distribution of the chain numbers Σ_i obtained during each of its colorings.

a4 - ***Calculating our example***

Let's return to the result of the Freudian numbering system we started with at the beginning of Chapter II. It reflects the alternation and non-alternation of the string passages in the crossings. This is how we obtained it in the general example already chosen in the presentation p as a numbering system for this fact.

We can now give it meaning.

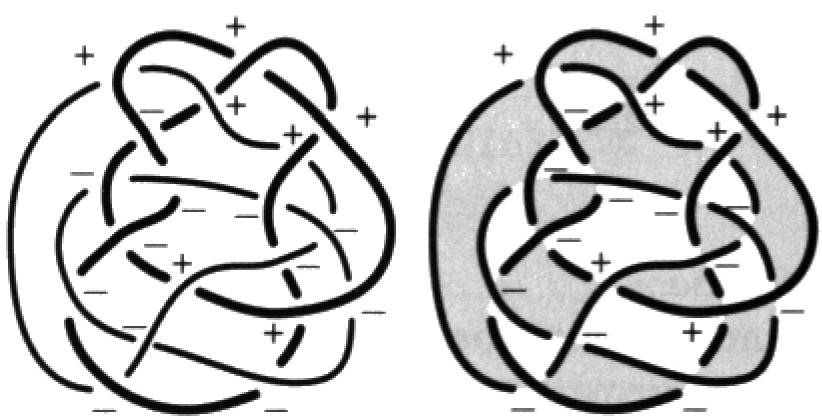


Fig. 8

The signs in the first figure correspond to the twist sign of each crossing in the second figure. These signs are defined when a surface area of the object is determined, and they take on meaning as soon as the first step of the algorithm is completed. It was still necessary to isolate, in the flattening, the opposition of the two torsions, noted as positive or negative, in the pairs of crossings equipped with this surface. Thus weighted by the solids and voids, the crossings are no longer all superimposable through the displacements on the plane. There are then two types of crossings.

Let's show how to obtain the quantifiable effect on this last figure from the other definitions we have just encountered. We retain the general example already chosen in the presentation p, now oriented thanks to the colorings *i*.

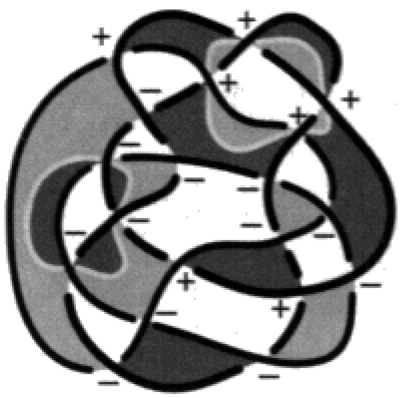


Fig. 9

Refer to the table that gives the values of the torsion and characteristic of the four types of crossings and take into account the proper or improper nature of each of them to determine the number of chains.

$_{cp}$ = –5, because there are seven positive twist crossings and twelve negative twist crossings (+7 – 12 = –5).

$_{vi}$ = –7, because among the positive twist crossings, there are three positive characteristic crossings where the cut does not pass and four negative characteristic crossings where the cut passes on the one hand, then among the negative twist crossings, there are nine crossings with negative characteristics where the cut does not pass and three crossings with positive characteristics where the cut does pass (+ 3 – 4 – 9 + 3 = –7).

$_{\Sigma i}$ = –3, because there are fourteen improper crossings and five proper crossings.

Three clean crossings are positive torsion in this case and two are negative torsion. Among the first three, one has positive characteristics—the cutoff does not pass through it—and two have negative characteristics—the cutoff passes through them. Among the other two, there is one crossing of each type.

We modify the previous calculation by removing these respective values (+2 – 2 – 8 + 2 = –6) since we only calculate this indicator on improper crossings.

The sum of the intertwining is by definition equal to half of this sum (1/2 (+2 – 2 – 8 + 2) = –3).

We can detail the distribution of the numbers of mutual interlacings of pairs of circles, naming the three circles with three distinct letters S, T, and J.

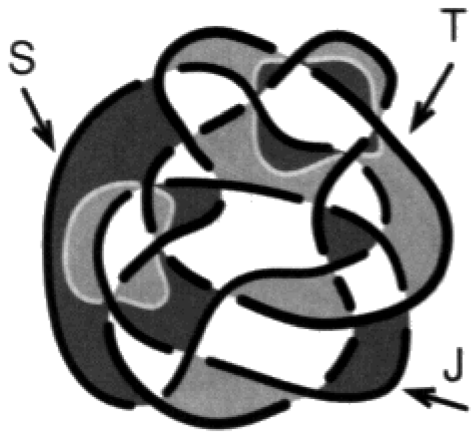


Fig. 10

We count the number of improper crossings of each type common to each pair of circles.

enl (S, T) = 1/2 (+ 2 – 2 – 2 + 0) = –1

enl (T, J) = 1/2 (+ 0 – 0 – 4 + 2) = –1

enl (J, S) = 1/2 (+ 0 – 0 – 2 + 0) = –1

In this calculation, the crossings are positive and negative due to the orientation produced by the characteristic.

In our example, we can verify that the sum of these three counts does indeed give the result obtained above, $\Sigma i = -3$, but we note that it is easier to find it directly by calculating the half-sum of the characteristics of the improper crossovers.

Armed with these definitions, in order to address the variation in the cut, let us now return to a remark concerning the difference between proper and improper crossings in the effect produced, at the level of these crossings, by a change in orientation or coloring.

4. **On the variation of the cut**

We have just clarified the distinction between proper and improper crossings. This nuance leads us to formulate a very practical principle in searching for the different cuts in a given presentation, when we already know one of them.

a1 - ***Difference between proper and improper crossings***

This principle depends on the different reactions of proper and improper crossovers to variation in the cutoff. Let's start by discussing this.

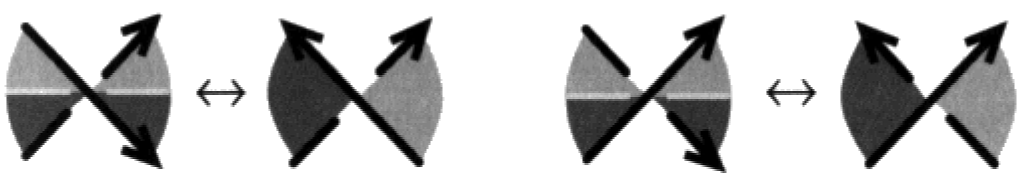
Improper crossings

At an improper crossing, if we change the orientation of one of the circles, the characteristic changes, and consequently the type of crossing also changes.

In fact, characteristic signs form, by definition, a simple pair, and its two terms differ when the direction of only one of the string elements constituting the intersection in question is changed. This change does not affect the twist.

Thus, in the case of an improper cross, when a single strand changes direction, the characteristic changes and, as a result, the type of cross is modified while retaining the same twist sign.

If it was a knotting type, with a cut, it becomes a non-knotting type without a cut, and two cases can occur.



(−1, +1) ↔ (−1, −1)   (+1, −1) ↔ (+1, +1)

Conversely, if it was a non-knotting type without cutting, it becomes a knotting type with cutting. To verify this, simply consider the same pairs of crossings given as examples retroactively.

Clean crossings

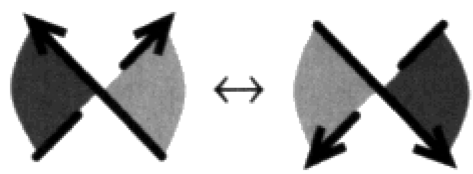
Clean crossings do not change their characteristics or type of crossing when the orientation of the round in question changes.

The two string elements of a clean cross participate in the same round. When the orientation of the round in question changes, the two string elements change direction simultaneously. There is therefore a double change of direction at this clean cross, which amounts to no change at all.

As we have just seen, due to the existence of a single pair of opposing characteristic signs whose terms are exchanged when the orientation of a single string element changes, a double change of direction at the level of a crossing produces an involution and does not modify its characteristic.

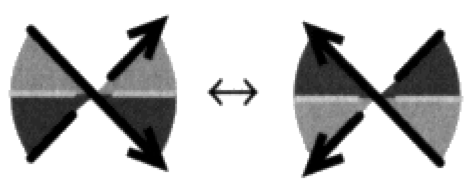
Consequently, at a proper crossing, if we change the orientation of the loop, the characteristic does not change, and since this change does not affect the twist, it follows that the type of crossing does not change.

If it was a non-knotting type, without a break, it remains so.



(−1, −1)   (−1, −1)

If it was a knot type, with a cut, it remains so.



(−1, +1)   (−1, +1)

We can therefore deduce a principle. a2

- ***Principle***

To obtain another cut from a given cut by changing the color or orientation of a circle, only its improper intersections with other circles change type.

This principle also states that the variation in the cut follows the change in type of the improper intersections.

Let us give an example of the use of this principle in the general case we have chosen.

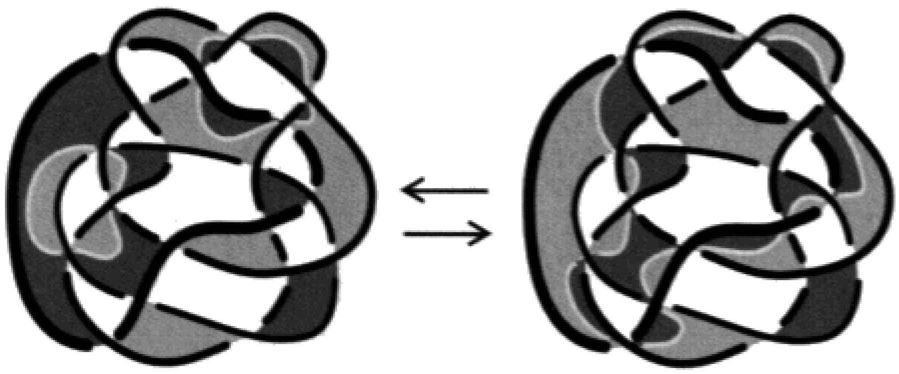


Fig. 11

If we follow the path of the circle that changes orientation, marked in bold, where the cut passed through the improper intersections in which it participates, it no longer passes through them, and it passes through those where it did not pass before.

a3 - ***Topological variation of the cut***

We can show more precisely what happens when a circle changes orientation, when the cut varies. According to our third algorithmic step, the variation in the cut is produced by a different choice of colors distributed along a circle in the chain.

Let's take the coloring from our example, which corresponds to a $_{k1p}$ cut.

Fig. 12

The change in coloring of a circle can be explained by a $_{k12}$ cut along that circle. In this case, we change the orientation of a circle, choosing a different circle from the previous example, still marked in bold, and accompanied along its entire length by the cut $_{k12}$.

Fig. 13

We connect this cut $_{k12}$ with the initial cut $_{k1p}$ to obtain the second cut $_{k2p}$.

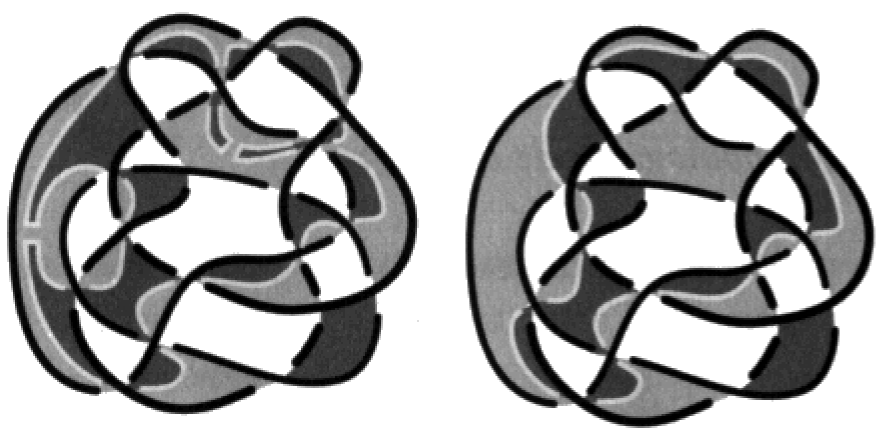


Fig. 14

We justify this process solely on the basis that it is consistent with the colorings[7].

This gives us an explanation, in the sense of an unfolding, of the variation in the cut, and we can consider calculating the difference between these two numbers of the cut in order to quantify this intermediate **cut k12**.

However, we will not pursue this avenue for the moment, and we will address a question of paramount importance in our discussion.

5. **On the completion of the cut**

With these conventions, we can write a definition and a main theorem. a1 - ***Main***

results

For a presentation of a node or a chain oriented by torsion, whose circles are unoriented, we will call the average of the numbers in the cut the number Σp obtained for any orientation i of the circles in this presentation:

$\Sigma p = 1/2\ (cp - vi) + \Sigma i$

Main result

i – For a given presentation *p* of a knot or link and for any orientation *i* of this presentation, we can write the following relation:

$cp - 2\Sigma p = vi - 2\Sigma i = \eta$

ii – The number Σp is oriented by the twist and is independent of the colorings (orientations) of the circles that form the knot or chain.

According to the definition of Σp, the relationship that forms the first part of this statement is an easy proposition to establish. We must demonstrate the independence of Σp with respect to coloring (orientation) *i* in order to establish the second part of this main result.

To demonstrate this, we will use the definition of the number h that appears in our formal expression and its independence from orientation *i* and presentation *p*.

Indeed, vi and Σi are invariants of regular isotopies, Reidemeister moves [8]M2 and T3.

Due to the definition of vi, by the characteristics of the set of all crossings of the presentation, and of Σi, by the characteristics of the improper crossings alone, the number η is well defined as the sum of the characteristics of the proper crossings alone.

We will call this number η□a proper twist, and we can note the following additional result.

Proper twist

1. The clean twist $\eta = {}_{vi} - {}_{2\Sigma i}$ is an invariant of knot or chain presentations, for regular isotopies performed on the sphere.

This means that η is independent of the presentation p, as long as we do not create or remove loops; it only changes as a result of regular isotopies composed of M2 and T3.

But in order to prove our main result, we use another partial result that follows from what we have already observed.

2. The proper twist η is independent of the colorings of the circles that form the knot or chain.

As we showed above with regard to proper crossings, these do not change their characteristics when we change the coloring.

Thus, thanks to this last lemma, we know that the number h is independent of the coloring. — since the number ${}_{cp}$ is itself independent of the coloring *i*, by definition it depends only on the torsion — we can conclude that the number ${}_{\Sigma p} = 1/2\ ({}_{cp} - \eta)$ is indeed independent of the coloring and depends only on the torsion, which is what we needed to prove.

a2 - ***Quantification of our example***

In order to practice reading this main result in the drawings, let's return to our example.

The values of ${}_{vi}$ and ${}_{\Sigma i}$ are calculated in the figures, using their definition, such as the value of ${}_{cp} = -5$, which does not depend on the change in coloring but only on the torsion, since the number of crossings does not change between these drawings.

It turns out that $\eta = {}_{vi} - {}_{2\Sigma i} = -1$, it does not vary in the four cases as stated in our second lemma, and in fact its value is justified here because there are three crossings with positive torsion and two crossings with negative torsion in this case.

The average of the numbers of cuts ${}_{\Sigma p} = 1/2\ ({}_{cp} - \eta) = -2$, by definition, is indeed independent of the colorings *i*, and our relation is verified for the four colorings.

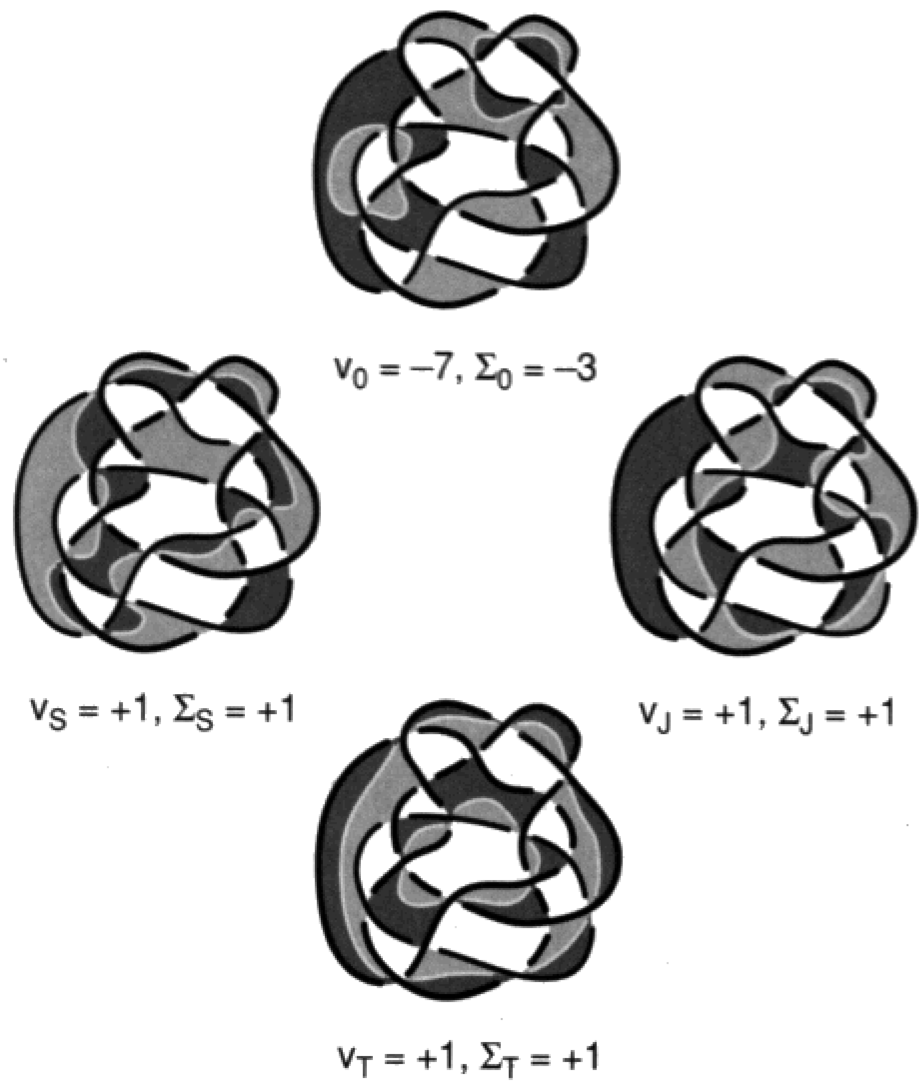


Fig. 15

But it is more interesting to justify the name "average of the numbers of the cut" given to Σp.

We will comment on this fourth figure after returning to the variations in the cut, i.e., the problem we started with.

a3 - ***The number of the cut***

We had drawn up the distribution of the numbers of crossings through which the cut passes, taking into account the non-alternation of the crossings. This non-alternation is now expressed now in terms of opposite twists, and our example has a greater number of negative twists. Our distribution oriented by the twist becomes:

$$\begin{array}{ccc} & +1 & \\ -3 & & -3 \\ & -3 & \end{array}$$

In the case of the coloring shown on the left, $_{kip}$ = –3, the cut passes through eleven crossings, four of which have opposite twists to the other seven. We propose to define the number of the cut $_{kip}$ by the difference between these two numbers:

$_{kip}$ = 4 – 7 = –3

This definition of the number of cuts based on the number of twists gives the expected result in the other three cases.

Thus, the $_{kip}$ cut number is the sum of the values of the twists oriented by the twist through which the cut passes.

In other words, to summarize this definition, we can say that the cut number is:

$_{kip}$ = q – b

where q and b are the respective numbers of (+1, –1) and (–1, +1) type crossings.

This number is related to two of the indicators defined above. The kip break number verifies the relationship:

$_{kip}$ = 1/2 ($_{cp}$ – $_{vi}$)

This relationship can be easily verified by a simple calculation, given the arrangement of the signs of the respective numbers of the four types of crossings in these two indicators:

$_{cp}$ = p + q – b – d

$_{vi}$ = p – q + b – d

Thus: $_{cp}$ – $_{vi}$ = 2q – 2b = 2 (q – b) = 2 $_{kip}$

q and b correspond to the crossings through which the cut passes, provided here with their torsion sign, hence the proposed relationship. We will call this expression (q – b) the knotting part; it is the number of the cut.

The non-knotting part is also characterized by a number resulting from an expression: $-k^*_{ip} = p - d$.

This is the opposite of the dual cut number[9]. In the dual presentation, the twist is thus reversed; for example, the number of crossings oriented by the twist in the dual presentation with its span surface is $\mathbf{c^*p} = c_{p^*} = -\mathbf{cp}$, and the non-knotting part verifies the same properties as the number of cuts in the dual situation:

$$k^*_{ip} = 1/2\,(c^*_p - v^*_i) = 1/2\,(-c_p - v_i)$$

The characteristic does not change in the dual situation and therefore the twist does not change, giving the proposed expression:

$$-2\,k^*_{ip} = c_p + v_i = 2\,(p - d).$$

a4 - ***Arithmetic variation of the cut***

This number of cuts depends on the twist and the colorings.

For an orientation *i* of a given presentation of a node or chain, our main theorem ensures that the cut number k_{ip} satisfies the following relation:

$$k_{ip} = 1/2\,(c_p - v_i) = \Sigma_p - \Sigma_i$$

We therefore have the following result, which solves our

problem. The number of cuts k_{ip} verifies the relationship:

$$k_{ip} = \Sigma_p - \Sigma_i$$

Thus, for a given presentation, the variation in the cut according to the colorings follows the variation in the value of Σ_i in terms of its number.

This consequence can be immediately deduced from our main corollary since Σ_p is independent of coloring *i*.

The distribution of cut numbers is therefore a transform of the distribution of string numbers Σ_i. The transformation between these distributions is given by the formula $k_{ip} = \Sigma_p - \Sigma_i$, which consists of inverting their signs and translating them by the length Σ_p.

In our example where $\Sigma_p = -2$, we can now interpret the distribution of the cut values.

$$\left[-3 \overset{+1}{\underset{-3}{\quad}} -3 \right] = (-2) - \left[+1 \overset{-3}{\underset{+1}{\quad}} +1 \right]$$

$$\mathbf{k}_{ip} = \Sigma_p - \Sigma_i$$

These results shed light on the fact that, when discussing the number Σ_p, we were referring to the expression of the average of the cut numbers[10], which is indeed what it is, as we will now demonstrate.

a5 - ***First interpretation of*** Σ_p

We interpret Σ_p as the average number of cuts varying across different colorings.

For a chain of r components (rounds), there are $n = 2^r$ possible colorings.

For the n colorings of a presentation of a node or chain, the expression for the average number of cuts is written as:

$$1/n \sum_{i=1}^{n} \mathbf{k}_{ip}$$

Let's calculate the value of this expression, using our corollary which states that $k_{ip} = \Sigma_p - \Sigma_i$. Thus, since Σ_p is independent of the colorings *i*:

$$1/n \sum_{i=1}^{n} \mathbf{k}_{ip} = 1/n \sum_{i=1}^{n} (\Sigma_p - \Sigma_i) = 1/n\,(n\,\Sigma_p) - 1/n \sum_{i=1}^{n} \Sigma_i$$

We then use a new proposition.

In any object, the sum of the chain numbers distributed among the colorings is zero.

$$\sum_{i=1}^{n} \Sigma_i = 0$$

This last relationship is easy to establish based on the definition given above for Σ_i, depending on the characteristics s_i, and knowing that changing the orientation of a circle reverses the characteristic sign of each of the improper intersections of that circle with the other circles.

Thus, each characteristic appears an even number of times, with opposite signs in each pair, which ensures their mutual cancellation.

Thanks to this proposition, we can deduce that:

$$1/n \sum_{i=1}^{n} \mathbf{k}_{ip} = 1/n\,(n\,\Sigma_p) = \Sigma_p$$

This is what we wanted to verify for our interpretation.

The average of the numbers in the cut Σ_p can still be interpreted through the intertwining and movements of the theory, which ensures the identity of objects through changes in presentation; we will study this in the following chapters, but to demonstrate and calculate it, we will first need to clarify a few definitions.

a6 - ***On the completion of the cut***

We have thus solved, in arithmetic, the problem of the variation of the cut, and we can then distinguish it from what we call its accomplishment.

What we call, in our title, the completion of the cut cannot be reduced to the variation in the number of cuts, for the reason that the variation in the number of cuts is the variation in the numbers of chains, or the distribution of interlockings, and, as we show below, it is the non-knot contained in the chain in the cases of one, two, and three circles. This is our first point.

Secondly, if we emphasize in our main relation:

$c_p - 2\Sigma_p = v_i - 2\Sigma_i$

Since each of its members is oriented respectively by torsion and by characteristic, we will say that the number of the cut is accomplished as an articulation between these two orientations.

This articulation is written by the condensed formula:

$$kip = 1/2\ (cp - vi) = \Sigma p - \Sigma i$$

The kip number depends on both orientations, unlike the proper twist η, which is independent of both orientations.

Thus, the arithmetic and formal accomplishment of the cut is this articulation where, to put it another way, there is a kind of oscillation carried by the cut, which draws on both of our two orientations and creates a pulsation between the two members of our main relationship.

If we associate the flattening of an object before performing our algorithm, the index $\$$ with the twisting of the presentation *p* of the fabric and the letter a with the coloring *i*, that is, with a possible orientation of the circles of the object, that is, with the characteristic of this
fabric. In this comparison, object a would be more like the color of emptiness, our main relationship proposes a writing homologous to that of the structure of fantasy, whose expression " $(\$ \lozenge a)$ " was coined by Lacan.

Our written expression in knot arithmetic articulates the relationship of the subject barred by the cut produced by coloring to this object called petit a, the cause of its desire. The number of the cut comes as the lack, noted $-\varphi$, oscillating between the two terms of this relationship, our two orientations, and the cut to be accomplished in our presentations of knots[11].

On the other hand, in our drawings, the accomplishment of the cut appears in its own register, which is topological. Indeed, the topological interpretation of the number of the kip cut is given thanks to our algorithm. The topological accomplishment of the cut, the knot, occurs in nodal space, for one, two, and three circles.

We must therefore distinguish between the completion of the cut in the node space and its notation in the formulas, even though this notation remains the best indication of this completion.

The most exciting question remains that of assessing the link between Lacan's categories once this presence has been observed in the field of the knot, a structure proven in the Freudian field. We return to this in our Appendices (chapter II) in relation to the calculation of knot polynomials.

We will then have to distinguish this accomplishment from the break in the movement of the break that accounts for the interpretation of Σp through the entanglements and movements of the theory, in the manner of Reidemeister's, in order to condense our discussion of the knot.

From a technical point of view, the constancy of the cut reveals the absence of entanglement, i.e., a type of original chains characteristic of the knot that escape the simple calculation of the number of entanglements, which remains blind to the knot.

6. ***Link*** **knots**

All of these considerations and findings lead us to adopt the term improper knots for chains with constant cuts, which can be summarized in this condensation of *link* knots that we will use to refer to them.

Here is a particularly basic example with the Borromean knot and its four cuts.

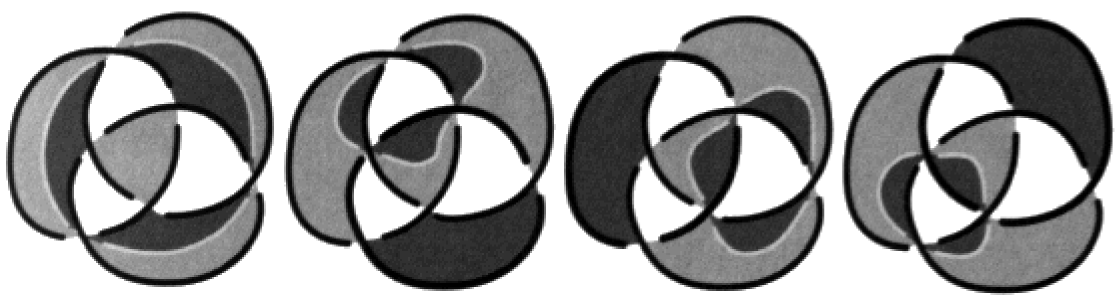


Fig. 16

These link knots contain no intertwining.

To justify our choice of words, we should point out that proper knots have constant breaks because they are made up of a single piece of rope. Consequently, we find it remarkable that certain chains, made up of several loops, have a constant number of cuts, which makes them analogous, if not homologous, to proper knots, thereby renewing the distinction between the one and the multiple, as we will study in our last chapter.

Our aim, in what follows, is to take this homology further and establish it as a structure.

7. **Distribution of interlocking**

We already have the formula $kip = \Sigma p - \Sigma i$ (main corollary), which expresses the arithmetic variation of the cut as a function of the variation in the number of chains.

We want to show that we can treat linkages in chains oriented only by torsion, i.e., independently of a chosen coloring.

It is necessary to start from chains oriented by a coloring in order to calculate, according to the correct definition of the number of entanglements, what entanglements there are in a chain.

The directed chain by torsion, on the left of our figure, corresponds to a distribution of chain numbers scattered among its different colorings.

In our example where r = 3:

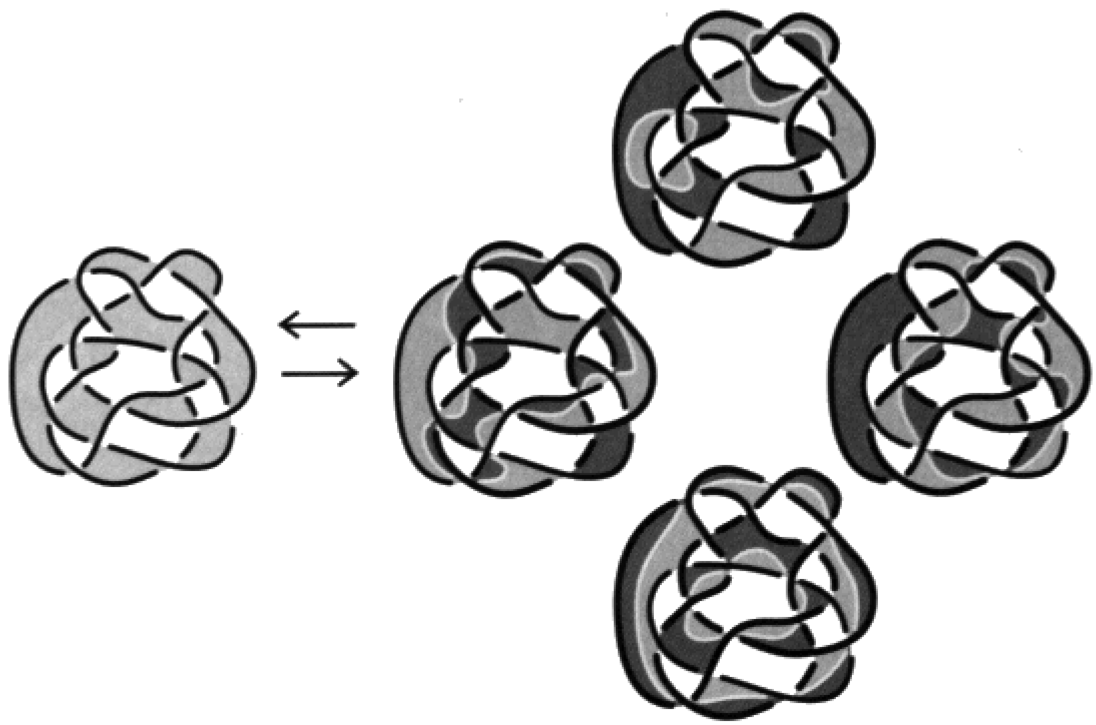


Fig. 17

Any of the colored chains on the right of our figure shows a distribution of the entanglement numbers between the pairs of components and a chain number obtained as the sum of these entanglement numbers.

Among these colored presentations, we move from one to another by reversing the direction of travel or the distribution of colors on one or more circles.

We propose to begin with a study of the relationship between the distribution of the numbers of interlocking circles for colored chains and the distribution of the numbers of chains for a corresponding uncolored chain.

It is easy to write down the entire set of numbers for these different cases in order to study the permutations of signs and the correspondences that can be deduced from them.

A chain of r uncolored circles corresponds to 2^rcolorings that give different results when calculating the entanglement numbers.

We denote by *C* the set (of cardinality r) of components and we index the chain numbers Σ_i by the subsets of C, that is, $i \in P(C)$, such that for any fixed orientation, denoted by ø, the orientation *i* is obtained by changing the direction
of traversal of the components belonging to *i*. The chain numbers thus obtained are indeed 2^r .

The colorings produce the same cut in pairs, with a simple global inversion of colors. We therefore have the relationship $\mathbf{\Sigma i} = S_{\neg i}$, where ¬ i denotes the set complement of i in C. There are therefore $2^{(r)} \, ^{(-1)}$ numbers of potentially different chains to distinguish, which are linear combinations with coefficients in {+1, –1} of the numbers of intertwining of all pairs of components.

Consequently, there are 2^{r-1} numbers of cuts to distinguish.

a1 - ***Chains oriented by the characteristic***

We will refer to a distribution of the numbers of intertwining for a given orientation as the distribution of the numbers of intertwining distributed among the pairs of circles.

This is the distribution of numbers defined for a given orientation *i* by the expression
:

$$\mathbf{enl}\,(X_i, Y_i) = 1/2 \sum_{x \in \mathbf{crois}(X_i,\, Y_i)} \mathbf{s_i}\,(x)$$

It is sometimes more convenient to write this number multiplicatively as X_iY_i.

Let's define an initial relationship between chains oriented by characteristic or colored.

Renl-equivalence

Two respective colored presentation chains S_{1i} and S_{2j} are said to be Renl-equivalent if and only if there exists a bijection **f** from the components of S_{1i} to those of S_{2i}, such that for each pair of components (X_i, Y_i) of S_{1i} and, consequently, for each pair (**f** (X_i), f(Y_i)) of components of S_{2j}, the equality:

enl(**f**(X_i), **f**(Y_i)) = **enl**(X_i, Y_i).

In this case, we say that they have the same distribution of entanglement numbers between their respective circles, and their respective chain numbers are equal: $\Sigma_i = \Sigma_j$.

This relationship is a relationship of equivalence between colored

chains. Here is an example of two Renl-equivalent objects:

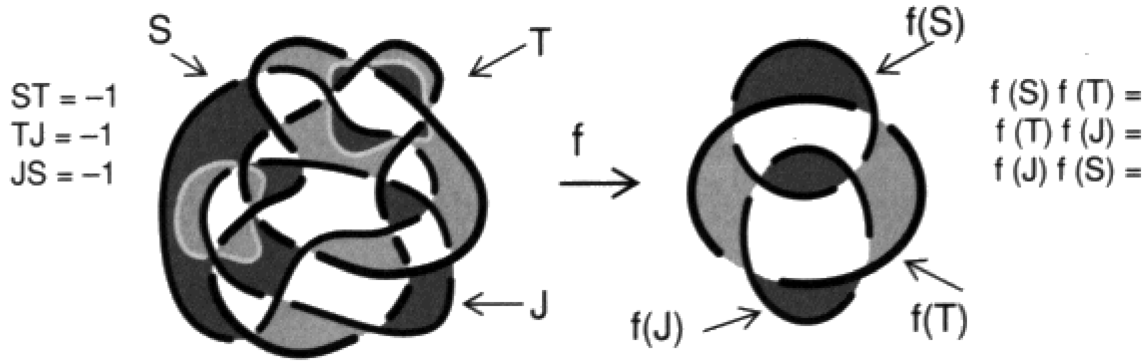


Fig. 18

For the same distribution of entanglement numbers between two colored chains, we find the same entanglement number between the pairs of circles in each of them and, at the same time, the same chain number for this coloring.

For different colorings

In the case where S_{1i} and S_{2j} are Renl-equivalent by a bijection **f**, then $S_{1i'}$ and $S_{2j'}$ are also Renl-equivalent for any coloring *i'* of S_1 and j' of S_2, where j' corresponds to *i'* through the bijection **f**.

Thus, the distributions of the number of interlacings and their respective number of chains will be equal in pairs.

All we need to do is note that we move from one coloring to another by inverting one or more circles and indicate this fact with a negative sign assigned to the letter of the circle that changes orientation.

For example, let's note:

enl ($X_{i'}$, $Y_{i'}$) = **enl** (− X_i, Y_i).

However, the number of intertwining algebraically respects this negative sign, allowing us to write:

enl $(-X_i, Y_i) = -$ **enl** (X_i, Y_i)

Thus, under these conditions, we can note the numbers of intertwining in relation to each other, for example:

enl $(X_{i'}, Y_{i'}) = -(X_iY_i)$.

In the case we are studying, the different colorings give the following sign permutations.

$ST + TJ + JS = -3 = \Sigma_ø$

$-ST + TJ - JS = +1 = \boldsymbol{\Sigma}_{\{S\}}$

$-ST - TJ + JS = +1 = \boldsymbol{\Sigma}_{\{T\}}$

$ST - TJ - JS = +1 = \boldsymbol{\Sigma}_{\{J\}}$

and to add that these permutations are respected by the correspondence **f**, because:

f $(-X_i) = (-$ **f** $(X_i))$.

Given the result stated above and the distribution of chain numbers that appears during these changes in orientation, we can now speak of an equivalence between chains oriented only by torsion, presenting the same distribution of chain numbers across the various respective orientations of their components.

They give rise to colored chains that will then exhibit the same distributions of entanglement numbers in the case of each respective orientation.

a2 - ***Strings not oriented by the characteristic***

For a given chain not oriented by characteristic, we call the distribution of chain numbers the distribution of its chain numbers spread across orientations by colorings. To see a figure of this, simply refer to our example.

This is the distribution of numbers defined for each coloring by the expression:

$$\Sigma_i = \sum_{\{Xi,\, Yi\} \in \mathcal{P}_i} \mathbf{enl}\,(X_i, Y_i)$$

Let us define a relation between chains not oriented by the characteristic.

RΣ-equivalence

Two presentation chains s_1 and s_2, respectively uncolored, will be said **to** be RΣ-equivalent if and only if there exists a bijection **g** between the possible orientations of their respective components such that for any orientation *i* of s_1 and, consequently, **g** (i) of s_2, the equality:

$\Sigma_i\,(s_1) = \Sigma_{g(i)}\,(s_1)$

holds true.

In this case, they are said to have the same chain number distribution.

This relationship is an equivalence relationship between chains oriented by torsion signs.

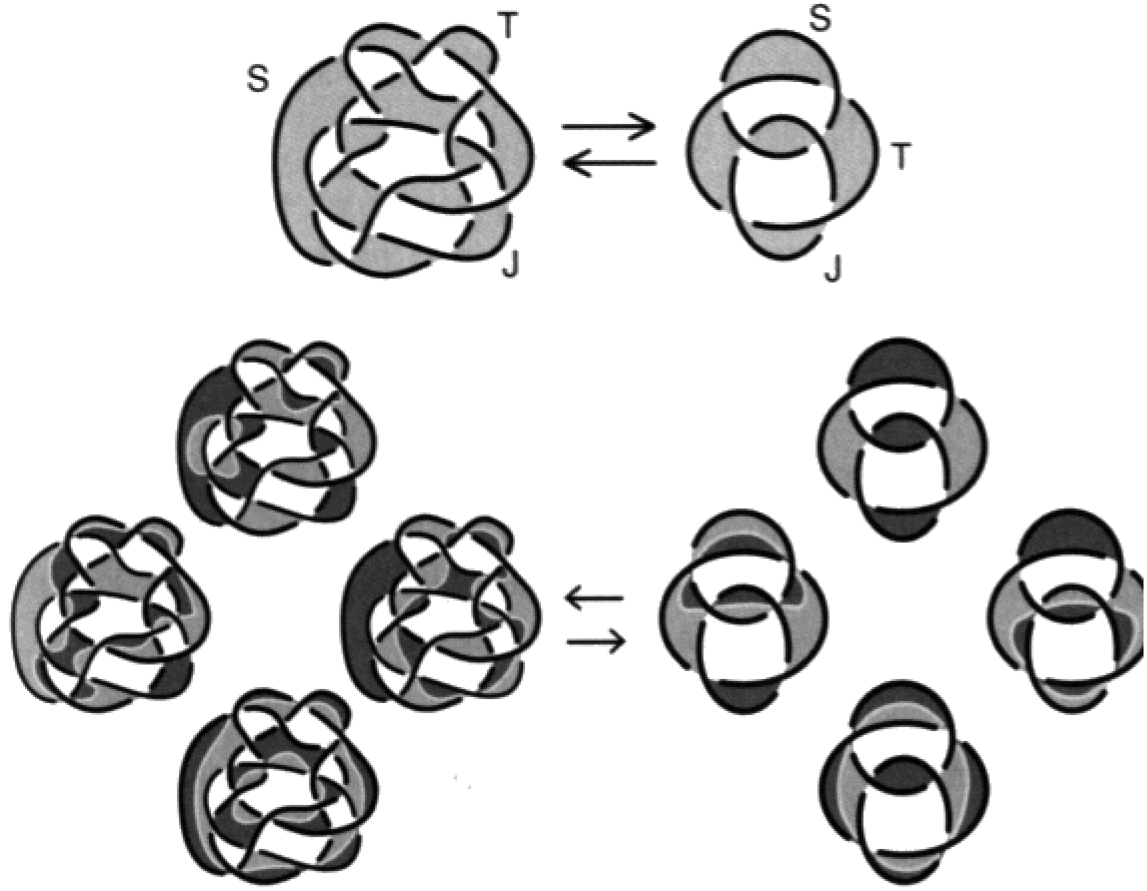


fig. 19

Transition from one relation to another

Two flat chains s_{1i} and s_{2j} oriented by the characteristic and Renl-equivalent will be RΣ-equivalent as chains not oriented by the characteristic.

This can be immediately deduced from the result stated above regarding different colorings, since the existence of the bijection g is ensured by the interplay of sign changes through orientation changes.

Transition in the other direction

Two uncolored and RΣ-equivalent presentation chains s_1 and s_2 will be Renl-equivalent as characteristic-oriented chains for any pair of respective colorings matched by **g**.

This can be demonstrated by simple linear calculation, based on the distribution of changes. sign between the colors. We can indeed find the C^2numbers n
of interlacing of the components taken two by two.

To express this result, we will use the chain number indexing presented above. With this notation, it is easy to write how to calculate the number of entanglements of any pair of components knowing the chain numbers. This number is given, in the most general case, by the expression:

enl (X, Y) = 1/4 $[(\Sigma_{ø} + \mathbf{\Sigma}_{\{X, Y\}}) - (\mathbf{\Sigma}_{\{X\}} + \mathbf{\Sigma}_{\{Y\}})]$

showing that the distribution of chain numbers accurately reflects the distribution of linking numbers.

In our example, which is unusual in that it only has three circles, knowing the If we can calculate the respective entanglement numbers of any two circles for the initial coloring using a simpler formula, namely:

$2\ XY = (\Sigma_{ø} + \mathbf{\Sigma}_{\{Z\}}) = - (\mathbf{\Sigma}_{\{X\}} + \mathbf{\Sigma}_{\{Y\}})$

For example:

$2\ TJ = (-3 + 1) = - (1 + 1) = -2.$

This expression verifies the previous formula if the reader takes into account the set complementarity between {Z} and {X, Y} in {X, Y, Z} and the fact that the chain numbers are equal when indexed by complementary subsets in the set of letters marking the circles.

We will therefore refer to the state of interlacing as both the distribution of interlacing numbers and the distribution of chain numbers.

a3 - ***On the variation of the cut***

According to the previous results, the distributions of the cuts of two flat patterns S_1 and S_2 are mutually translated.

With or without cut

Two chains that are not oriented by the characteristics in the respective presentations S_p and $S_{p'}$ will be RΣ-equivalent if and only if there exists a bijection **g** between their colorings such that for any orientation *i* of the circles of S_p and consequently g (i) of Sp':

$k_{ip}(S_p) - k_{g(i)\ p'}(S_{p'}) = \lambda_{pp'}$

where $\lambda_{pp'}$ is a constant independent of *i*.

This follows from the definition of RΣ, the expression for the cut, and our main result:

$k_{ip}(S_p) = \Sigma_p(S_p) - \Sigma i(S_p)$

Given that S_p (S_p) is independent of orientation *i*, the number:

$k_{ip}(S_p) - k_{g(i)p'}(S_{p'}) = \Sigma_p(S_p) - \Sigma_{p'}(S_{p'}) = \lambda_{pp'}$

is indeed constant when *i* varies.

8. **To conclude**

Let us clarify, as we proposed, the terminology specifically concerning improper nodes.

We have grouped the distinctions encountered in this chapter into a new table, refined from the previous one.

<table>
<tr><td colspan="2">nodes (one)</td><td colspan="3">Chains (multiple loops)</td></tr>
<tr><td>Cut

(knots)</td><td>Not of cut (non-knots)</td><td colspan="2">Cuts

(nodes)</td><td>No of cut (non-knots)</td></tr>
<tr><td>Constant cut

Nodes</td><td rowspan="2">knots
L
acan

<u>trivial knot</u></td><td>constant cuts

Knots</td><td colspan="2">Variation in cut-off

Chains</td></tr>
<tr><td>Clean knots</td><td>Chain knots

Improper knots chains

with constant cuts</td><td>Any chains</td><td>Entanglements

<u>non-standard knots</u></td></tr>
</table>

Terminology for chains and alternate knots of 1, 2, and 3 rounds in their minimal alternate presentation.

The concept of Knot, written with a capital letter, corresponding to the non-variation of the number of cuts, was introduced in this chapter.

Proper knots are Knots, and we refer to them as proper knots when they require a cut.

Lacan's nodes are very classically called this because they consist of only one circle.

We will therefore refer to the chain knots encountered in this chapter as improper knots, since they are chains, in the classical sense of the term, with constant cuts.

We now return to the term Chain, with a capital letter, to refer to chains that contain intertwining.

So, among what we call improper knots, there are chains that we refer to as arbitrary, because they represent hybrid cases containing entanglements and knots, due to the necessity of cutting.

And there are Entanglements, which are chains but also non-knots.

We will subsequently isolate standard non-knots, the trivial knot made up of a simple embedding of a circle among Lacan's knots, and the distributions of standard entanglements among the Entanglements.

On the other hand, we have encountered new results concerning the distributions of entanglements.

Given two chains that are not oriented by the characteristic, it suffices that there exist two orientations of their respective circles for each of them that make their distribution of entanglement numbers equivalent, as chains oriented by the characteristics, for them to be equivalent for any pair of respective orientations of their components with regard to the distribution of their entanglement numbers.

Their chain numbers Σ_i will be equal for each pair of respective orientations of their rounds. We can then talk about the distribution of the different chain numbers of a chain that is not oriented by the characteristic.

This distribution translates into a variation in the cut, which becomes a criterion for the distribution of interlacing in a chain. As a result, this distribution can be seen in the drawings, thus establishing the reading of the nodes that we proposed to discuss at the beginning of the previous chapter.

9. **Exercises**

e1 - ***Variation in cut and number of links***

1. Calculate the number of enlacements of two circles by taking the half-sum of the improper characteristics, in the following case, with the convention:

Fig. a

2. Find the coloring and possible cut for this same case.

3. Note that in an alternating case, the crossings of the non-knot part have the same sign, and the crossings of the knot part have the other sign.

4. As a result, the calculation is faster on the colored figure. How can we formulate the principle of this coloring interpretation?

5. Study the other cuts to see that the variation in the cut is equal to twice the number of intertwining, round to round.

e2 - ***Same exercise***

Study the variation in the cut and the number of interlacing by repeating the five steps of the previous exercise in the following cases:

Fig. b

e3 - ***Non-alternating case***

In a non-alternating case, check that the variation in cuts oriented by the characteristic does indeed give the number of interlocks.

Fig. c

Chapter IV

Deformation in the node

If I assert that the characteristic of each alternable node is a completion of a cut and that there are no alternable nodes other than nodes with a cut, I know that I will encounter irreconcilable opposition. The following objection will be raised: the fact that it is possible to construct a surface that is not orientable on nodes is not new; a whole series of authors have pointed this out for a long time [32].

But to say that there are only knots of cut accomplishments is an unjustified generalization that can easily be refuted. Too many knots surround an orientable surface without the slightest trace of a cut accomplishment.

The author who most clearly opposes this view is undoubtedly the Orientalist mathematician H. Seifert. He states this in his *Uber das Geschicht von Knoten* [36]. There are very good reasons to favor orientable surfaces, especially if we are aiming to classify three-dimensional manifolds, the space that surrounds our objects, manifolds of knots or chains.

Less pessimistic observers point out that the orientation of the surface is more relevant in space than non-orientation. Those who have studied knots from the point of view of knot variety tend to give this predominance of the orientable surface a differential expression. According to these authors, knots are only capable of receiving an orientable surface, and Seifert has produced a very elegant algorithm for constructing it([1].

Apart from non-knots with an even number of crossings, which continue the surrounding space in the surface, there are also odd non-knots that intrigue us, such as Lacan's knot, where the gusset surface coincides with Seifert's surface, and non-alternable knots, which can have an orientable gusset surface without requiring a cut. And it is precisely in the alternable cases, in the cases where we have found these knots performing a cut, that it is particularly relevant to note that the surface can be oriented without resorting to a cut.

It would seem that non-alternating nodes contradict the law of cut completion that we believed we could deduce from the examples in the previous chapters, and that they render our attempt at generalization absurd.

However, it is not difficult to respond to these objections, which appear so convincing. We need only remember that our theory is based on an examination, not of non-alternating presentations of the knot, but of alternating presentations when they exist, and that the work of deformation by changes in presentation is discovered heuristically. We contrast arbitrary surfaces with the minimum span surface in alternate cases. If we call this fact "deformation in the knot," a second question immediately arises: what causes this deformation of the knot?

It is true that there are non-alternating knots whose minimum surface area is orientable, but the non-alternating nature of these objects is equivalent for us to the presence of the cut in alternating cases, and has anyone ever tried to locate these knots, to discover their relationship and connection to alternating cases? If not, all objections fall away, for is it not also possible that non-alternating knots whose surface is orientable may, upon closer examination, turn out to be knots after all?

When the solution to a problem presents difficulties, it is often useful during research to move on to the next problem; it is easier to crack two nuts together. We will not attempt to resolve the question of how non-alternating knots with an orientable surface can perform cuts; we will first focus on another problem that also arises from what we have seen so far: why non-knots?

The knot we studied, which we explained at length, was not interchangeable; after calculation, the variation in the cut appeared to us to accurately reflect its entanglements. But why is analysis necessary? Why does the knot not immediately reveal its presence? In fact, this knot containing intertwining did not at first glance give the impression of performing a cut like improper knots or chain knots. The reader will have noticed that the variation in cuts corresponds to the state of intertwining of the object, which we ourselves did not know before analyzing it.

At first glance, various answers could be imagined. For example: it would be impossible, through the deformations, to find the alternating presentation of the knot each time. But the presentation of another theory, the theory of entanglements, will allow us to give this deformation another conception. This is what we will show by returning to the same example. It will require us to reformulate the arrangement of the intertwining it contains, but this additional effort will be compensated by a graphic resolution that will clarify the reading of the drawings.

Starting with this chapter and continuing through the next two, we will divide the material into two parts that follow each other from one chapter to the next. The first part will deal with the graphical description of alternable objects in classical knot theory. The second part will present nodal plasticity, beginning with the definition and development of the theory of non-knots with one to three loops, which is the core of our knot theory.

I. Graphical description

0. **Change in presentation**

We said that the theory begins when we have an equivalence relation between flat diagrams. This equivalence is rendered in the drawings by the deformations or changes allowed in these presentations.

The changes in presentation that thus ensure the identity of the object can be broken down into elementary movements.

These deformations of any object are called isotopies in classical theories. They are performed according to Reidemeister's elementary movements.

a1 - ***Reidemeister moves***

There are three types of Reidemeister moves:

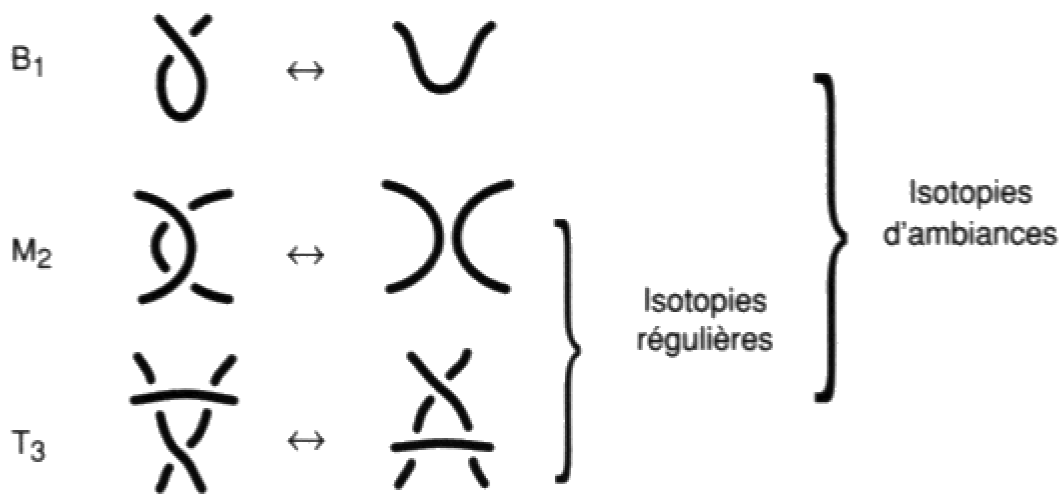


Fig. 1

Let's look at an example of how these movements are used when changing the

presentation. We start with a flat pattern, in this case non-alternating.

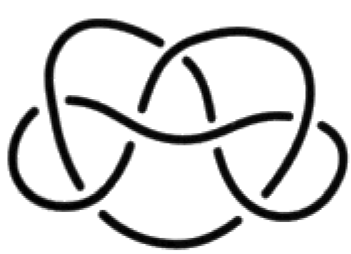


Fig. 2

We can practice different basic movements in succession.

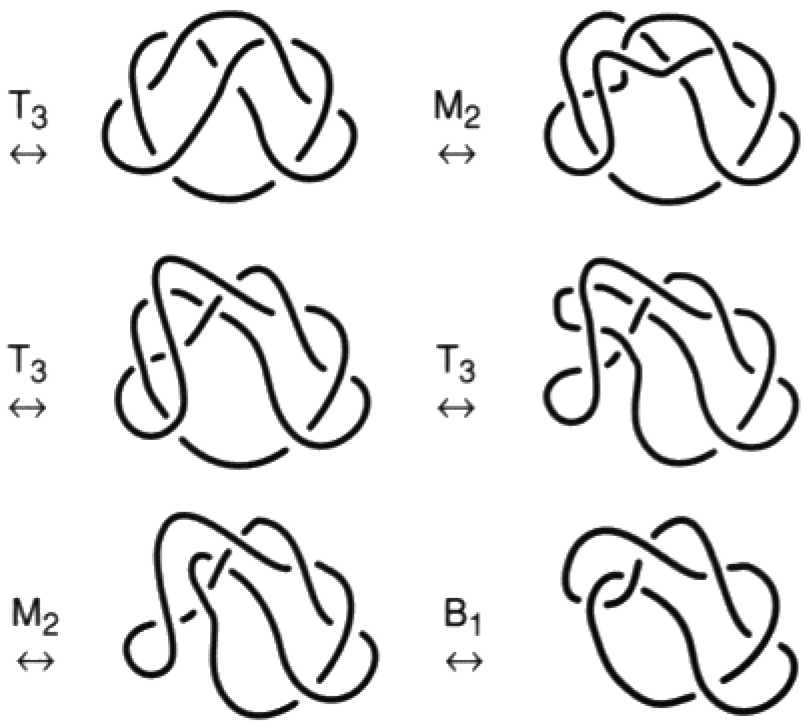


Fig. 3

Until we obtain an alternating presentation of the same object. This object was therefore

alterable. a2 - ***Degree of movements***

Each of these three types of movements is characterized by its degree. The degree of a basic movement is defined by the number of crossings involved in that movement.

The first movement is degree 1, the second is degree 2, and the third is degree 3. In the first two types of basic movements, the cross(es) used appear or disappear, while in the third type, the number of crosses used remains the same.

There are also degree 0 changes.

For example, the continuous deformation of an arc that does not involve any crossings is a type of elementary movement of degree 0.

Here we encounter a puzzle that we want to clarify in this chapter. The transition from an object with n crossings to its dual representation on the plane is a degree n movement, whereas on the surface of the sphere, the transition from the same object to its dual representation is a movement of degree 0. Let us explain this fact by breaking down the problem using more precise definitions.

We will call the dual representation of a given representation the flat diagram obtained by modifying the path of a peripheral arc in the drawing so that it now goes around the figure on the other side of the flat diagram.

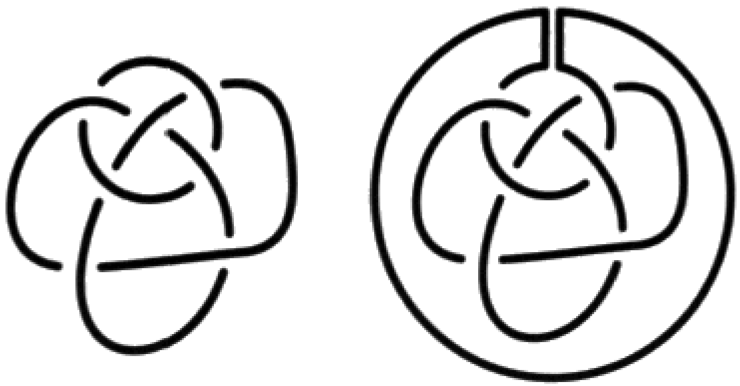


Fig. 4

We can describe this transition to dual representation by assuming that a peripheral arc has been connected by a cross section with a circle surrounding the figure, but this way of looking at things is too discontinuous due to the cross section. However, it is indeed the same topological object.

To clarify what we are doing, we need to consider the surface that supports the drawing.

Reminder

If we make a hole, like a break in the surface, in a sphere [2], we obtain a fabric that is no longer the same surface as the sphere in its entirety. We call this other surface, distinct from the sphere, a holey sphere.

The sphere with a hole is equivalent to a disk (pill) with its edge. This means that the disk with its edge, which we will also call a closed disk, is a sphere with a hole.

In this space with a rim, the part of the disc that is separate from its rim is an open set for a standard topology of this surface. This open set is the interior of the closed disc, which we will call an open disc.

Based on the above, we can say that the open disk is the interior of the sphere with a hole, interior not as a container but interior in the sense of general topology.

Now, to conclude this brief review, the infinite plane is equivalent to the open disk.

We can therefore formulate that the infinite plane is the topological interior of a sphere with a hole.

Let's return to our problem concerning the dual representation of the same object, the result of which is obtained by continuous deformations of order zero from the previous definition.

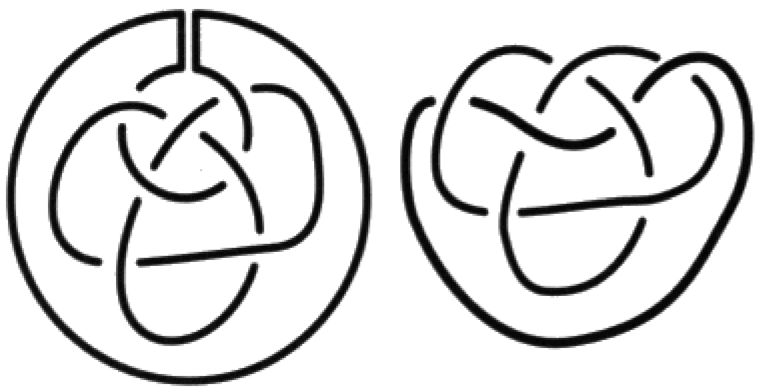


Fig. 5

What intrigues us can be formulated as follows. On the infinite plane or on the open disk (without edges), equivalent to the topological interior of the holey sphere, changes of presentation of degree 1, 2, and 3 must be made to move to the dual presentation of the same node, whereas on the non-holey sphere this movement does not involve any crossings.

Let us explain this nuance, which is invisible in the result.

Here we encounter a problem that, to our knowledge, has not been addressed anywhere from a topological point of view, with the distinction that must be made among regular isotopies, which extends the distinction between ambient isotopies and regular isotopies.

a3 - ***Ambient isotopies and regular isotopies***

When all three types of movement are allowed in our drawings, we say that the objects are subject to ambient isotopies; this amounts to placing ourselves in the ambient space in order to theorize about the objects thus determined.

If we retain only the last two types of movement from the graphical presentations, we would say that the objects are determined by regular isotopies, which is equivalent to subjecting them to additional constraints that appear to be linked to the surface of the drawing without them being immersed in that surface. In this case, we would refer to the object being placed on a

. Regular isotopies are a convenience that has an effect on the mathematical writing of the theory.

The question of the structure of the surface that supports the drawings in these so-called regular conditions is not sufficiently taken into account, yet we encounter it in relation to the degree of transition from one presentation to its dual presentation. This means that it is necessary to take into account a theory of flattening.

To this end, let us return to the definition of duality already encountered in Chapter II, during the first stage of the algorithm.

1. **Duality**

We call duality the inversion of the quality of solids and voids for a given presentation.

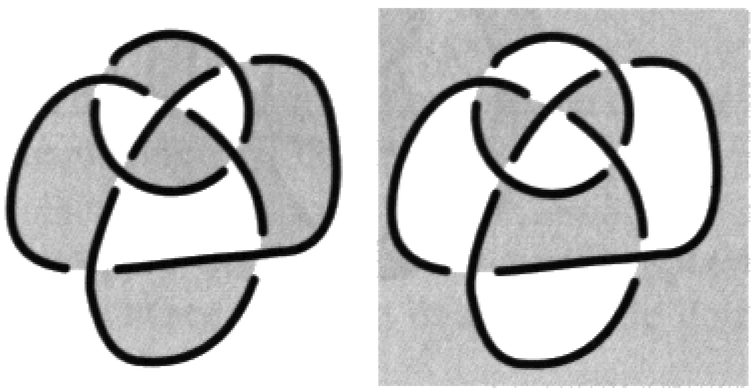


Fig. 6

The question then arises as to whether we are indeed obtaining a span area and what the area obtained is. What presentation is the span area of?

We know, having agreed on it, what the span surface of a presentation is. We define it as the surface composed of the solid areas of the presentation deduced from the first step of the algorithm. These areas are joined by half-twists. This means that we attach the notion of solids to the given presentation.

The convention we have adopted is to choose the solid areas of the presentation in such a way that the empty areas corresponding to them, by contrast, have the same sign as the peripheral area of this presentation. The sign attached to each area is produced by the first step of the algorithm.

In the figure above, the span surface of the given presentation, the solids attached to this presentation are given by the drawing on the left.

Thanks to duality, we construct what appears to be another surface, whose set of filled areas includes the peripheral area, the one around the flat diagram.

This surface is not the span surface of the given presentation. Is it the span surface of another presentation? Deciding which presentation this surface is the span surface of would allow us to answer this question.

To answer it, we will show that it is indeed the span surface of another presentation and thus show which presentation and which surface it is.

Let's draw a circle around the figure to delimit the peripheral area.

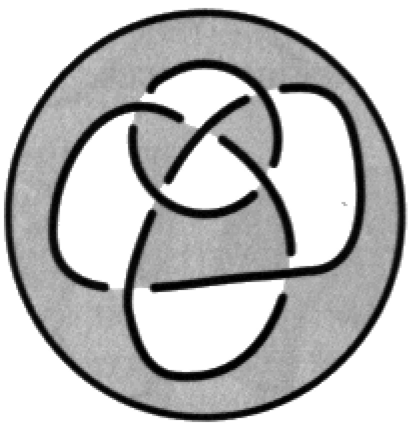


Fig. 7

By deforming the surface thus obtained, we can answer the question posed. We deform the surface by following a series of eight changes in presentation:

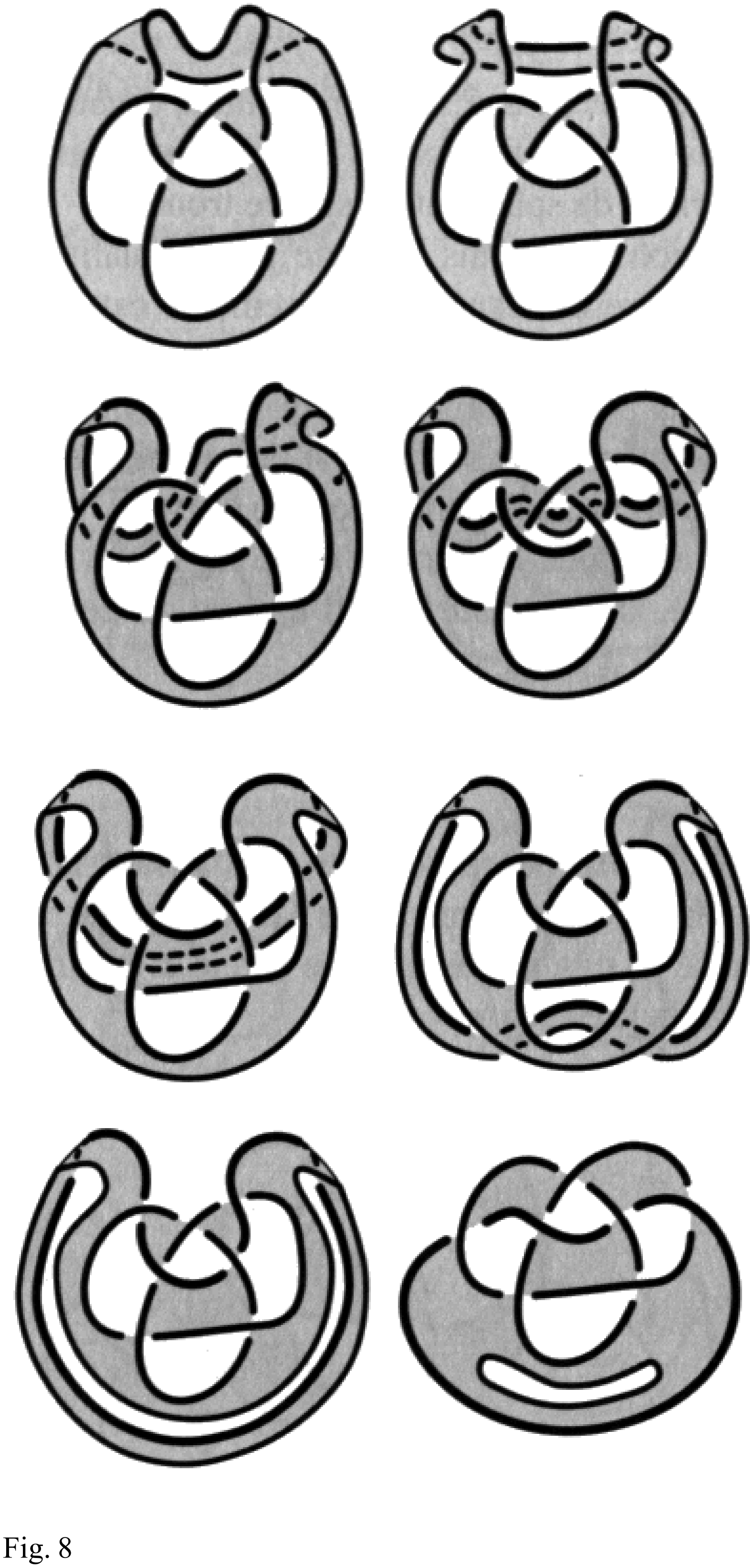


Fig. 8

The deformed surface has a hole that counts as an additional circle, distinct from the object.

We close this hole with a spherical disc to obtain the dual surface of the one we started with.

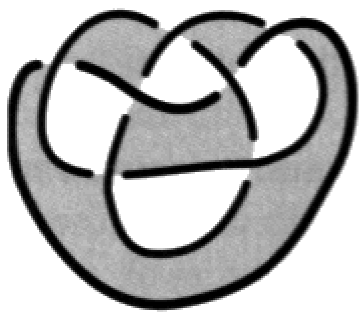


Fig. 9

The dual surface of a given span surface is a span surface modulo the question of this hole, which we will explain in terms of a sphere and a sphere with a hole.

The collection of solids obtained by duality, joined by the half-twists corresponding to the crossings, is indeed a span surface.

We previously defined the dual presentation of a given presentation. This surface is the span surface of the dual presentation of the given presentation.

We therefore know how to construct this surface from the dual presentation; it is the gusset surface of this dual presentation.

The duality of presentations takes its name from the duality of span surfaces. This notion comes from graphs, as we will see.

We can verify this by considering the dual surface bounded by a circle, as we have already encountered. By creating a continuity between this circle and a peripheral arc, we move to the dual presentation.

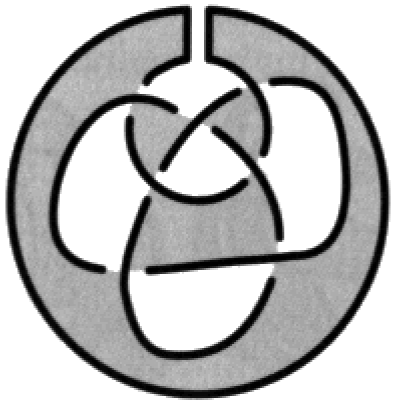


Fig. 10

This continuity is achieved by a transverse cut in the fabric in question, and this process replaces the need to close the hole in the surface with a spherical patch.

Now that the reader has begun to experience the difference introduced by an imaginary hole as a break in the surface, we can return to the problem posed by the degree of transition to the dual. We will find this hole again on the sphere of the flattened object.

2. **Regular isotopies on the sphere with or without a hole**

We formulated this curiosity in the case of the transition through the change in the presentation of a given flat diagram to its dual presentation. The transition from an object, which has n crossings, to its dual presentation on the plane, is a movement of degree n + 2, whereas on the surface of the sphere, the transition from the same object to its dual presentation is a movement of degree 0.

a1 - ***If the sphere has no holes***

We can explain the transition to the dual presentation:

Fig. 11

This transition is like a zero-degree movement that borrows the face of the sphere hidden from our view, as shown by the following change in presentation.

This deformation does not actually involve any intersection. a2 - ***If the***

sphere has a hole

Things appear as they do in the flattened plane.

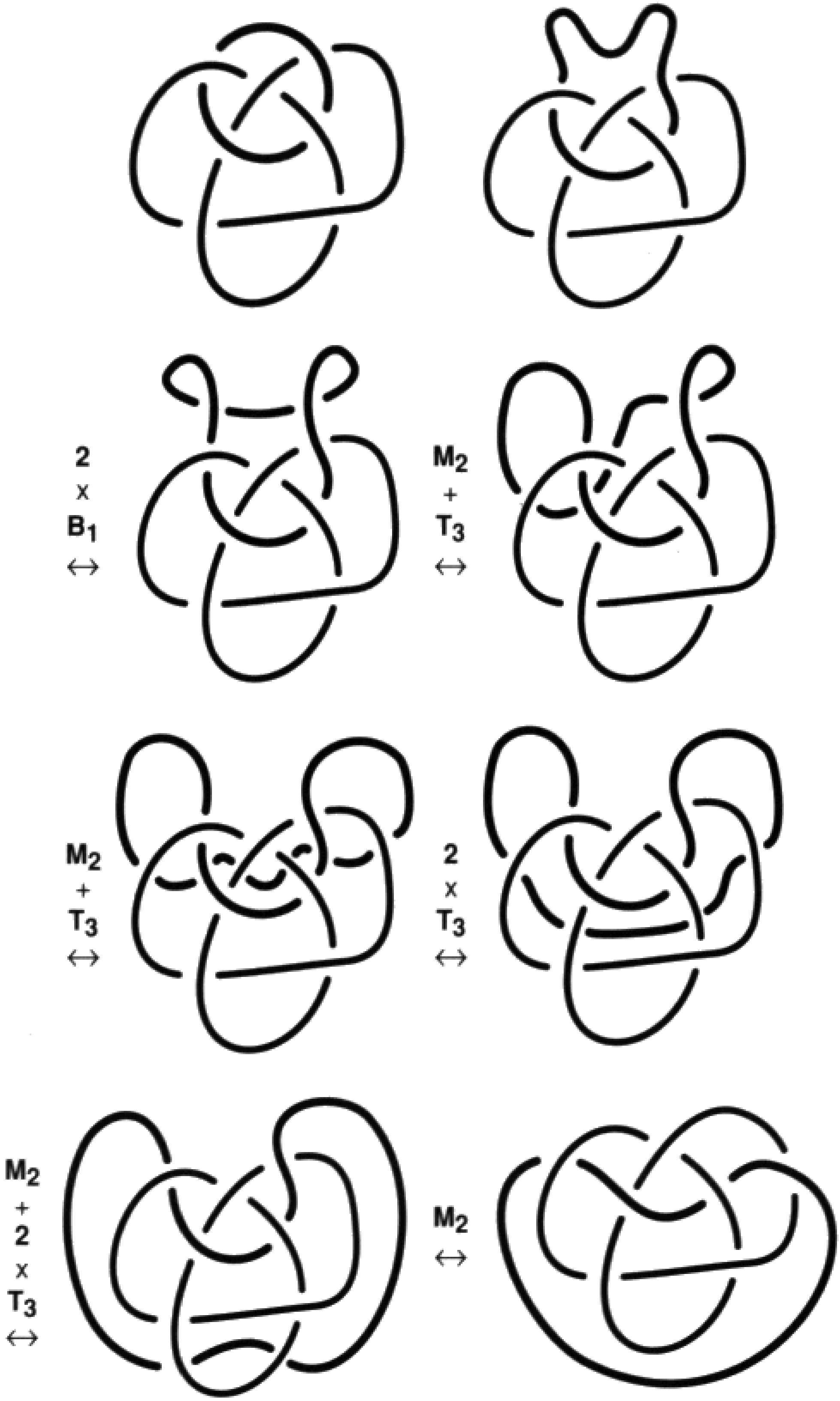

2 x B_1 ↔
M_2 + T_3 ↔
M_2 + T_3 ↔
2 x T_3 ↔
M_2 + 2 x T_3 ↔
M_2 ↔

Fig. 12

In the graphic register, we can explain the transition to the dual presentation as a series of T3 movements of degree 3, each composed of two B1 movements of degree 1 and a few M2 movements of degree 2, as shown by the following change in presentation.

Since the same crossings are used several times in as many T3s as there are crossings, and since two B1 movements must be performed, we will say that this change in presentation is of degree 3n + 2 or n + 2.

This method of counting is still crude, but we will improve it later in this chapter by specifying the orientation of the movements using the twist sign of each crossover.

3. **Graph of solids and graph of voids**

Let's stay with this relative imprecision for now to see that the notion of duality of span surfaces and presentations comes from the duality of graphs[3]. This translation is done using graphs defined on flat schemes based on the result obtained in the first step of our algorithmic series. We deduce immediately readable graphical properties, expressed in the form of elementary arithmetic formulas, thanks to results already encountered in relation to surfaces.

a1 - ***The duality of graphs***

A graph embedded in a surface has vertices and edges and determines faces. These faces must be disks whose portions of the graph are the edges.

These different elements, vertices, edges, and faces, are respectively of dimension zero, one, and two.

We define duality very generally in an ordered series, when we reverse the order of the series of dimensions.

0 1 2

2 1 0

Given a graph:

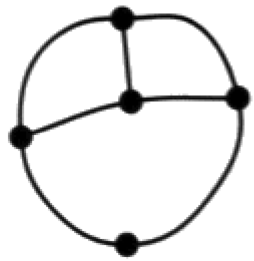


Fig. 13

The dual graph of a given graph is the graph consisting of the points that we can place in each face of the first graph; these are the vertices of the dual graph. The segments that join these vertices and cross the edges of the given graph are the edges of the dual graph.

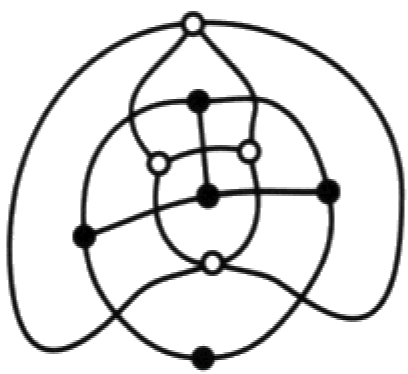


Fig. 14

If we isolate the graph constructed in this way, we can already see that we have indeed obtained a graph.

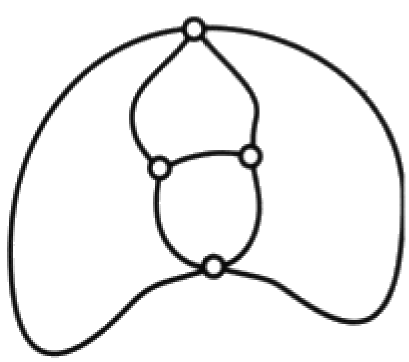


Fig. 15

This dual graph determines faces that correspond to the vertices of the initial graph. a2 -

Associated graphs

We consider a given presentation. In the first step of our algorithm, we determine a binary opposition pair of areas, the full and empty areas of the presentation.

By placing a point in each solid area, if we connect these points with edges that pass from one solid area to another at each intersection, we obtain the graph of the solid areas. It surrounds a void in the presentation in each of these faces.

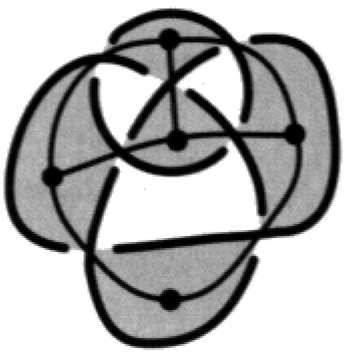


Fig. 16

The dual graph in the plane (or sphere) of the graph of the solids will be the graph of the voids.

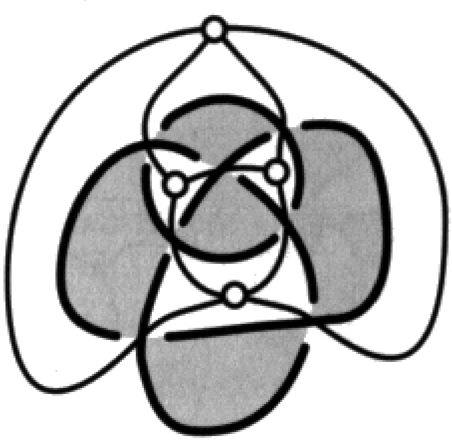


Fig. 17

It consists of vertices placed in each empty area, joined by edges passing through each intersection and crossing each edge of the graph of the full areas.

This is the graph constructed previously in the example we gave.

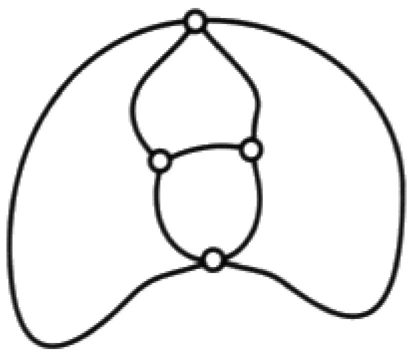


Fig. 18

This is not surprising since the full graph of this object was identical to the graph chosen to illustrate the definition of the dual graph.

Let us now give the graph of solids and the graph of voids for our general case. They are dual to each other.

Fig. 19

Here is the presentation of this general case. We can construct its full graph by placing a vertex in each area determined as full by our first algorithmic step.

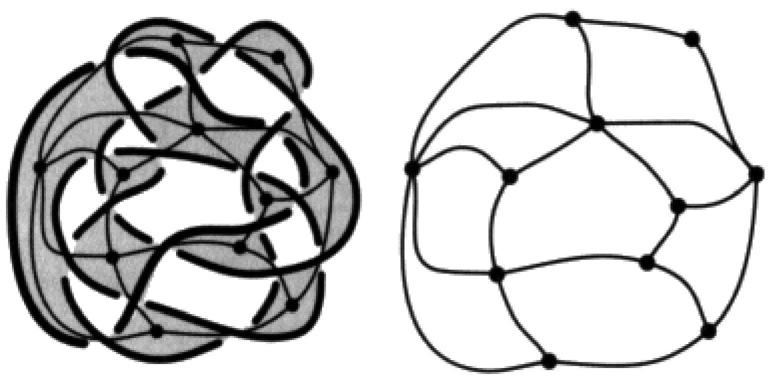


Fig. 20

It is obtained by joining its vertices with as many segments as there are arc segments in the given object, since each of its edges intersects an arc segment.

We proceed in the same way to obtain the void graph, placing the points in the areas deemed to be voids according to our algorithm.

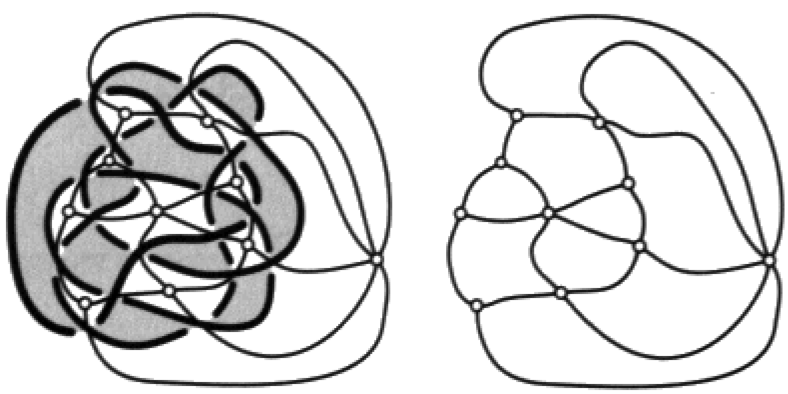


Fig. 21

We leave it to the reader to verify that these two graphs are indeed dual to each other.

a3 - ***The formulas***

These are results from surface theory that translate into literal regularities of flat nodal schemes when objects become literal. They apply to alternating presentations, or constructions (*universes*), with positive orientations in terms of torsion. It is interesting to compare them with the results already produced in the previous chapter regarding entanglements and cuts. They do not have the same scope.

The first relates to the spherical surface of the flattening already known to Descartes; it is the oldest result in this discipline and was demonstrated by Euler. It states in later and more condensed language that the Euler-Poincaré indicator[4] of the sphere is 2.

This indicator d is defined in a surface by means of a graph drawn in that surface, provided that the faces it delimits are disks (spheres with holes).

Its expression is written as the number of vertices of the graph, minus the number of edges, plus the number of faces:

δ = S – A + F

In the case of a sphere, $\delta = 2$, he gives the theorem that the number of vertices of the graph, minus the number of edges, plus the number of faces, equals two:

S – A + F = 2

Translated into terms of flat diagrams using the graph of solids, or using the graph of voids, this is irrelevant due to the median aspect of this expression with regard to duality; this result states that the number of solids, minus the number of intersections, plus the number of voids equals two:

P – C + V = 2

That is, by a slight conversion that makes the dual symmetry of solids and voids around the intersections legible, it is a proven fact that the sum of the number of solids and the number of voids is equal to the number of intersections increased by two units. Or, as they say, the sum of solids and voids equals intersections plus two.

This can be written as: **P + V = C + 2**, our most basic formula.

This explains what we can see in our example, as in any other case.

P = 11, V = 10, C = 19

Fig. 22

Provided, as always, that we do not forget to count the peripheral area, just like any other area, among the empty areas. Indeed:

11 + 10 = 19 + 2 = 21.

There are two other dual formulas that further specify a graphic property of our flat diagrams.

To demonstrate them, refer to exercise [5] in our previous work, concerning multitoruses presented as spheres with handles and tubes.

The first formula states that the sum of the number of blanks and the number of circles has the same parity as the number of the denomination. When the denomination is odd, this sum is odd; when this sum is even, the denomination is even. This sum is congruent modulo two to the number of the denomination.

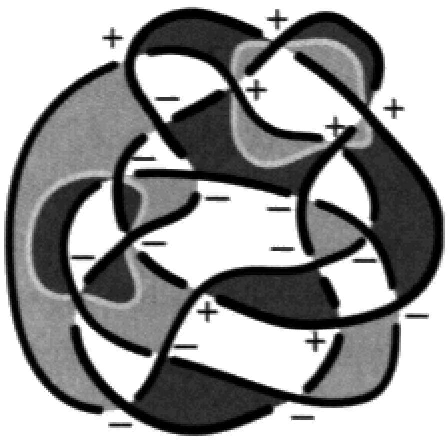
The second formula is the dual of the previous one: there is congruence modulo two between the sum of the number of solids and the number of hollows and the number of the dual break.

This result can be written in condensed form in the following two formulas:

V + R = kiS (mod 2)

P + R = k*iS (mod 2)

And can be read in the following figure:



$P = 11, V = 10, R = 3,$

$_{\mathrm{kiS}} = +4 - 3 = +1$ and $_{\mathrm{k*iS}} = +3 - 9 = -6$

Fig. 23

We verify that in this case, the sum of the number of blanks and the number of circles is odd, as is the number of cuts.

$V + R = 13 \cong 1,\ _{\mathrm{kiS}} \cong 1,$

The character that marks equivalence $\cong$ writes the congruence modulo two; for this relation, even numbers are equivalent to zero and odd numbers to one.

Similarly, the sum of the number of solids and the number of circles is even, as is the number of cuts in the dual surface.

$P + R = 14 \cong 0,\ _{\mathrm{k*iS}} \cong 0$

We will pause here, with these first elementary formulas of knot arithmetic, in our approach to duality. This notion will occupy us for a long time, but we must now address the question of theories that modify and extend the experience of deformations beyond the physical invariance of objects in three dimensions. We must now move forward into spaces of unknown dimension according to the same principle adopted in classical theory, simply telling ourselves that the deformation is stronger there.

II. Nodal plasticity

Here we address a first non-classical theory. It is specific to the Freudian field and begins, as we said in our first chapter, once we have given the definition of the movements of this theory. It therefore begins in the same place as classical theory with Reidemeister movements, but in a different way since it is determined by two non-classical movements.

Let's return to what we learned at the end of the previous chapter to introduce it before defining it.

1. **Lacing**

From the previous chapter, we already know the linking numbers of a chain whose loops are oriented, and we have learned how to calculate them. Their distribution provides us with a good analysis of the state of linking in the chain under consideration. In this regard, we have defined an equivalence relation **Renl** between oriented chains.

The number of chains Σi, for a given orientation, is the sum of these entanglement numbers of pairs of loops. It is an invariant of ambient isotopy and, a fortiori, of regular isotopy of oriented chains.

In the case of chain presentations whose circles are unoriented, we agree to associate them with the span surface (1st step of the algorithm) that leaves the space around the node empty. They are therefore necessarily oriented by torsion thanks to this convention.

We will refer to these chain presentations as uncolored chains, as long as coloring is not involved (2nd and 3rd steps of the algorithm).

Under these conditions, we will not take into account the presence or absence of the span surface, but we will consider uncolored chains as oriented by torsion.

We then considered, in the case of uncolored chains, the distribution of chain numbers according to changes in the orientation of its components. In this regard, we defined an equivalence relation **RΣ** between uncolored chains.

There is a correspondence between the distribution of chain numbers in the uncolored case and the distributions of entanglement numbers in each of the oriented cases. This correspondence is summarized in the following statement.

From entanglement to chain number

Two uncolored chains s_1 and s_2 will be RΣ-equivalent if and only if they are, as oriented chains, Renl-equivalent for each of their respective colorings related by **g**.

This proposition summarizes the last two statements relating to the properties of these relations in our previous chapter.

These considerations give us the simplest proof that chains are effective chains; they hold together and their components interlock in pairs.

On the other hand, we know of chain knots that hold just as well and whose chain numbers are zero for all orientations, as in the case of Borromean rings or Whitehead's chain. These chain knots, or improper knots, are chains with constant cuts.

Proper knots, made of a single component, always have zero chains and are therefore constant cut.

If we want to write a theory of knots and chains that preserves only the **Renl** relations between oriented chains and **the RΣ relations** between uncolored chains, and therefore respects only the state of entanglement, we can add regular homotopies and Gordian moves to the Reidemeister moves that generate the ambient isotopies of the classical theory **Tc** of knots and chains. We will immediately define these two types of additional moves.

Conversely, we are not certain that two Renl-equivalent oriented objects, or uncolored RΣ-equivalent **objects**, with the same entanglement state are identical in this theory. Proof is needed to be sure. This will give rise to a theorem.

2. **A theory of entanglements**

In this theory **T0** of entanglements, the set of movements leaves the entanglement state unchanged.

Here we move within the set **P** of presentations of chains and uncolored knots in terms of components, from which we choose any coloring when the calculations require it and take into account the set of colorings for the correct definition of the entanglement numbers.

We have the set **E0** of moves that are divided according to a set **TE0** of types:

TE0 = {**B1**, **B1***, **M2**, **T3**, **G**, **H**, H*}

Let us specify the types of transformations that respect the entanglement state within a chain, i.e., that preserve the relation **RΣ**.

a1 - ***Definitions of the types of movements in this theory***

1. We have Reidemeister-type moves to change the presentations.

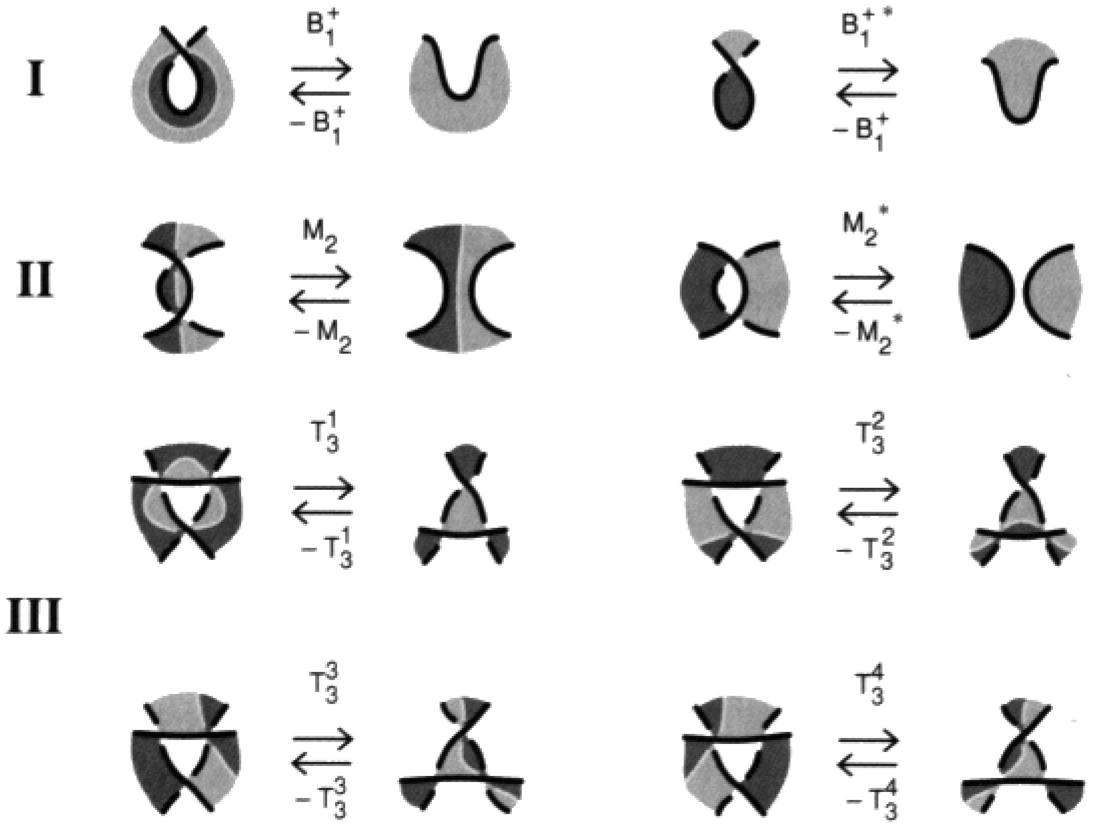


Fig. 24

2. Regular homotopy undoes proper knots and certain chains, those containing proper knots, by acting at each proper crossing by inverting the top and bottom.

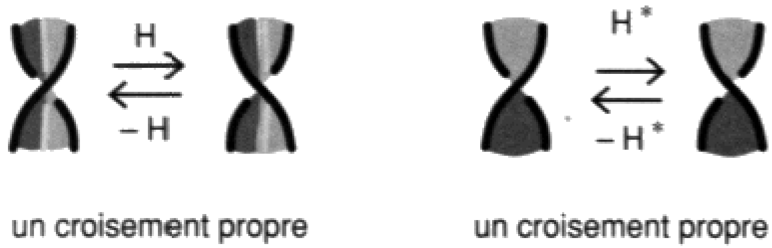


Fig. 25

We will call these regular homotopies proper Gordian movements.

3. Improper Gordian movements undo certain chains, those containing improper knots, by acting at the level of a pair of improper crossings that must involve the same two circles, have the same torsion, and have the same crossing signs.

opposites. The latter distinction is characterized by the presence of a cut for one and the absence of a cut for the other of the crossings that are unsuitable for any coloring.

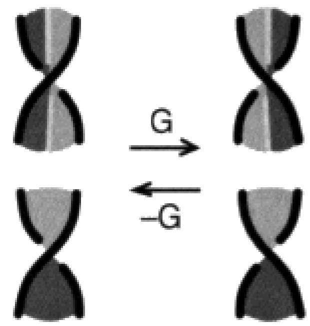


Fig. 26

This set **TE0** of transformation types defines the topology of nodes and chains with ambient Gordian knots, i.e., the theory of Enlacements.

A series of Gordian moves ***$*** is a transformation of the presentation S into the presentation S',

$*:** P → P, such that ***$ (S) = S'

defined by a series of movements $\mathbf{x_i}$,

($\mathbf{x_1}$, $\mathbf{x_2}$, …, $\mathbf{x_i}$, …, $\mathbf{x_n}$) with $\mathbf{x_i} \in$ **E0**

where each of the movements $\mathbf{x_i}$ is an elementary change of presentation taken from the types in the set **TE0**.

Thus: ***$*** (S) = $\mathbf{x_n}$ (… $\mathbf{x_i}$ (… $\mathbf{x_2}$ ($\mathbf{x_1}$ (S)))), where the bar above the letter designating this change of presentation, i.e., a series of movements, serves to remind us how much these movements cause the isolated cut in our drawings to drift, until it disappears momentarily. We will refer to this as the movement of the cut.

We then define, thanks to these changes in presentation, a relation $\mathbf{R_0}$ (s_1, s_2) on the set of node or chain presentations.

That is: $\mathbf{R_0}$ (s_1, s_2) ⇔ ∃ ***$*** (***$*** (s_1) = s_2).

This relationship is one of equivalence. We will sometimes denote it as S1 $\approx_0$ S2.

We will call the equivalence classes defined by this relation Enlacements. These classes of presentations constitute the objects of this theory.

The theory **T0** of Enlacements is indeed the theory of these equivalence classes, and it is easy to show, by careful calculation of the crossing signs of any orientation, when dealing with improper crossings, that these movements respect the numbers of enlacements, and therefore the relation **RΣ**.

Identical objects in this theory have the same linking.

For two presentations of chains or knots s_1 and s_2:

R0 $(s_1, s_2) \Rightarrow$ **RΣ** (s_1, s_2)

Reidemeister moves do not change the sum of the signs of improper crossings. This is because only move **B1** removes or adds a crossing, but it is proper.

Homotopies act only on proper crossings, so the number of entanglements is irrelevant.

Gordian knots act exclusively on improper crossings; they are designed not to change the number of intertwining of a pair of components. These movements reverse the twist sign and the crossing sign of two crossings involving two identical rounds with the same twist and opposite crossing signs. Their action on the crossing signs therefore does not change the total number of interlacing, i.e., the number of chains, because a +1 sign becomes -1 and a -1 sign becomes +1.

a2 - ***Effectuations***

Let us give two examples of these transformations and this equivalence relation. We propose to carry them out in the most general case we have chosen so far, using the rings of the Borromean family.

Let's return to the general case we have been studying since the beginning:

Fig. 27

and let's break down its knot using these movements[6]:

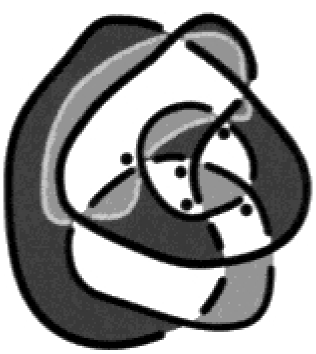
G

T_3

$2M_2$

M_2

Tş



S

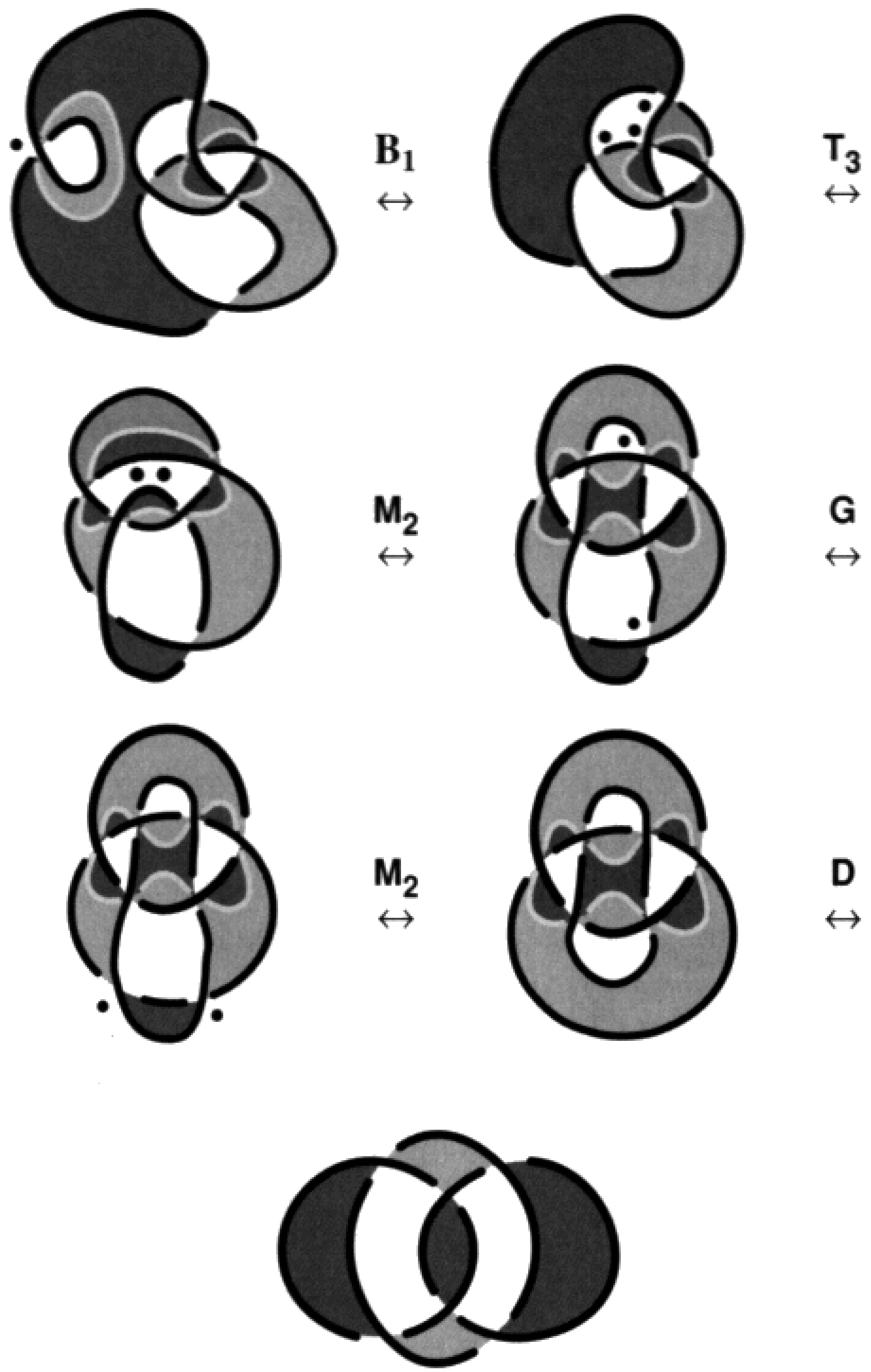


Fig. 28

This change in presentation $ is defined by the series of movements:

(**G**, **T3**, 2 x **M2**, **M2**, **T3, S, B1, T3, M2, G, M2, D**)

where S stands for crossing over a circle forming a junction, making five junctions, i.e. (**M2**, 5 x **T3**, **M2**), and **D** stands for duality in this case, i.e.:

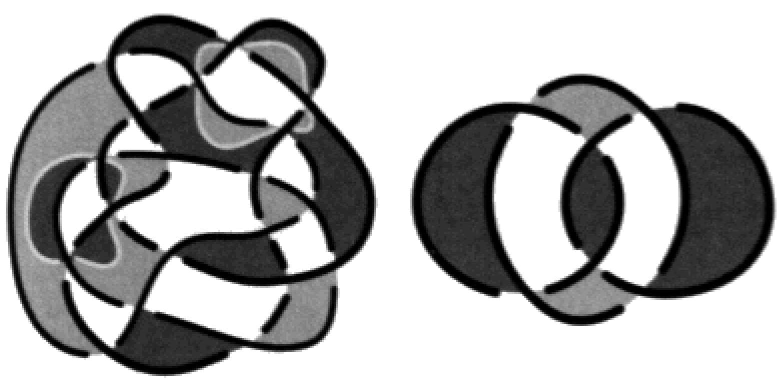
(2 x **M2,** 2 x **T3,** 3 x **M2,** 4 x **T3,** 2 x **M2,** 2 x **B1**)

that is:

(**G, T3,** 2 x **M2, M2, T3, M2,** 5 x **T3, M2, B1, T3, M2, G, M2,**

2 x **M2, 2 x T3,** 3 x **M2,** 4 x **T3,** 2 x **M2,** 2 x **B1**)

In the case of the chosen coloring, let's compare its chain number to that of the result obtained.



Σi = 1/2 (– 9 + 5 – 2) = –3□Σi = 1/2 (– 6) = –3

Fig. 29

The Borromean rings

We undo the Borromean knot by performing a series of movements consisting of a Gordian movement **G**, followed by a Reidemeister **T3** movement, then three **M2** knots.

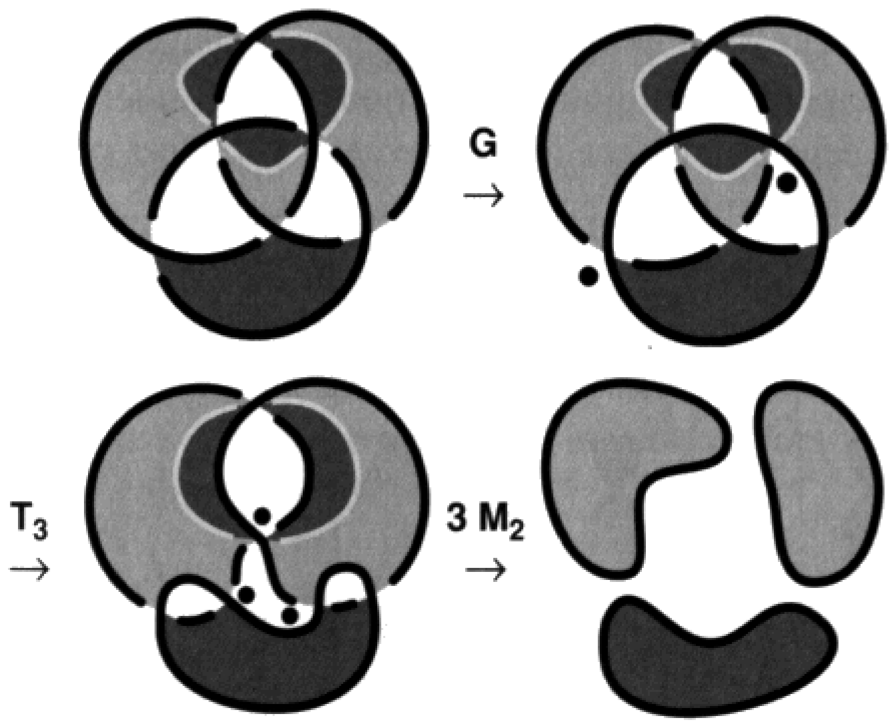


Fig. 30

Let us consider the series: (**G**, **T3**, **M2**, **M2**, **M2**), where it can be verified that the Borromean rings do not contain any intertwining.

a3 - ***Gordian knots with different twists***

Let us define a type of movement that is composed of two previously encountered movements. The non-alternating Gordian movement acts at the level of a pair of improper crossings that must involve the same two circles and have opposite torsion signs and opposite crossing signs. This latter distinction is characterized by the fact that the chosen crossings with different torsions are of the same type with respect to the cut. Either it is present in both crossings, or it is absent.

We will index them by their torsion sign.

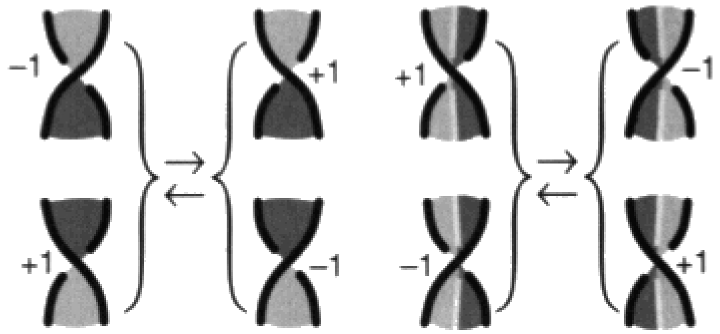


Fig. 31

Let us show how these movements between non-alternating crossings are composed of our generating movements.

In the case of a pair of crossings involving two identical circles, we can always create a link between these two circles using movement **M2**. This produces two new crossings in relation to each other.

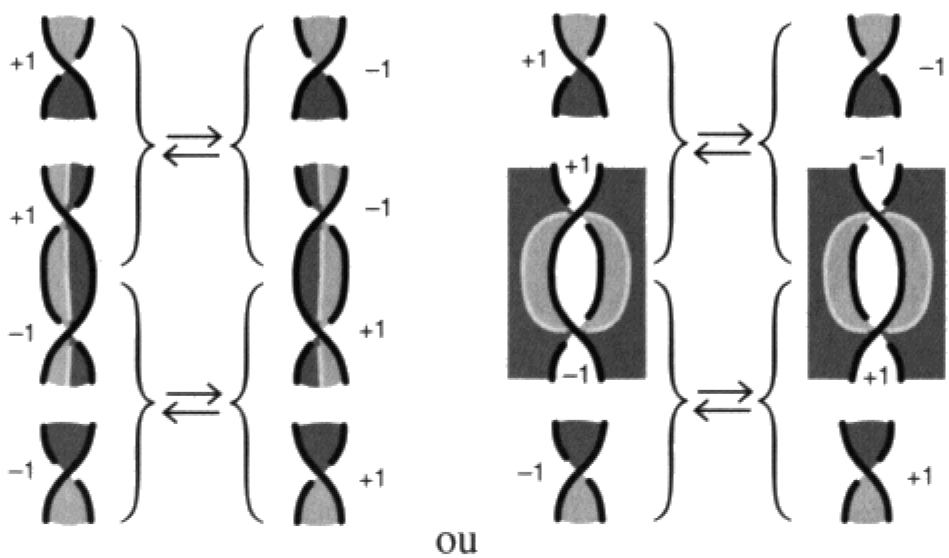


Fig. 32

However, it is necessary here to produce crossings of a different type relative to the given cut, which can always be achieved by suitably choosing the arcs that produce a new mesh and adding a loop if necessary.

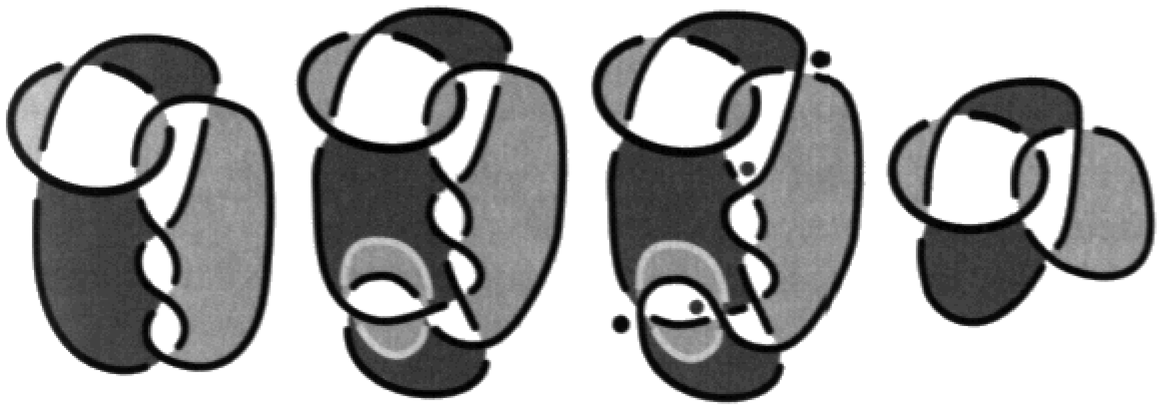


*Fig. 33

If our initial crossings have inverse twist numbers, they can be paired with the new crossings. In this case, we can perform two alternating Gordian knots defined above. We then simply reduce the mesh by movement **M2** to obtain the desired result.

We use these non-alternating Gordian knots in the

following. a4 - ***The different interlacings***

At the beginning of this chapter, we said that we did not have proof of the reciprocal implication that two chains with the same analysis of the entanglement state, RΣ-equivalent, would be identical in entanglement theory.

This raises the question of whether, for two chains s_1 and s_2 **that** are RΣ-equivalent, there is always a series ***$*** of movements between them such that:

S2 = (S1)

Answering this question in the affirmative proves the following theorem.

Two equal enlacements are identical objects

If two chains s_1 and s_2 are RΣ-equivalent, then they are R0-equivalent: **RΣ** (s_1,

s_2) ⇒ **R0** (s_1, s_2)

Proof

Let us define a composition mode, denoted #, between chains such that their entanglement numbers or chain numbers add up.

This composition mode consists of connecting the span surfaces by as many bilateral ribbon immersions as there are chain components. These ribbons must respect the bijections **f** and g of our relations **Renl** and **RΣ** and the coloring of each of the composed surfaces.

It may be necessary to twist these ribbons in order to satisfy this last condition.

This composition also connects the respective components of each chain, respecting their chosen orientation.

The use of bilateral ribbons ensures that we do not create entanglements, as their edges are traversed in both directions. Consequently, where their edges enter, they also exit.

Let us use this method of composition based on two S_1 and S_2 chains, which are RΣ-equivalent, choosing an example to aid in understanding.

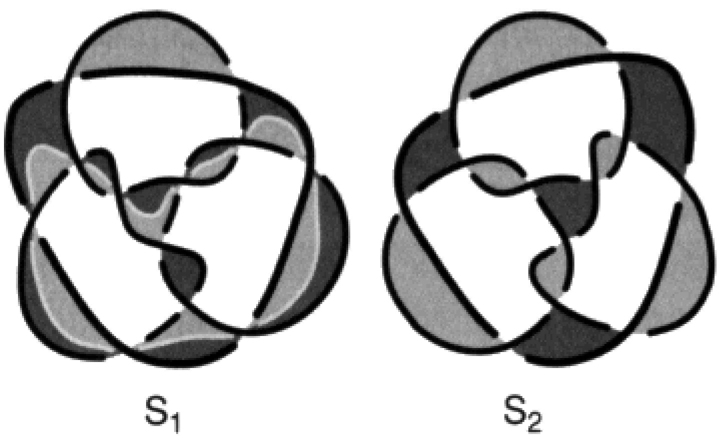
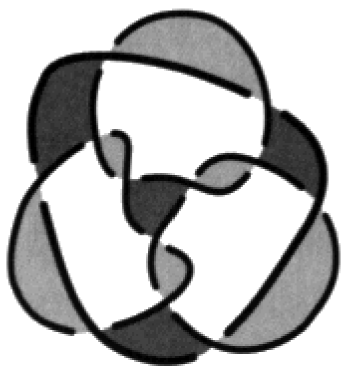


Fig. 34

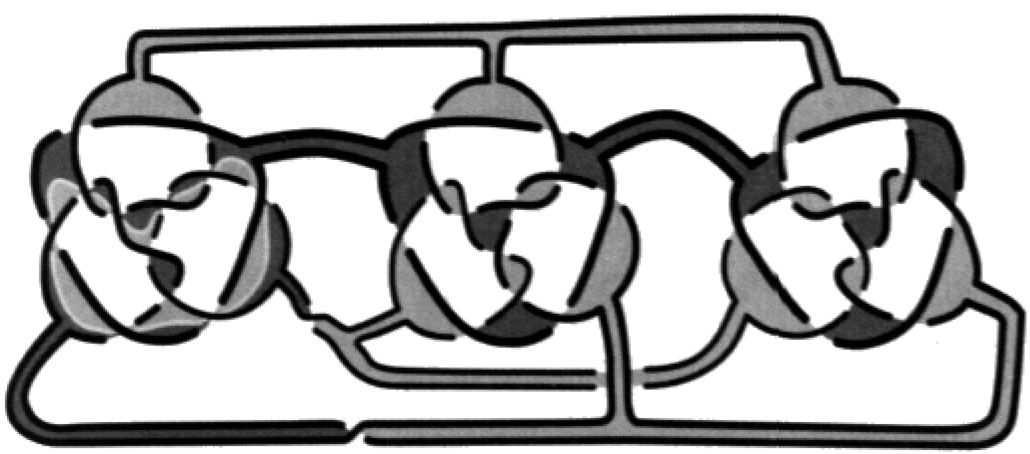
Let's call S_2^{-1} the presentation obtained by inverting all the crossings of S_2.



S_2^{-1}

Fig. 35

And we construct the compound $S_1 \# S_2^{-1} \# S_2$ as $(S_1 \# S_2^{-1}) \# S_2$ or as $S_1 \# (S_2^{-1} \# S_2)$:



S_1 S_2^{-1} S_2

Fig. 36

There is a series of Gordian moves between this compound and S_1 on the one hand, and between this compound and S_2 on the other.

This compound differs from S_1 by the chain $S_2^{-1} \# S_2$, and from S_2 by the chain $S_1 \# S_2^{-1}$.

These two compounds are indeed knots, since their linking numbers are zero for all orientations. Indeed, the composition by # adds the linking numbers for each respective orientation, and S_1 and S_2 have the same distribution of linking numbers

, which are simultaneously opposite to the entanglement numbers of s_2^{-1} by construction.

Knot chains have no entanglement

A chain knot can always be reduced, by a series of movements from the **T_0** theory, to a trivial chain made up of scattered rings.

We demonstrate this by noting that in a chain knot we can always stack the loops by acting with alternating Gordian knots and Gordian knots of different torsion at the height of the improper crossings.

It suffices to note that in a chain where Σ_i is always zero, by definition, its improper crossings can be paired, either by pairs of elements with the same twists and different colorings, or by pairs of elements with opposite twists and the same colorings. They therefore exhibit
therefore exhibit symmetry between them, without remainder since there is no entanglement.

Let's verify this fact in the chosen example by reducing the chain s_1 # S_2^{-1} to a trivial chain:

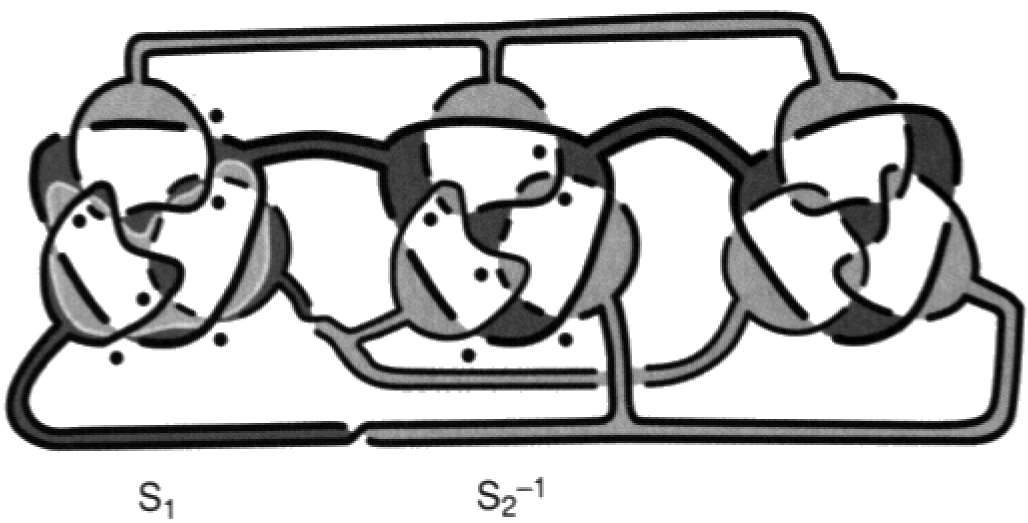


Fig. 37

Let's reduce the chain s_2^{-1} # s_2 to a trivial chain:

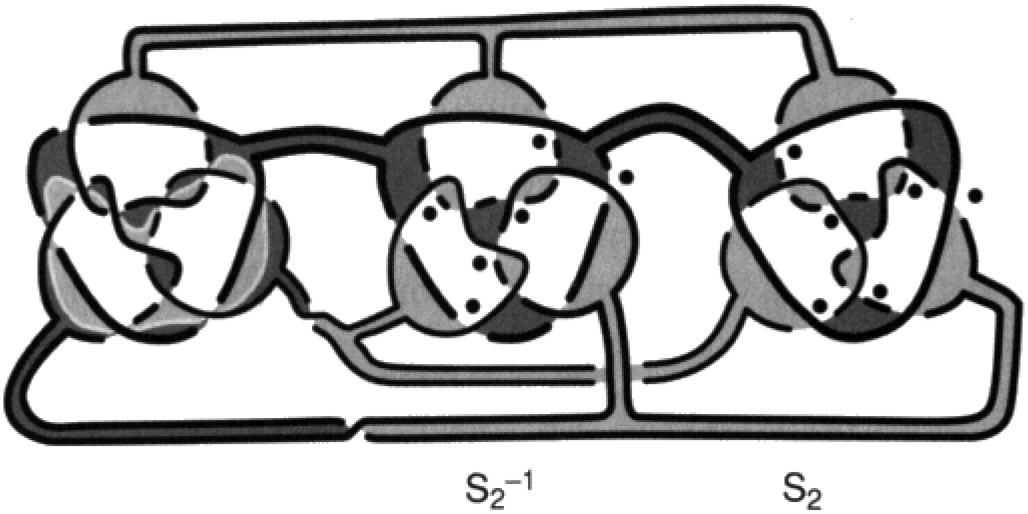


Fig. 38

By doing this when the circles are stacked, we can use Reidemeister moves to disperse the circles that can still form proper knots, if necessary.

In this case, we will need to undo the proper knots by homotopies to prove our lemma.

Thanks to homotopy, we already have the following result.

Proper knots are trivial

Proper knots form a single class for the relation **R0**.

This second point is easy to prove using homotopy. Any proper knot can be reduced to a stack of loops. This mesh is equivalent, using Reidemeister moves, to the trivial proper knot consisting of a simple embedding of the circle in space.

This completes our proof, because:

$s1 \# (S12^{-1} \# s2)\square \approx_{\mathbf{0}} s1 \# 0 \approx_{\mathbf{0}} s1 (s1 \#$

$s2^{-1}) \# s2 \approx_{\mathbf{0}} 0 \# s2 \approx_{\mathbf{0}} s2$

The verification consists of counting improper crossings, but can always be constructed in the case of any two objects.

Due to the equivalence between the relations **RΣ** and **R0**, this theory effectively groups RΣ-equivalent entanglement states into stable equivalence classes for **R0**. In each class, we can choose an exemplary case that condenses its entanglement state and represents that class. This choice allows us to begin interpreting, using these exemplary cases, the average number of cuts ΣS in our main theorem.

We will continue along this path in the following chapters, but not without first providing the means to compare these results with the graphical data of our objects.

6. **Gordian and homotopy**

We now have two extreme and opposing theories. The classical theory of knots and chains **TC,** which deals with physical objects that we can make with string, and the theory of entanglements **T0,** which deals with the easiest chains to imagine, since the rings pass through the holes of other rings, and easy to write, since the state of the entanglements can be presented as a table of linear combinations and calculated on the drawing. This theory reduces all objects in classical theory to these.

We will summarize this situation with the following diagram:

T0

|

TC

and it is by refining this with other alternative theories that we will attempt to clarify our progress. These theories give rise to groupings of objects characterized by specific nodal structures, such as proper knots, entanglements, Whitehead chains, Borromean chains, and reductions, rendered by theorems specific to each. We will develop some of these in studies ancillary to this series of works according to their effect on the overall structure, in order to orient ourselves within the multitude of objects.

a1- ***Lacan and Soury***

We can begin by situating Lacan and Soury's respective statements as their two respective theories.

Those who attended Lacan's seminar still do not know what he was trying to show during the 1970s. Our work answers this question, despite the diagnosis often made by most, without having looked closely, of a search for totality.

This is politely presented, thanks to a literary reference, as a search for the absolute. However, Lacan does not seek the absolute for the simple reason that, for him, absolute means

means separate, and that when it comes to detachment, others have already said all there is to say about what can be achieved.

Children who produce a transitional object make a discovery of the absolute that the Western adults around them cannot fit into their categories.

Lacan points to the place of a theory of chains and knots with improper Gordian movements, which we will refer to by the acronym TG, since it performs this type of movement. It is true that he uses it to account for the transition from four to three. We will return to this question in our last chapter.

For Soury, things are more straightforward. Soury studies chains with homotopies, which he also calls Borromean chains, in a theory that we will refer to by the acronym TH. We know, having heard him say so, his main argument in this investigation. He reasoned as follows:

— Lacan, studying knots, speaks insistently of the Borromean knot.

— Milnor [29. a and b], studying these same objects, encounters the Borromean knot, which he calls almost trivial.

— Is it for the same reasons?

As a result, Soury focuses on Milnor's work and embarks on calculations of noncommutative groups and Magnus's calculations. He seeks to make Milnor's number of an object more accessible through drawing to those who do not practice algebra.

These different theories, in particular TH theory, are of additional interest to us because they model in three dimensions what happens in four dimensions. Or, if you prefer, they deal with objects immersed in three dimensions, which remains a fairly good mental picture of what happens in four dimensions.

It is mainly this question of dimensions that will concern us. At the end of the following chapter, we will show how a structural break can be detected, with regard to non-knots, between three and four dimensions. This depends on the number of entanglement states.

It is clear to anyone who works with these topological spaces that, to date, there is no construction of space or dimensions that provides a continuous model for TG theory. The fact that this theory is not topological in three dimensions does not rule out the possibility of a topological model in other dimensions. Or else it must be demonstrated.

We can draw up a graph of the relationships between these different theories, which gives the following diagram that we will enrich as we progress.

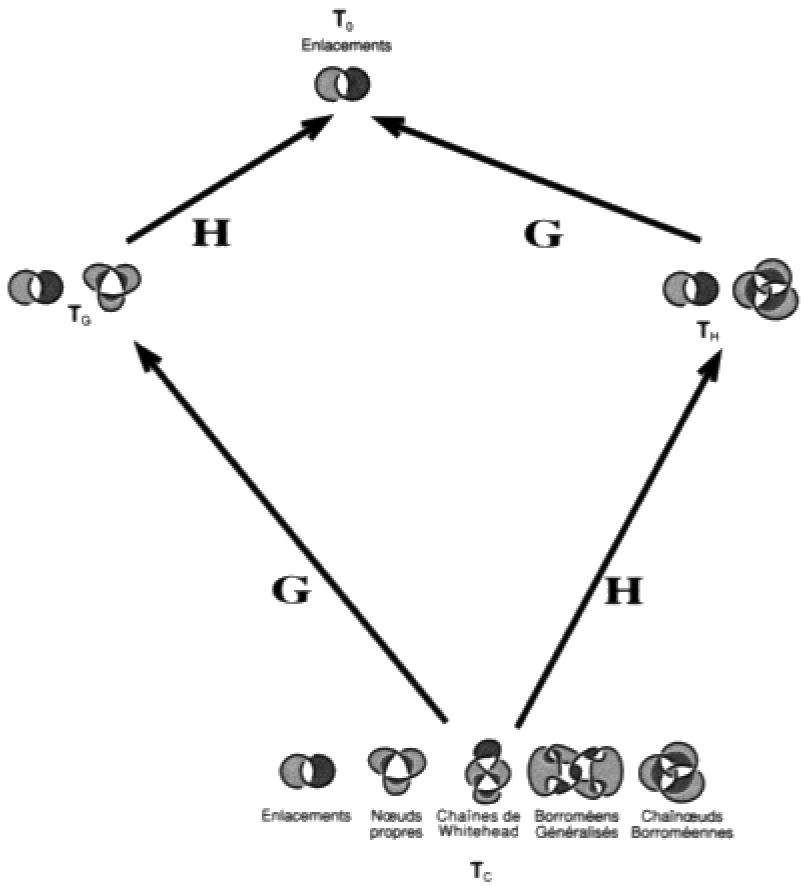


Fig. 39

The two theories, **TG** and **TH,** deserve to be considered separately, and the relationship between them needs to be established.

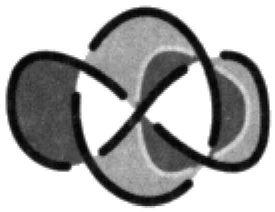
This study will establish the relationship between improper Gordian movement and homotopy (proper Gordian movement).

This will be an opportunity to address the relationship between proper, a single circle, and improper, multiple circles.

We will introduce this study of the relationship between the one and the many with a well-defined relationship, known as homology, at the end of this book.

However, Soury identifies a difficulty by highlighting the existence of Borromean rings that are undone by homotopies, but—and this is what characterizes him—homotopies performed on different circles. This property is therefore difficult to see in group calculations. In addition, he presents an example that has an alternating presentation made up exclusively of improper crossings.

Note that there are objects that are no longer visible from these two theories. This means that they can be undone by both Gordian cuts and homotopies, such as Whitehead's chain.



La chaîne de Whitehead

Fig. 40

a2- ***The generalized Borromean knot***

I intervened in this debate with the Borromean knot, which Lacan said was the generalized Borromean knot.

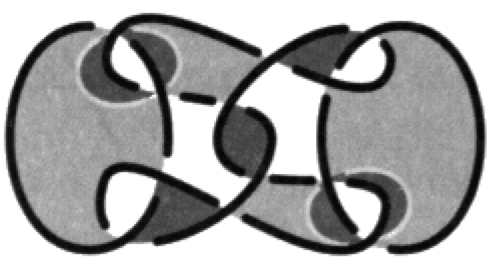


Fig. 41

The function of this knot condenses the complications that theories based on Gordian movements, whether proper or improper, are blind to. Let us therefore introduce an intermediate theory based on generalized Borromean and Whitehead.

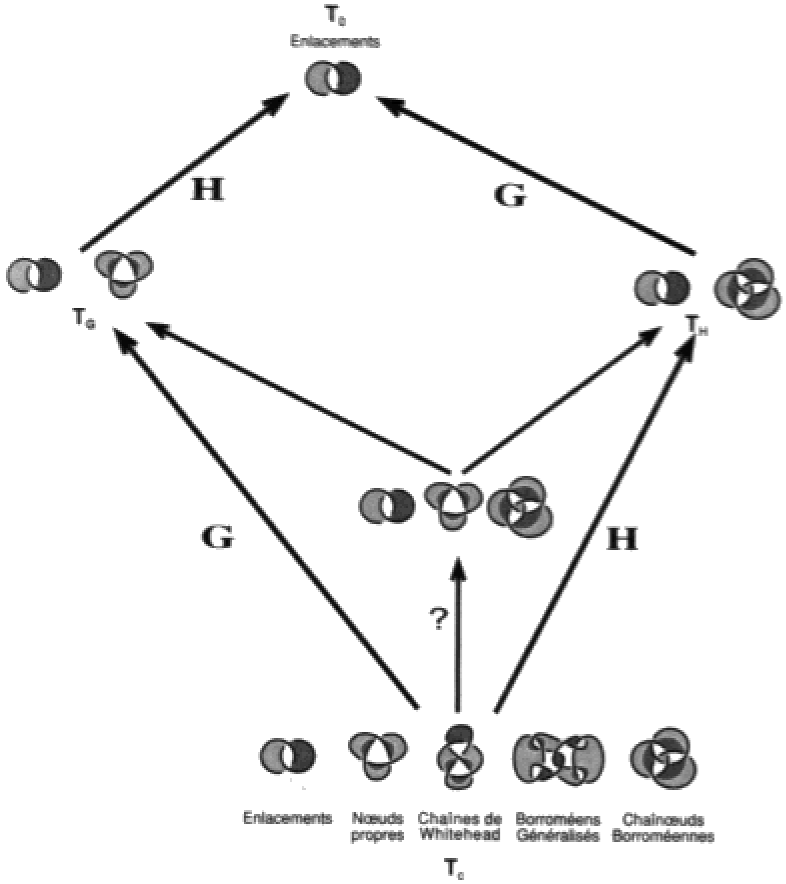


Fig. 42

At this point, we do not know what kind of movements could be used to define it. We will only define it in chapter seven.

We will therefore only deploy the network of these different theories in the last chapter of this book.

But first, we will draw other conclusions from the comparison of the two theories **T0** and **TC**, through the analysis of the average number of cuts ΣS.

7. **Exercises**

e1 - **Change of presentation and dual presentation**

1. Change the presentation of the following objects until they are reversed, if possible.

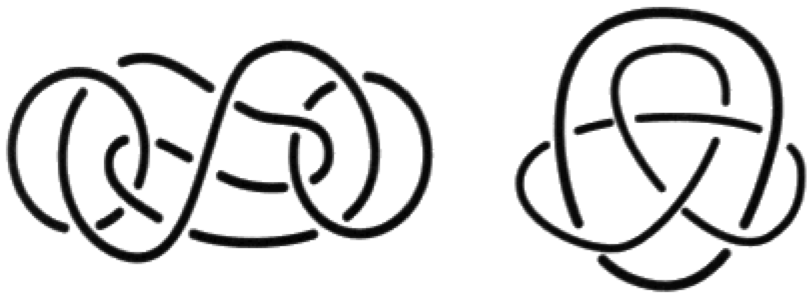


Fig. a

Show that the following knot can be re-alternated as in the second drawing.

Fig. b

2. These two presentations are dual to each other:

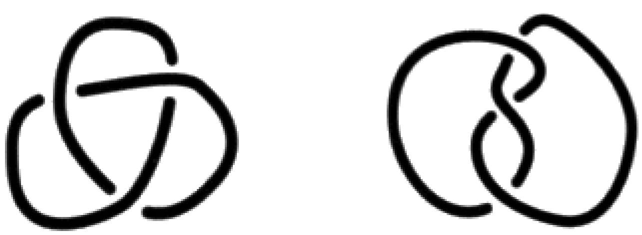


Fig. c

We move from the first to the second by turning an arc on the sphere, or by placing it in continuity with a circle that surrounds it.

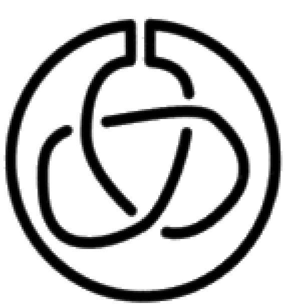


Fig. d

Deform the presentation above to obtain the second figure of the clover.

3. Check by switching to the dual representation that the Listing knot drawn here is autodual (although inverted):

Fig. e

4. Note the regular duality of the following objects

Fig. f

e2 - **Dual graphs of solids and voids**

Draw the graph of the voids and the graph of the solids for the following objects.

Fig. g

Check that they are mutually dual to each other. e3 -

Calculating elementary formulas

Check the elementary formulas for the following cases:

$P + V = C + 2$ $\quad$ $V + R = k_{iS} \pmod 2$ $\quad$ $P + R = k^*_{iS} \pmod 2$

Fig. h

e4 - **State of entanglement**

Using Gordian moves and homotopy, reduce the following chains to their linking state, in the form of a non-knot.

Fig. i

Chapter V

The material and sources of the knot

The first question we asked ourselves, after analyzing the node chosen as an example and determining that it was a cut, was whether this was a general characteristic. During this interpretation work, other questions came to mind. Now that the first point has been clarified, we can address these issues, even if it means losing sight for a moment of the reason for the cut, the study of which is by no means complete.

We know, thanks to our interpretive work, that we can discover in knots a knotted part and a non-knotted part. We must hasten to reexamine one by one the various problems posed by the knot and thereby seek to resolve enigmas and contradictions which, as long as we knew only the knot in its entirety, seemed insoluble to attempts at classification.

In our first chapter, we presented the mathematical results of authors who have studied the knot in terms of the relationship between the knot and the space that gives rise to it, without questioning the origin of the knot's material or the composition of that material. In doing so, we will answer the question of what causes the complexity in knots and chains.

I. Graphical description

Let us specify here three particularities of the description of objects, never observed before us, not yet explained:

1 — Knots and chains have two parts, called knotting and non-knotting, which may themselves be made up of several components.

2 — Knots and chains choose the components of the knotting part from among pure knots, and those of the non-knotting part from among non-knots.

3 — Knots and chains arrange their parts according to a very specific composition method among the various possible composition methods. We will refer to this as regular and define this type of regular assembly.

These particularities in the choice of object elements were, of course, drawn from observations made on the different parts of the knot.

In this chapter, we will study the source of these parts taken separately. We will study the regularity of the composition of its parts in the following chapter.

0. **The first node**

Fig. 1

The first primary knot [2, Sém XX, p. 111], the Trefoil knot, is a knot in the mathematical sense of the term, since it is made by tying a single loop. For us, in our way of speaking, it is a proper knot.

a1 - ***The concept of the first knot***

Let us now define the method of composing nodes that leads to the concept of the primary node[1].

We can connect two nodes by joining their respective span surfaces with a single ribbon.

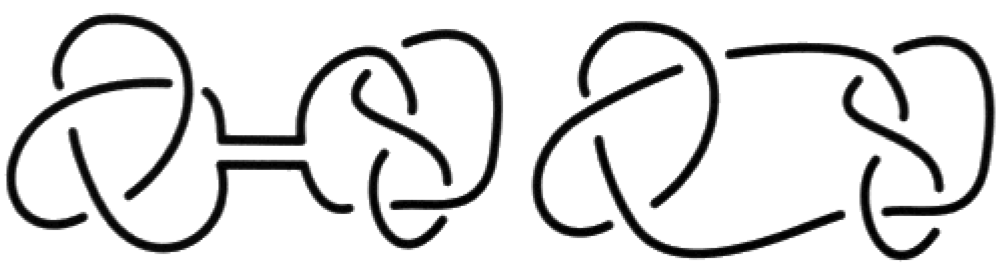


Fig. 2

From the point of view of knot theory in its relation to surface theory, knots made with string are homologous to knots on the edge of a surface.

This reference to surface theory is a convenience, in order to define things clearly. In this way, from another point of view, the knots are arranged on a string that forms a circle. An indefinite number of knots can be composed in this way.

Fig. 3

If we close the circle by joining the two ends of this figure, we see that we have knots arranged in circles distributed along the same chord.

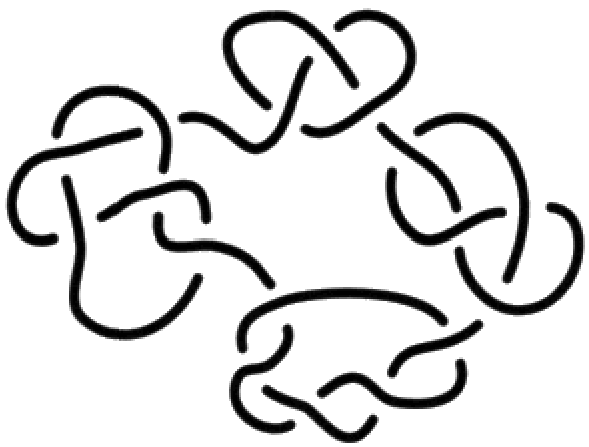


Fig. 4

The general form of this composition consists of joining the knots by means of a continuity junction. For reasons of convenience, this junction can be designed as a junction of ribbons connecting their respective span surfaces.

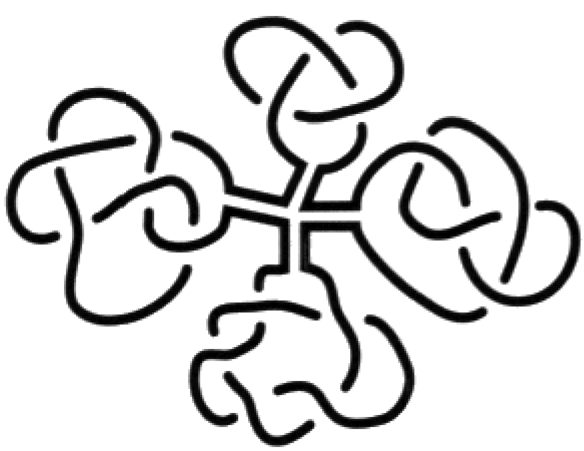


Fig. 5

With this mode of composition, we can speak of the decomposition of an object into elementary knots. These are the prime knots.

a2 - ***The monoid of primary nodes***

This method of composition can be misleading, giving the impression of a certain relevance due to the fact that it presents a structure that has a name in algebra, namely the structure of a commutative unitary monoid, but this algebraic structure is fairly rudimentary. It is associative in the sense that the composition of two nodes, itself composed with a third, is equivalent to the composition of the first with the composite of the other two.

This is easy to write, in such a way that the associativity can be seen in the use of parentheses:

(N1 # N2) # N3 = N1 # (N2 # N3)

whereas it is not apparent in the presentation of the three nodes on a string.

It is more remarkable to note that this mode of composition is commutative. The nodes can effectively be moved in their order along the string that joins them in a circle. The reader can experiment with this by constructing a composite of nodes and trying to change their circular order. They must be passed through each other, which is always possible.

This commutative monoid has a neutral element since the simple circle, also known as the trivial knot, can be composed with other knots. The result of the composition does not change the initial compound.

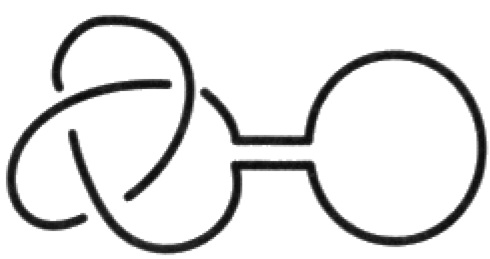


Fig. 6

However, there is no inverse element, which is why we refer to a monoid structure and not a group.

The prime nodes are those that remain indecomposable into simpler elements according to this mode of concatenation.

This point of view establishes an analogy with prime numbers that cannot be decomposed by multiplication, among positive integers, which themselves have a monoid structure when composed by both addition and multiplication.

This mode of composition does not take our colorings into account. It is contemporary with the first step of our algorithm and is restricted to introducing only one ribbon for each node.

In Chapter VII, we will ask ourselves whether this mode of concatenation can be generalized by using several ribbons between each node, especially in cases where we have several circles in a knot.

The node tables list the prime nodes and prime chains defined in this way using this approximate and algebraically rather amorphous mode of composition. We have seen internally within each object that a richer arithmetic is already at work, and that externally, the composition of the intertwining also already allows for more developed calculation.

For now, with our graphical description, we continue to specify the internal properties of our objects.

1. **Cut nodes and combs**

a1 - ***The trefoil knot***

This first node, known as the cloverleaf node, is a specific toric node in that it is produced by a regular path on the torus[2].

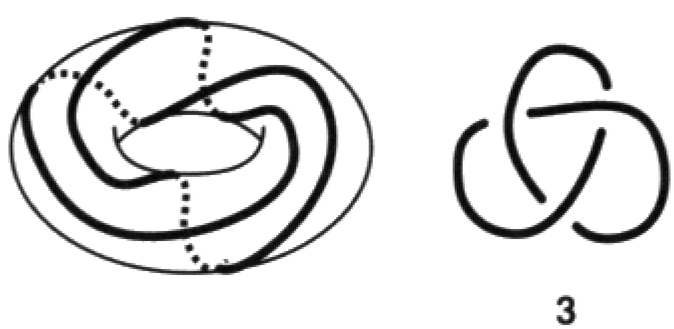


Fig. 7

Before studying these various flattened representations, we emphasize this initial toric representation. It justifies the name given to it. It is the flattened knot that has the smallest number of crossings. Note that the smallest knot can only be achieved with a minimum of three crossings. Given the first step of our algorithm, this presentation corresponds to the smallest number of voids. Its minimum span surface here is a Möbius strip with three half-twists([3].

There are two cloverleaf knots in space that are inverse to each other in the sense that they are inverted upside down at the level of the crossings. These two trefoil knots are not superimposable, or to put it another way, there is no series of Reidemeister moves that can transform the flattening of one of them into the flattening of the other.

This knot can be presented as the edge of a bilateral surface in another presentation, which we will return to below.

a2 - ***The family of trefoil knots***

This cloverleaf knot initiates a series of alternating knots:

Fig. 8

These are toric knots that always make two longitudinal turns and only two, with any odd number of meridian turns[4]. We count them among the clover knots. These are cut knots because they are qualified by our colorings with an odd cut.

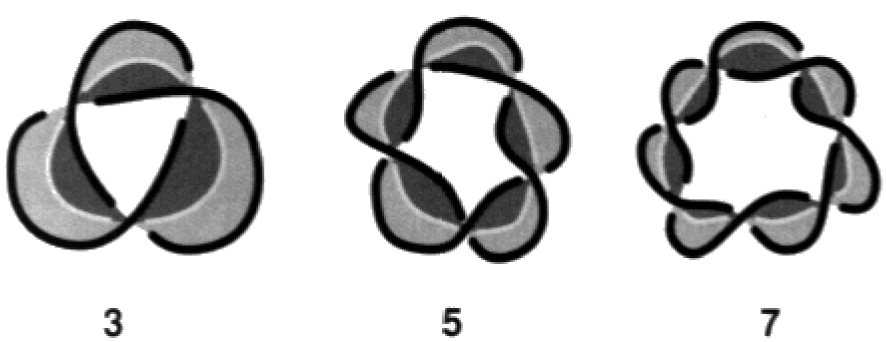


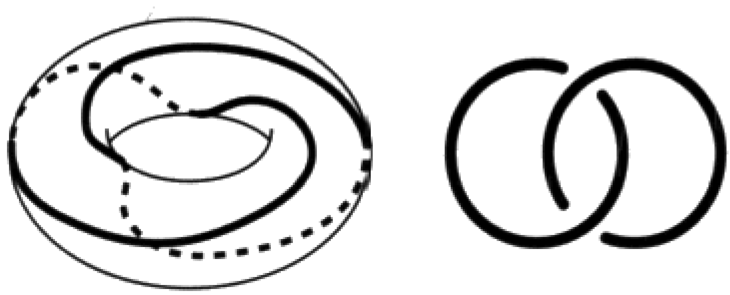
Fig. 9

These knots are part of the cut knots. Cut knots are alternating knots in which all crossings are crossed by the cut. These cut knots will be used in the composition of regular assemblies that will allow us to define and describe any knot.

a3 - ***Interlacing***

There is a configuration with only two crossings. In this case, it is not a proper knot but a chain made of two loops.

There is a simple interlacing with two crossings, which is a toric chain.



2

Fig. 10

In this presentation of the entanglement with the smallest number of crossings, we can stretch a surface of span using the first step of our algorithm. There are then two solids and two voids.

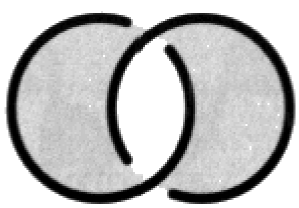


Fig. 11

The second step of the algorithm indicates that this is a non-knot because there is a coloring that does not require cutting. The minimum surface area is an orientable ribbon with two half-twists. It is a sphere with two holes and twisted by a full twist.

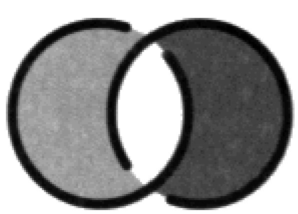
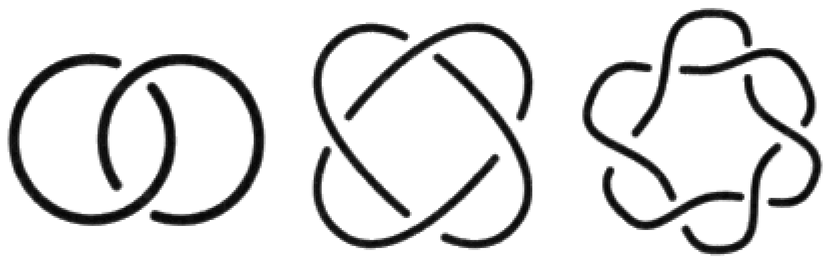


Fig. 12

Our aim is to contrast within the structure what is a knot, of which the trefoil knot is the simplest expression, and the non-knot, of which the simple entanglement is the smallest example.

a4 - ***The family of toric interlacings***

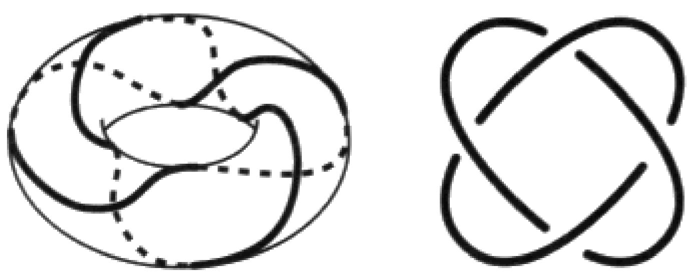
This first case develops in the elementary linkages of two circles that are chains.



2 4 6

Fig. 13

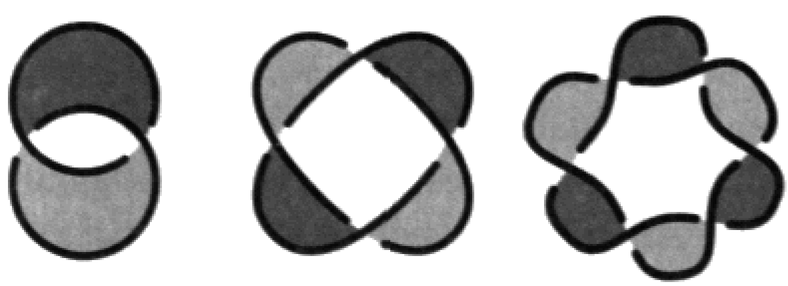
They complete two longitudinal turns and only two, and an even number of meridian turns on the torus.



4

Fig. 14

These are non-knots since they can be colored without interruption.

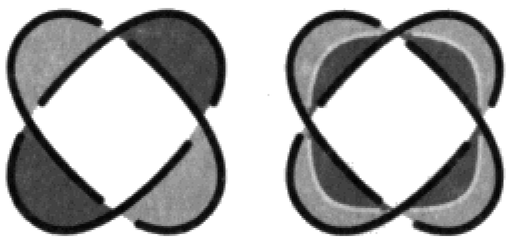


2 4 6

Fig. 15

These toric chains are so named because they consist of two circles due to the even number of meridian turns on the torus. Since there are several circles, there are therefore several colorings.

By changing the choice of colors on one of the circles, the two-color coloring of such a continuous interlacing will present a break.

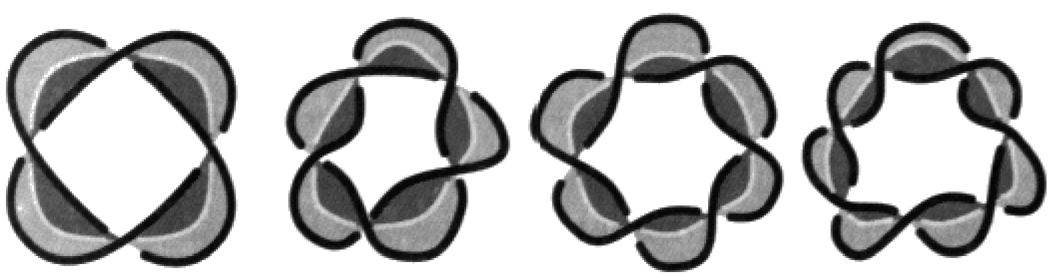


4 Fig. 16

These toric chains with an even number of crossings will serve as prototypes in regular assemblies in the case of breaks passing through an even number of twists. We will use them as such in the definition and description of arbitrary knots in the following chapter. This observation does not contradict our definition of non-knots, since to be considered as such among alternating objects, it is sufficient that there be a coloring without breaks.

a5 - ***Cut knots***

We call the toric knots we have just discussed, which make only two turns of longitude, cut knots when their coloring requires a cut.



4 5 6 7 Fig. 17

a6 - ***Combs***

The dual presentation, defined in Chapter IV, of any cut node of the clover family or the entanglement family we have just considered, are combs, odd or even.

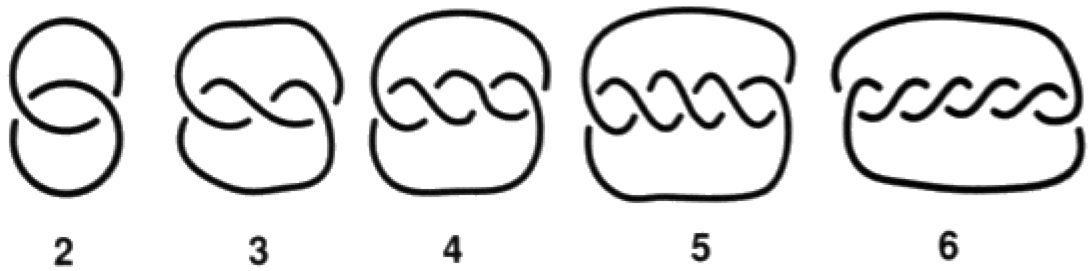


Fig. 18

This dual presentation is obtained by inverting a peripheral arc of our previous toric presentations.

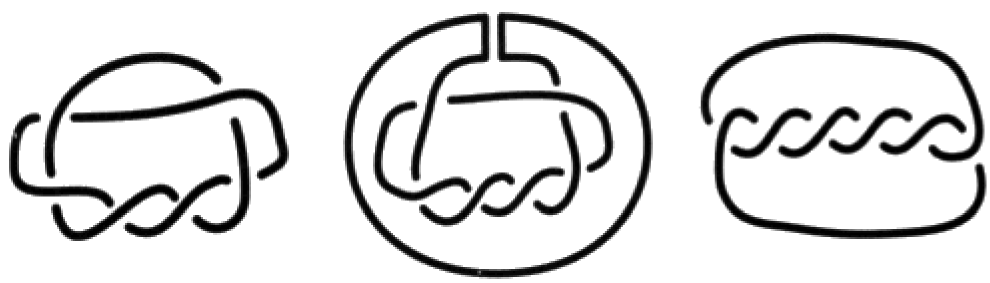


Fig. 19

We call these dual presentations of cut nodes, from the family of entanglements and the family of clovers, combs. These combs are always susceptible to cut-free coloring. This fact does not contradict our distinction between trefoils as cut nodes and enlacements as non-nodes, because these definitions take into account the minimum span area of each object, thus presentations with the smallest number of voids, which is no longer the case for dual presentations.

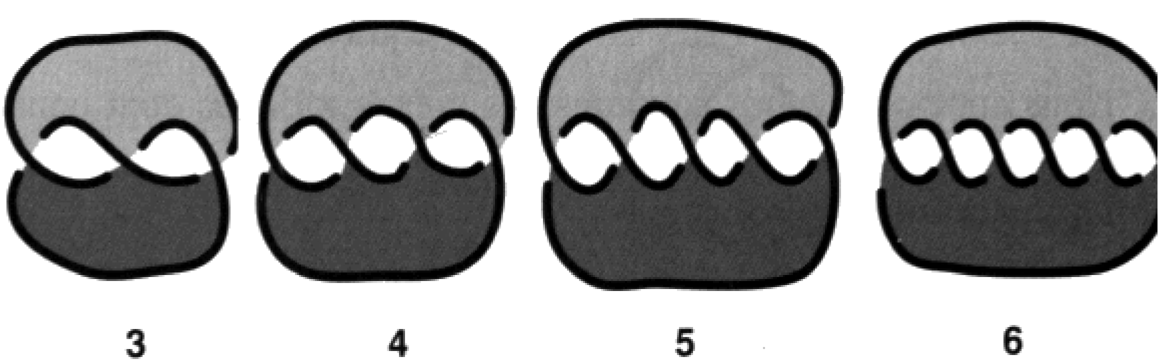


Fig. 20

Odd combs have a knot from the clover family as their edge. There is therefore only one cut-free coloring because it is a clean knot.

On the other hand, even combs have an edge that is a loop, which is a chain of two circles. There is therefore another coloring, this time with a cut.

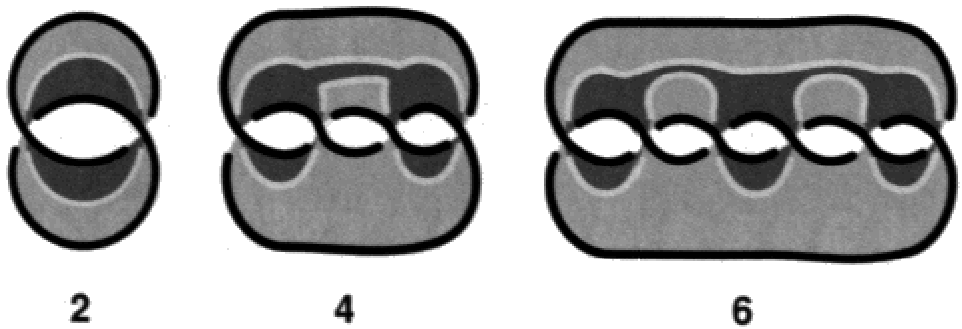


Fig. 21

Even and odd combs are obtained by braiding two strands. These are all two-strand braids that are closed to form chains and knots. These even and odd combs will allow us to produce any alternating non-knot.

2. **Non-knots and pure knots**

a1 - ***Cross sections***

We now extend the description of objects by defining the process that allows us to obtain, respectively, from combs, the multiplicity of non-knots, and from cut knots, the multiplicity of pure knots.

There are two types of cross sections, which are dual to each other: cross cuts in non-knots and cross straps in pure knots.

The existence of these cross-sections causes the real difficulty of knot theory, as we announced at the beginning. Without them, there would only be braids, and knot and chain theory could be reduced to braid theory.

Braiding theory can be transcribed into algebraic group theory using a very simple process. This result, due to Artin, resolves the question of their classification.

Transverse cuts

Let's return to combs. These are alternating objects, with two solid parts and capable of two-color coloring without cuts.

If we make transverse cuts in the span of a comb, we obtain a non-knot. A transverse cut is a cut that can be made with scissors in the span. A transverse cut always goes from one void to another void.

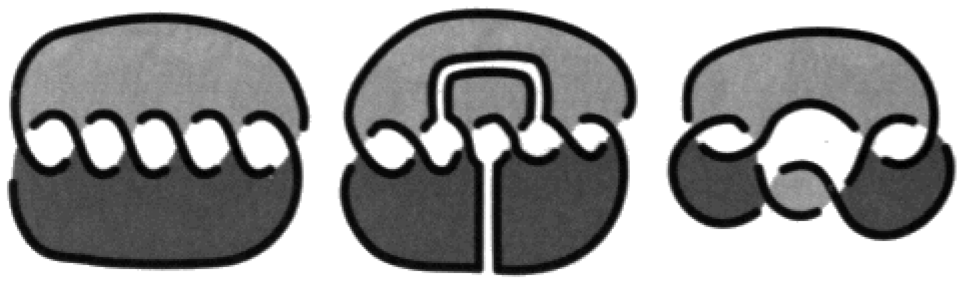


Fig. 22

Transverse cuts of combs produce non-knots.

It is noteworthy that even toric (even) non-knots, which produce combs by duality, can be obtained by transverse sections of combs. However, it is important to note that the twist of the crossings is reversed in duality, whereas it remains unchanged in this process.

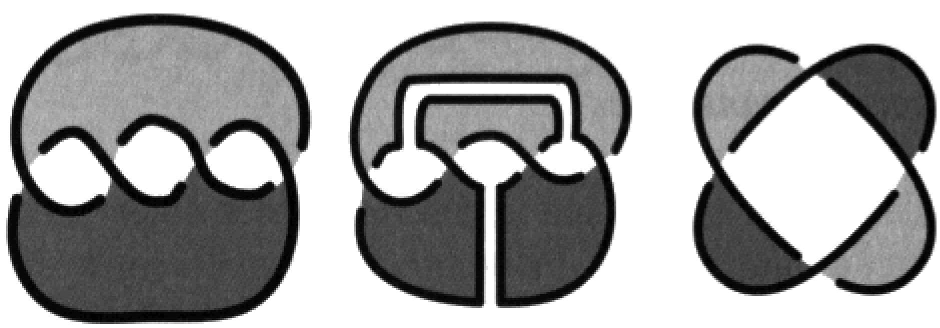


Fig. 23

The important result to remember is that, conversely, any non-knot is always the product of cross-sections in a comb. But we will have to prove this.

Note that the transverse cuts never cross the cut that isolates the knots, since we make them exclusively in the non-knots.

Transverse straps

The dual concept of the transverse cuts that we make in non-knots is the transverse strap, which is a ribbon stretched across a gap in a pure knot.

A set of straps added to a cut node produces a pure node.

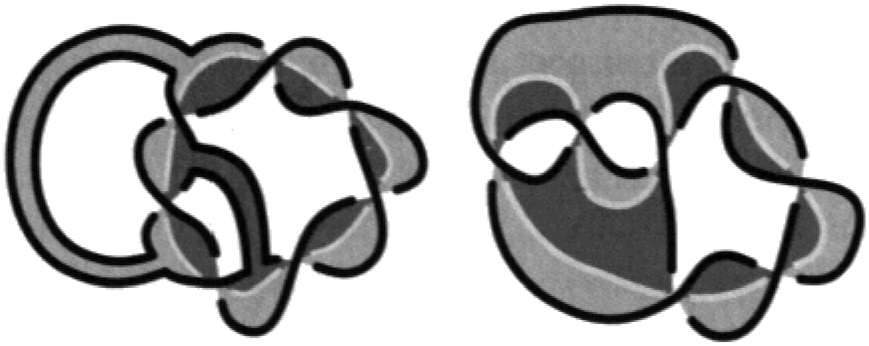


Fig. 24

Note that, unlike transverse cuts, which always respect the coloring if they do not encounter the cut, the ribbons placed across the gaps in the pure knots must respect the coloring at the point where they attach to the surface.

We demonstrate by duality the same result as that which defines non-knots as always obtainable by transverse cuts in combs. This result then states that a pure knot can always be obtained by a set of transverse straps added to a cut knot.

a2 - ***Non-knots and pure knots***

If a set of transverse cuts in a comb always yields a non-knot, we are not yet certain that a non-knot is always obtained from a comb by a set of transverse cuts. We will now demonstrate the following result.

Any non-knot is always obtained from a comb by various sets of transverse cuts.

We will prove this in the opposite way to what we have done so far, starting from any non-knot and showing that it always gives rise to a comb thanks to transverse ribbons.

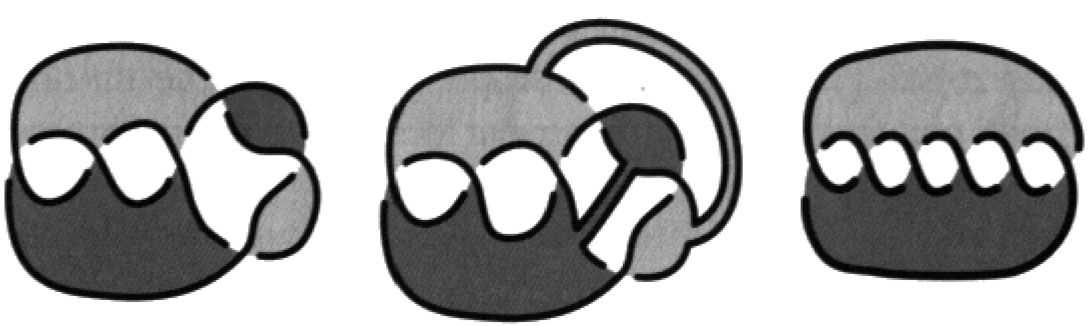


Fig. 25

A non-knot, apart from being alternating, has a set of voids that are always of even valence, as assured by the principle deduced from the second step of our algorithm.

In the dual presentation of a non-knot crossing, without a cut, a cut occurs if we do not change the orientation of the circles corresponding to the coloring.

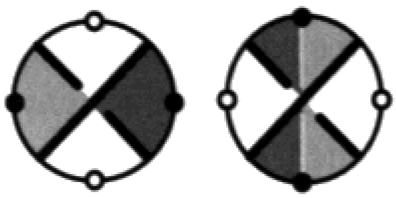


Fig. 26

This means that a cut crossing is the dual of a non-cut crossing.

But since the empty areas of a non-node have even valence, we can connect these different crossings of the dual presentation in such a way that the crossing forms a circle.

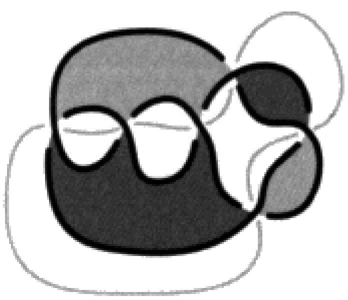


Fig. 27

In fact, the cut entering an area of even valence can always exit[5] this area.

Finally, it is remarkable that the cut of the dual presentation of a non-node closes in a circle that delimits two areas in the plane, one of which contains and supports all the areas of the same color as the starting non-node. All areas of the other color are located on the other side of the cut, in the other area delimited by the circle formed by the cut.

Therefore, based on our algorithm that establishes the parity of the empty areas of an object that appears as a non-node, we can always use monochrome ribbons without twists that do not encounter the cut in question to join the areas of the same color and transform them into a single area. The same applies to the areas of the other color.

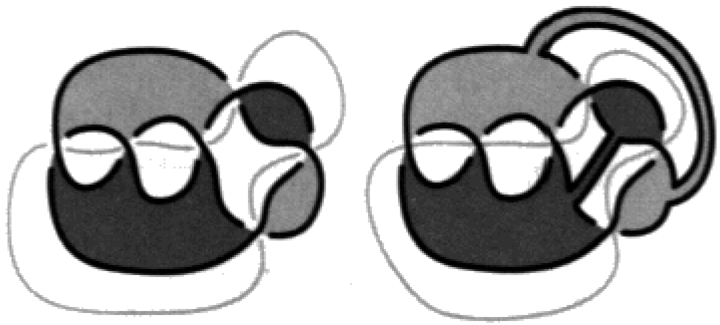


Fig. 28

These ribbons, which join the monochrome areas on either side of the cut circle in the dual, do not cross, do not interfere with each other, and do not obstruct each other because they transform these areas of even valence into areas of valence two.

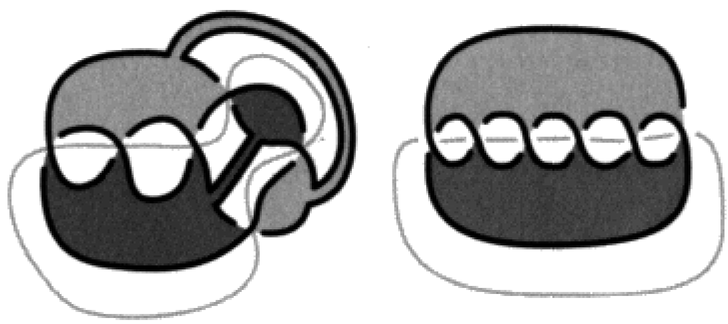


Fig. 29

We can conclude that it is always possible to reconstruct a comb that has a surface area consisting exclusively of two solids and two valence voids, with a number of crossings equal to that of the given non-knot.

We have therefore demonstrated the result we announced.

We perform the same transformation in a richer case, i.e., one that is more complicated in appearance but has the same simplicity because it has the same structural property.

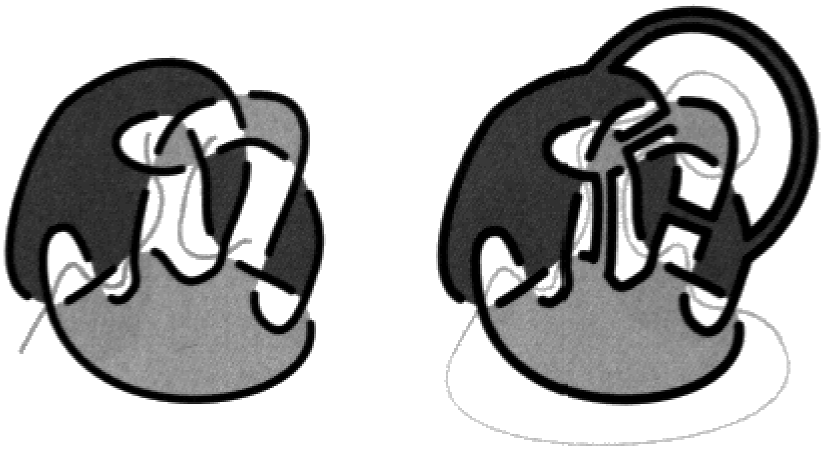


Fig. 30

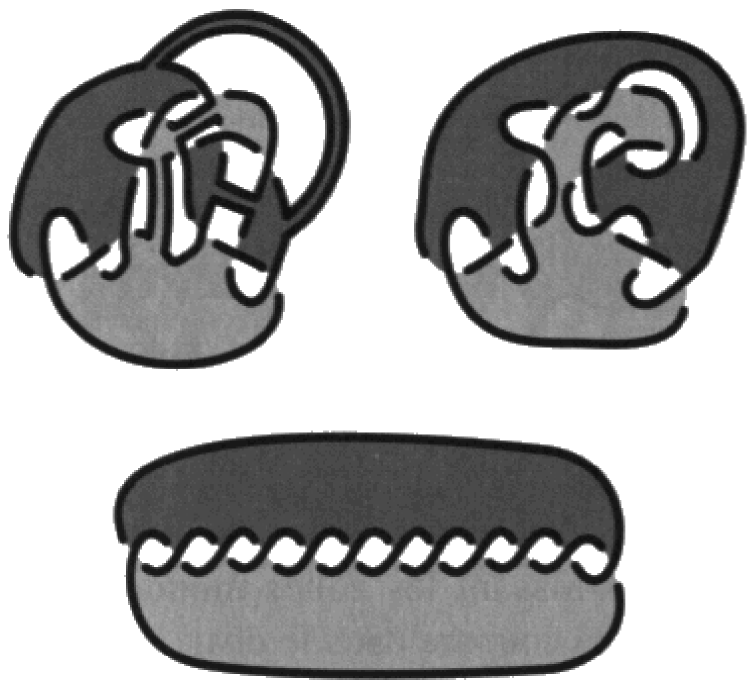


Fig. 31

This structural property becomes even more apparent, to the point of seeming trivial, if we perform the inverse transformation, producing a non-node from a comb in the presence of the cut in the dual.

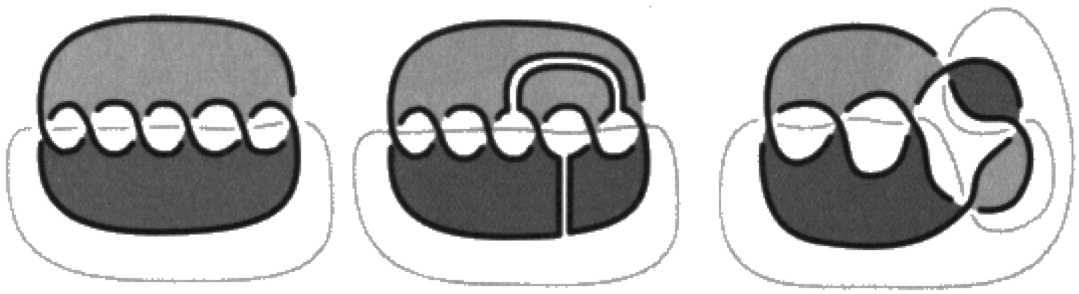


Fig. 32

The reader will note, however, that we have not performed the transverse cuts that exactly cancel out the added straps in the presentation of the inverse transformation in Figures 25, 28, and 29. As a result, the cut traced in the dual of the non-knot in question does not traverse the voids along the same path. This shows that it is necessary to perform a calculation on the drawings that is not trivial. If we want to obtain the previous path of the cut, we must cut the comb in a different way.

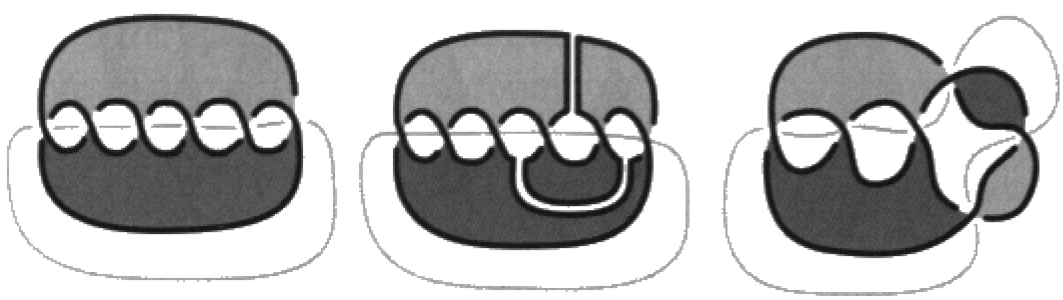


Fig. 33

The dual result is as follows: any pure node can always be obtained by transverse ribbons from a cut node.

In fact, a pure node is an object such that the cut passes through all the crossings.

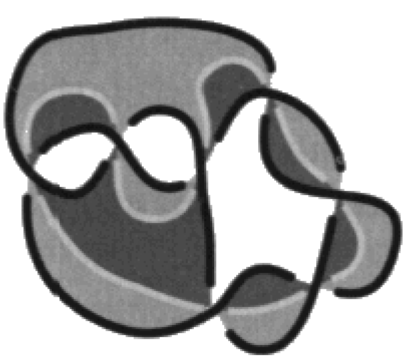


Fig. 34

We can always reconstruct a cut node from a pure node by as many transverse cuts as necessary that do not encounter the cut.

Fig. 35

These transverse cuts are simply transverse ribbons added to the dual non-knot, as in the previous proof.

Thus, the dual result announced, any pure knot can always be obtained by transverse ribbons from a cut knot.

Fig. 36

Now that we know how to describe all non-knots and pure knots from combs and cut knots, thanks to cross sections, we will be able to show in the next chapter how any alternating objects we want to theorize about are obtained from these elements through a regular mode of composition. All we need to do is apply these cross sections to regular assemblies made exclusively from combs and cut nodes. We will therefore refer to assemblies constructed exclusively from combs and cut nodes as regular assemblies.

II. Nodal plasticity

Non-alternable objects will respect these structures (*universes*). These structures correspond to immersions in the plane, although they pose a broader plastic and topological problem, not just a graphical one. Non-alternable objects represent an additional difficulty since the knot accomplishes a non-alternability that we will resolve from a nodal point of view. The arithmetic study of the previous chapters begins to explain reason. In this plastic approach, the source of our objects is different since the movements of the theory of Enlacements allow us to re-alternate any object with one, two, or three circles, and even more, as we will now show. The theory of entanglements becomes the theory of non-knots with one to three circles. We will continue the arithmetic of knots in the next chapter when we have completed the graphical description of the structures that follow the alternating presentations.

1. **Theory of non-knots with one to three loops**

We call a knot or alternating chain a non-knot if its minimum girth surface is bicolorable, i.e., it does not require cutting.

These non-knots are always alternating by definition.

We can talk about a theory of non-knots in the context of the theory of Enlacements in cases consisting of one, two, or three circles.

Under this condition on the number of circles, $r \leq 3$, knowing that all presentations of knots and alternating and non-alternating chains belong to a Linkage, we now call this class a non-knot, because we can advance
:

The existence of standard non-knots

Each class, constituting what we have called a linking, has a standard non-knot that represents it.

This fact is demonstrated by the formal study of the distributions of entanglements by the characteristic table of entanglement states, the movement of negative signs that mark the change in orientation of circles as well as the inversion of the characteristic signs of crossings in the formalism that we introduced in the study of **Renl** and **RΣ** relations and the effective construction of non-knots.

Let us consider, for $r \leq 3$, the two tables presenting the richest complexity in this domain. Simpler entanglement states are obtained when certain values in these tables are zero or certain letters are absent:

$$ST + TJ + JS = \Sigma_{\emptyset} \qquad - ST - TJ - JS = \Sigma_{\emptyset}$$
$$- ST + TJ - JS = \Sigma_{\{S\}} \qquad ST - TJ + JS = \Sigma_{\{S\}}$$
$$- ST - TJ + JS = \Sigma_{\{T\}} \qquad ST + TJ - JS = \Sigma_{\{T\}}$$
$$ST - TJ - JS = \Sigma_{\{J\}} \qquad - ST + TJ + JS = \Sigma_{\{J\}}$$

We know how to construct, as we will show immediately, a copy of each entanglement state corresponding to each table and therefore to each non-node.

This instance offers a minimum number of crossings in each case and is purely inappropriate.

We call this non-node representing a given class the non-node contained S_0 in any presentation belonging to that class.

a1 - ***Construction of non-nodes with one to three circles contained in a node or chain***

One - Let's start with nodes consisting of a single circle. There is only one non-node, which is the trivial node:

Fig. 37

Two - In the case of chains made up of two circles, apart from the trivial chain:

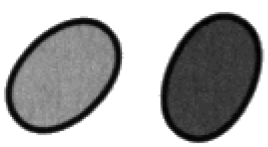


Fig. 38

The non-knots are made up of the series of positive torsion toric non-knots:

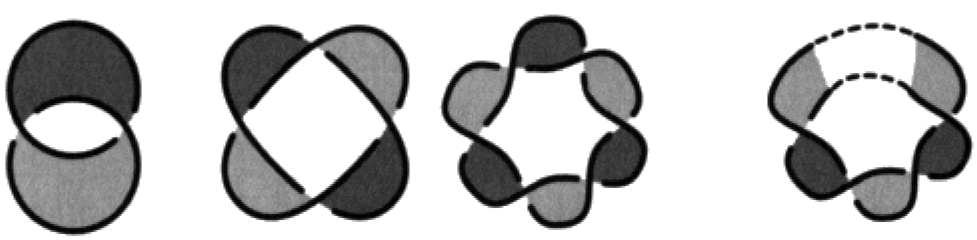


Fig. 39

Three - In the case of chains made of three rings, apart from the trivial chain and the previous interlacing accompanied by a free ring, the non-knots consist of alternating positive twist chains made of three rings:

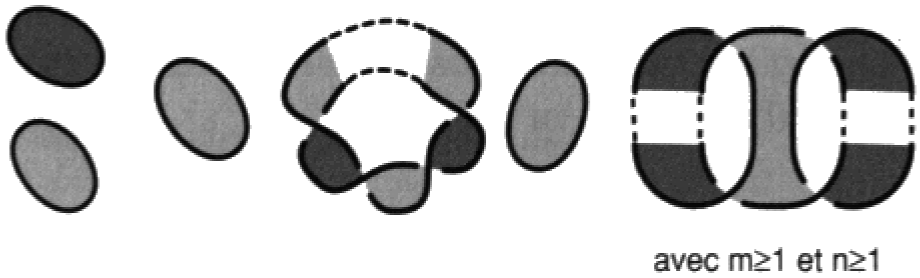


Fig. 40

and the two series of Olympic chains, one the inverse of the other, of which we give the first examples:

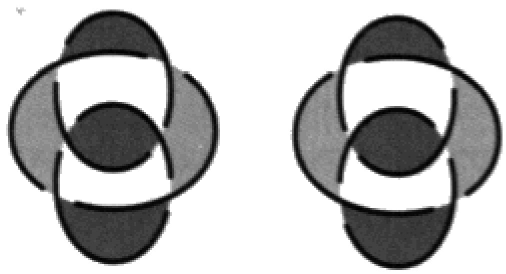


Fig. 41

These two different series are summarized in the following figures indexed by three integers:

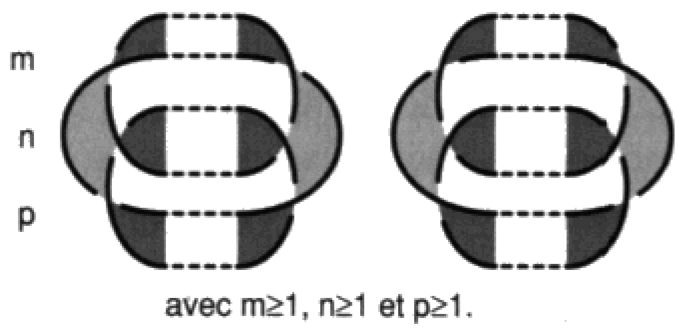


Fig. 42

Here we have established the prototypical representation of each non-node in our theory as a standard non-node.

We do indeed obtain all types modulo R0 and RΣ. a2 -

Another calculation of entanglement

For a given presentation, r ≤ 3, we construct the entanglement state that belongs to it, in terms of its standard non-knot. The distributions of these entanglements correspond to a distribution of chain numbers that allows us to designate this standard non-knot of this entanglement state.

We can then refer to the non-knot s_0 contained in a knot or chain.

The average of the cut numbers Σ_0 of the contained non-node s_0 of the equivalence class of the given presentation provides a new calculation of the entanglement contained in the chain as a function of torsion, independently of colorings.

It should be noted that the non-standard nodes contained in the strings are purely inappropriate and that, as a result, a simple calculation is required.

Opposition of dual numbers of average cut

For standard non-nodes, the average of the cut numbers calculated in the dual Σ^*_0 is the opposite of the average of the cut numbers Σ_0, i.e.:

$$\Sigma^*_0 = -\Sigma_0$$

In standard non-nodes, the proper twist is zero, h = 0, because there is no proper crossing. Thus, according to our main theorem:

$$C_0 - 2\Sigma_0 = 0$$

$$\mathbf{C}^*_0 - 2\Sigma^*_0 = 0$$

However, the equality $\mathbf{C}^*_0 = -\mathbf{C}_0$ is justified by the inversion of the torsion signs in duality, hence our proposition.

But this change of sign has no effect on the definition of the characteristic number of the linking thus calculated, if we stick to the non-knot constructions proposed.

Only one of the Olympic series has negative linking numbers. This is due to the existence of the two tables in the only cases where all the linking numbers are non-zero.

2. **Theory of linkages based on four circles**

Beyond three circles, we still know how to construct examples of some of the interlacing distributions.

They correspond to a state of entanglement rendered by a table that is only more extensive than in cases with fewer than three circles:

SJ	+	ST	+	SZ	+	JT	+	JZ	+	TZ	=	+6
−SJ	−	ST	−	SZ	+	JT	+	JZ	+	TZ	=	0
−SJ	+	ST	+	SZ	−	JT	−	JZ	+	TZ	=	0
SJ	−	ST	+	SZ	−	JT	+	JZ	−	TZ	=	0
SJ	+	ST	−	SZ	+	JT	−	JZ	−	TZ	=	0
SJ	−	ST	−	SZ	−	JT	−	JZ	+	TZ	=	−2
−SJ	+	ST	−	SZ	−	JT	+	JZ	−	TZ	=	−2
−SJ	−	ST	+	SZ	+	JT	−	JZ	−	TZ	=	−2

This table corresponds to this alternating presentation.

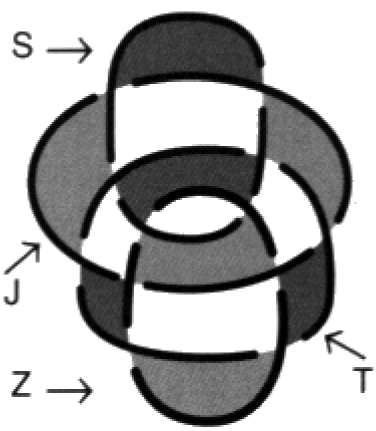


Fig. 43

These examples appear as non-knots, they offer the minimum number of alternating crossings and are purely improper, they are therefore alternable.

Based on these structures, we can construct examples of other states of interlocking. When they are not re-alternable, the distribution table does not contain any entirely positive or negative lines. The existence of these non-alternating cases, which cannot therefore be represented by a non-node, is confirmed by calculation. This existence is due to a combinatorial constraint that applies to the distribution of positive and negative signs. They appear without interruption but are non-alternating, and they offer the minimum number of crossings if these are counted in absolute value regardless of orientation.

They are purely improper. Here is an example:

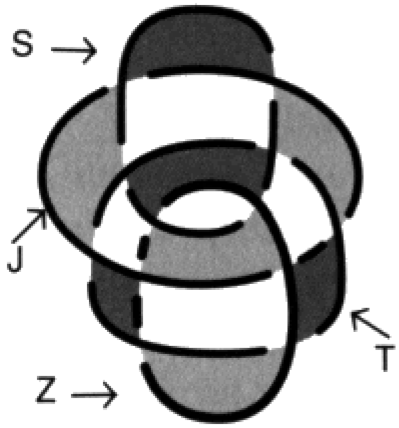


Fig. 44

accompanied by a table showing its entanglement

status.

SJ	+ ST	+ SZ	+ JT	− JZ	+ TZ	= +4
− SJ	− ST	− SZ	+ JT	− JZ	+ TZ	= −2
− SJ	+ ST	+ SZ	− JT	+ JZ	+ TZ	= +2
SJ	− ST	+ SZ	− JT	− JZ	− TZ	= −2
SJ	+ ST	− SZ	+ JT	+ JZ	− TZ	= +2
SJ	− ST	− SZ	− JT	+ JZ	+ TZ	= 0
− SJ	+ ST	− SZ	− JT	− JZ	− TZ	= −4
− SJ	− ST	+ SZ	+ JT	+ JZ	− TZ	= 0

However, we can use Gordian movements to construct alternate versions of these various states of intertwining. In this case, they present a break due to the alternation, and these prototypes are therefore not non-knots. They offer the minimum number of alternate crossings and are purely improper.

In the previous case, for example, we obtain the following alternating presentation.

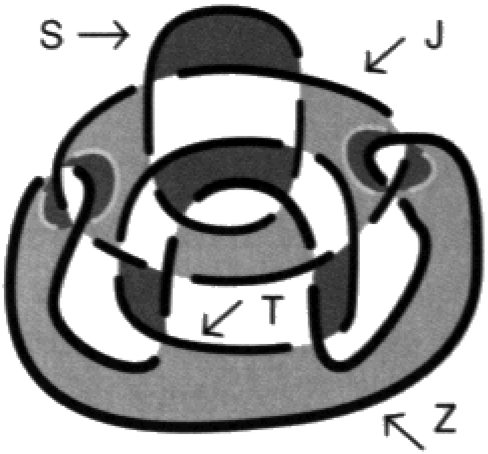


Fig. 45

We will call this type of example a minimal alternating presentation.

Consequently, we can construct examples of each entanglement state. Starting with four circles, there will be two non-knots and non-alternating cases without breaks, the latter of which can be re-alternated by means of a break.

This fact justifies that we cannot speak of a theory of non-knots for the theory of linkings when the number of circles is greater than or equal to four.

But in all cases, we obtain alternating presentations of the distribution of link numbers, which we choose as an exemplary case of the distribution.

Starting with four circles, the knot is no longer a cut accomplishment, since this is also the case for certain entanglement states.

There is therefore a difference between three and four, since all exemplary cases of order less than or equal to three are non-knots (alternating), and the first exemplary alternating case presenting a necessary cut appears at order four.

3. **Return to the theory of dimension and the completion of the cut**

The necessary presence of the cut from four circles onwards, in the exemplary alternating presentations of certain states of entanglement, is an indication linked to the dimension of the space in which the circles are immersed. Working in three dimensions, a crowding effect occurs from four circles onwards.

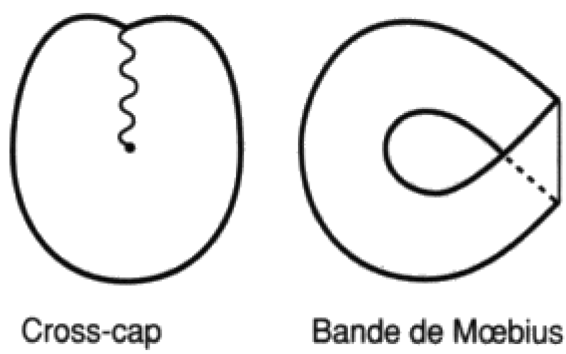
The dimension of space

This gives rise to a new definition of the dimension of space. A space is said to be three-dimensional when, starting with four circles (each one-dimensional), it no longer admits a theory of non-knots.

This can be expressed in another way, by comparing this fact to the impossibility of immersing the projective plane in three-dimensional space.

We can consider the cut necessary for the realternation of four-chain links as a singularity, which reveals a knot, analogous to the immersion line or the hole in non-orientable surfaces.

Indeed, we know how *to immerse* these non-orientable surfaces in three dimensions, using a line of multiple points, known as an immersion line; this is the cross-cap, for example:



Deux présentations du plan projectif en dimension trois.

Fig. 46

Or we can also *immerse* them in three-dimensional space using a hole that can be imagined as a surface break. This is the Möbius strip, if we stick with the same example. In the case of the projective plane, a hole transforms it into a Möbius strip.

The immersion line or this immersion hole are characteristic singularities required by the representation of these surfaces in three dimensions.

This observation can be linked to a new function of the cut made by the knot up to three loops.

The first function of the knot, revealed by the cut, produces the multiplicity of embeddings of the entanglements regardless of the number of loops. This function still exists in three plus one dimensions, as proven by chains that cannot be undone by homotopies, but it is more reduced there. This is accessible to us because the theory of knots and chains up to homotopy in three dimensions is a model of the theory of these objects in four dimensions.

Thus, the new cut function ensures the embedding of certain linkages starting from four circles in three dimensions.

We want proof of this in the fact that the completion of the cut already slips in from three circles in the dual presentation of non-knots, whereas non-knots of two circles do not even require a cut in the dual presentation. Thus, the knot revealed by the cut begins to appear, in the dual of non-knots, starting from three.

Similarly, starting with two circles, uncut chains containing knots always see the knot undo itself by homotopy. These chains are also non-knots, but are not standard. We will specify at the end of this book in which theory this singularity disappears.

The number of circles and the completion of the cut are inversely proportional.

In surface theory, the two remarkable singularities of the presentation of the projective plane are:

— on the one hand, the immersion line required by this immersion (cross-cap), or the hole required by the embedding (Moebius strip).

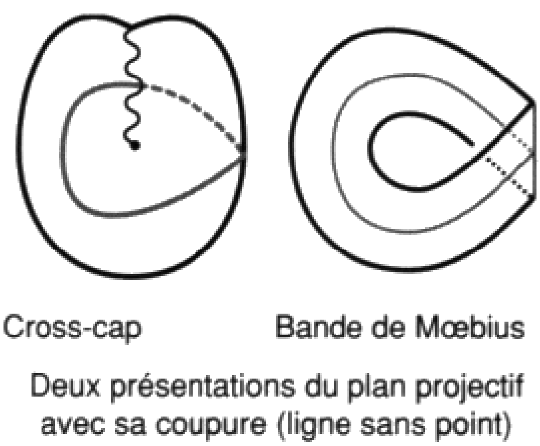


Fig. 47

— on the other hand, the line without a point [7] characteristic of the structure.

Thus, in knot theory, these two remarkable singularities of the presentation of the projective plane are homogenized by the cut of the span surface of the chains from four circles.

It is clear that this deserves further elaboration, since here lies the main reason Lacan encountered for moving from the topological surfaces intrinsic to the knot. This reason concerns the homogenization of the phallic function, the horror of castration as opposed to its representation.

We will return to these questions later, but for now we refer to the theory of chains in four dimensions (codimension three), studied by Milnor and explored by P. Soury in the case of a theory in three dimensions admitting only homotopy in addition to Reidemeister movements (**TH**), and to the work of M. Bertheux, who expanded on all of their results.

4. **Exercises**

e1 - ***Non-knots as comb cuts***

What is the knot obtained by cutting the following 4-comb:

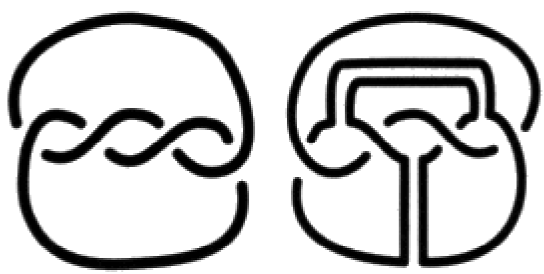


Fig. a

Change the presentation of the last drawing.

Based on this result, describe the relationship between these two chains in terms of comb cuts.

Fig. b

Can you find the relationship between these two chains in terms of comb cuts?

Fig. c

Generalization: from which comb does this non-knot originate, and through which transverse cuts?

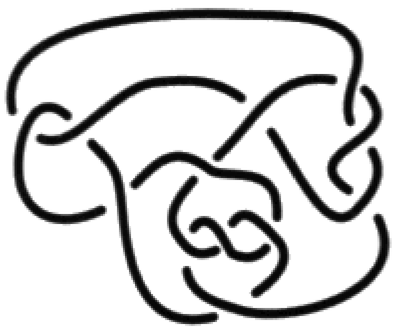


Fig. d

e2 - ***Pure nodes as cut nodes connected by straps.***

What is the relationship between these two nodes?

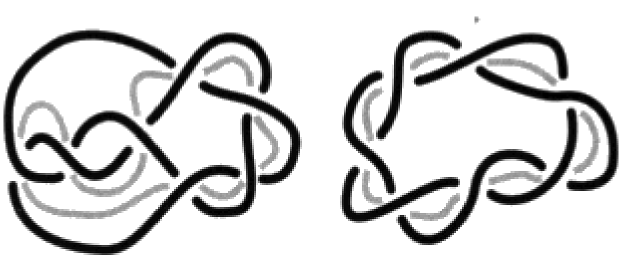


Fig. e

Chapter VI

The work of the knot

Most attempts made to date to elucidate the problems of the knot have focused on what is known as the variety of objects, as provided by the complementary space around string constructions, and have sought to classify these varieties in three dimensions. Even when they abandoned classification, they still relied on these immersions, with Seifert surfaces, for example.

We are the only ones to have taken into account something else noted in what preoccupied Lacan with hesitation and determination in those earlier periods: for us, between the embedding and the results reached by our study, new nodal material must be inserted, the piecewise-oriented span surfaces, or colorings, highlighted by our reading process. It is from these colorings in the minimal presentation of the alternable cases of one to three circles—with an inversion in the non-alternable cases—and not from the varieties that we seek the solution.

This means that we have a new task ahead of us. We must investigate the relationships between the different parts, cut and non-cut, more precisely between their components, in the minimal presentations of the alternate cases of one to three circles first, and examine the mode of composition by which these are assembled with those, without forgetting the non-alternate cases and the fact of immersion.

The minimal presentations of the flattenings and the nodal embeddings appear to us as two registers of the same facts in two different realizations; or rather, the minimal presentation appears to us as a graphical description, in relation to the two-dimensional, of the nodal embeddings in three dimensions, in another mode of inscription, whose features and algebra we will only be able to know once we have compared the transcription and the embedding.

I grasp the embeddings intuitively as soon as they are drawn. The minimal presentation of the alternable node is found heuristically. Its features must be determined successively in the graphical description of the alternating presentations, when it exists. We will obviously be mistaken if we want to read these descriptions as nodal and not according to their graphical dimension.

Suppose I look at space as having reduced constraints; it represents a void whose vastness we perceive and within which we believe we can move freely, etc. I could state that in this set, its different parts have no qualities of their own that differ from the set.

I will only be able to judge space accurately when I give up trying to assess the whole and the parts in this way. I will endeavor to replace each graphic description with a knot or a chain which, for a reason summarized in Reidemeister's movements, can be represented by this graphic presentation. Articulated in this way, the presentations will no longer be devoid of nodal dimension, but will be able to explain some knot or entanglement.

The knot is determined by space just as it determines space; better still, it opens us up to properties of space that we could not imagine or conceive without it.

It seems to us that there are only two ways to circulate in space, to pass through it, either in the manner of entanglement (intrinsic chain) or in the manner of the knot.

As Lacan puts it: in entanglement, which forms a chain, one circle uses the hole of another circle; in the knot, none of the circles uses the hole of another circle [2 Sem XXII, lesson 15.05.75].

Our predecessors made the mistake of wanting to interpret space as a uniform container. That is why it appeared to them to be multiple and indeterminate.

The graphic work of the knot is a work of composing simple elements together. It provides a description.

The nodal work of the node is a work of composition between simple movements. It gives a topology.

In graphic terms, it consists of composing the node parts and non-node parts of alternating nodes using a tool called the Terrasson graph, which will be defined shortly.

This brings us to the concept of regular assembly. A regular assembly is a composite, using a specific application of the Terrasson graph, of cut nodes and combs.

Any case of a node or chain that is not a regular assembly is a compound, according to the same assembly process, known as the regular process, of pure nodes and non-nodes. Or, to put it another way, any case is obtained from a regular assembly by cross-sections, either ribbon straps or cross-cuts as discussed in the previous chapter and as the Terrasson graph will allow us to specify the definition, performed respectively in the node parts and non-node parts of the regular assembly.

In the nodal register, the work consists of the action of proper (homotopies) and improper Gordian movements on the non-knots that represent the different states of the entanglements. This results in a numbering of the cut part, which corresponds to the number of knots, and of the non-cut part containing the numbering of the entanglements, in terms of the non-knot contained, for a particular coloring.

From here, a unique coloring can be assigned to each chain presentation containing interlacing. This is the coloring that corresponds to the non-node content with its uncut coloring. Clean nodes do not contain non-nodes, but they only have one coloring, except for a color inversion. Chain nodes will have several cuts, but they will have the same number.

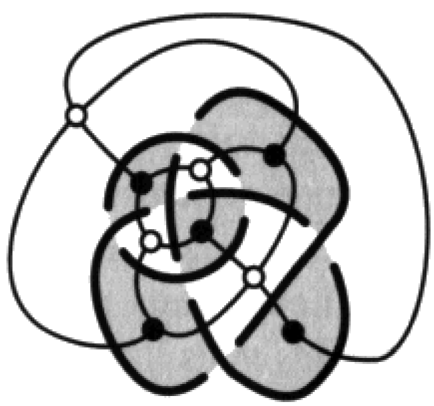
We can therefore talk about the uniqueness of the cut number, definitively solving the problem of cut variation, i.e., the use of object orientation by the characteristic of crossings, thus approaching the solution we want to give to the treatment of interlacing in the rest of this book. We will clarify this in the last chapter.

I. Graphical description

0. **Terrasson's graph**

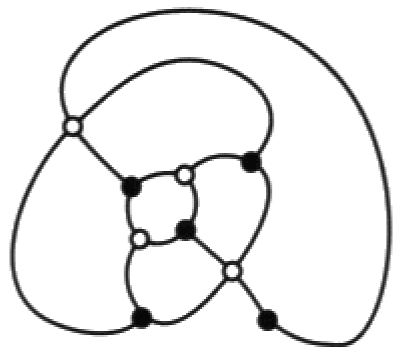
a1 - ***Definition***

In a given flat diagram, we mark the areas distinguished by the first step of our algorithm with solid points (black dots) and empty points (white dots). Remember that this first step of the algorithm resulted in the distinction between so-called full areas and so-called empty areas. These points are the vertices of the Terrasson graph. Maintaining the distinction established between these vertices, we join them with edges that cross, in a straightforward manner, each arc fraction adjacent to each area.



Le graphe de Terrasson d'un schéma plat Fig. 1

This gives us the Terrasson graph for the given presentation.



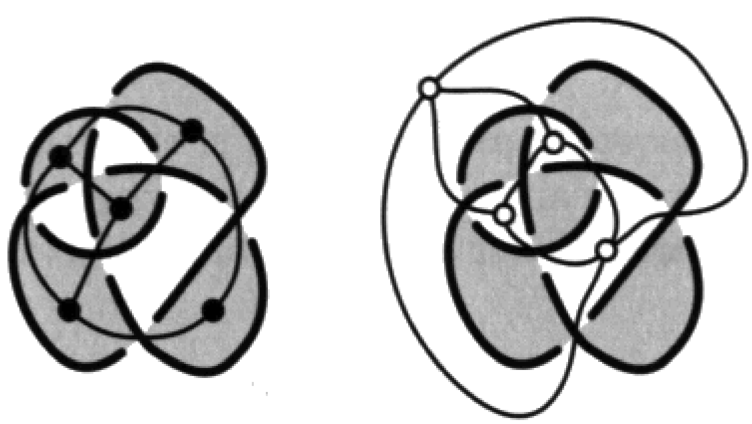
Le graphe de Terrasson d'un schéma plat

Fig. 2

Each edge thus connects a black point to a white point. The Terrasson graph is an even-valency graph (even number of vertices) that alternates between solid vertices and empty vertices, which we represent by these black and white points, respectively.

We will also sometimes refer to this type of graph associated with a presentation as a T-graph.

The set of vertices of this new graph is the union of the sets of vertices of the two graphs, which are dual to each other: the graph of solids and the graph of voids. We defined them in Chapter IV and here we give the examples attached to this presentation of a 2-chain that we are using as an example.



Graphe des pleins

Graphe des vides

Fig. 3

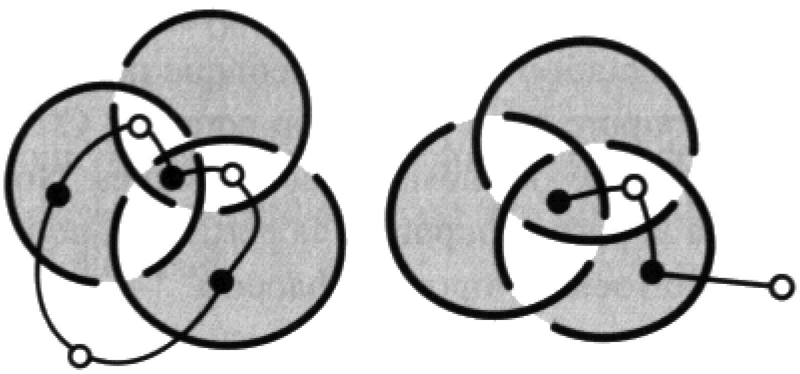
It is quite certain that the reader may encounter difficulties in distinguishing between these three different graphs, which have elements in common. This is the case with the T-graph, since we take into account the set of vertices of these two graphs to form the latter. In the T-graph, we continue to distinguish their respective vertices as solid points and empty points.

But this is not the case for these two graphs, the solid graph and the empty graph. They are disjoint—they have no common elements and are dual to each other (see Chapter IV).

In these three graphs, the edges are always different.

The Terrasson graph is not a composite of the full graph and the empty graph.

In the rest of our study, we mainly use subgraphs of this Terrasson graph. Here are some examples where the span area is only determined by the first step of our algorithm, before being colored.



Un cycle du T-graphe   Un chemin du T-graphe

Fig. 4

Here are other examples where the span surface is colored by the other steps of our algorithm.

Deux chemins non connexes du T-graphe   Un cycle du T-graphe   Un arbre du T-graphe

Fig. 5

These portions of graphs will be used to compose and decompose node or chain presentations. They will also be used to perform transformations on object presentations. This is done while always taking into account the existence of any possible colorings of the object.

This graph and its logic will help us clarify the definitions of what we are accomplishing with these presentations, show the sometimes surprising unity of the actions to be performed according to the needs produced by these objects, and gain greater confidence in the practice of flat patterns.

a2 - ***Punches***

A punch is delimited by a cycle of the Terrasson graph in the presentation of an object. This cycle is the edge of the punch.

In the example of the Borromean ring, we can break it down into two parts using a cycle that separates it into two punches. Any colored presentation can be broken down into its cut part and its non-cut part. Each of these parts can be made up of several components. Here we give an example where the cut part and the non-cut part consist of only one component each.

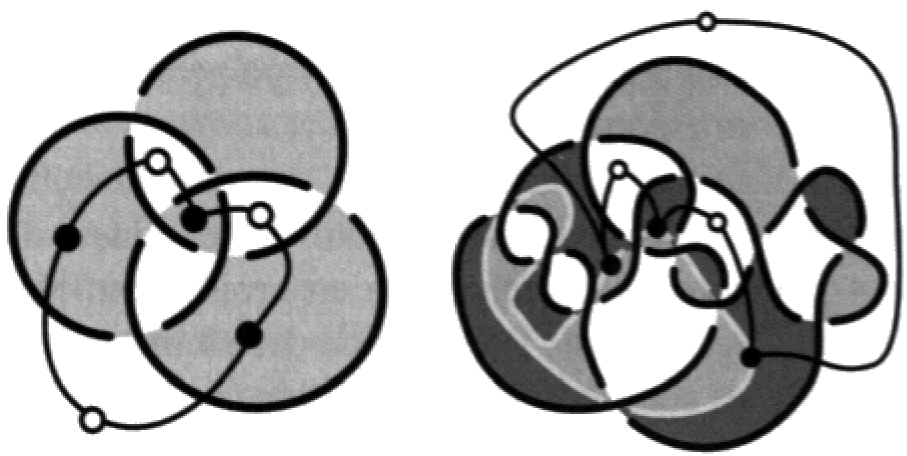


Fig.6

A component of the cutout part can be presented in its punch by deformation on the sphere. We obtain the punch object delimited by its edge extracted from the Terrasson graph.

Fig.7

Punches always have an even number of vertices, alternating between solid vertices (black dots) and empty vertices (white dots). Here, our two examples show the presence of six punches, the characteristic of the punch being given by the number of vertices, which equals the number of edges of the punch's edge.

The punch is an object that generalizes the notion of entanglement (*tangle*), well known since Conway integrated it into a calculation [18. b, pp. 59-69], unfortunately too restricted to cover the multiplicity of chains and knots. The closure of *tangles* also presents a difficulty that will be resolved later by our graphical description[1].

The 4-punch

The 4-punch has two empty vertices and two full vertices. It is the smallest punch we will have to consider in the work of the knot.

It should be noted that at the extreme of its decomposition by Terrasson's graph, a knot consists of 4-punchings, each surrounding a crossing.

When we distinguish between solids and voids, there are two types of 4-punch patterns containing only one intersection. Note that the twist, depending on the opposition between solids and voids, is produced here by the vertices of the punch edge.

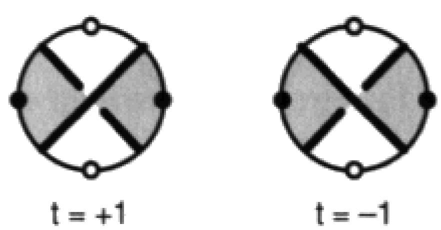


Fig. 8

With coloring that takes into account the characteristic of the crossings, there are four types of elementary 4-punctures:

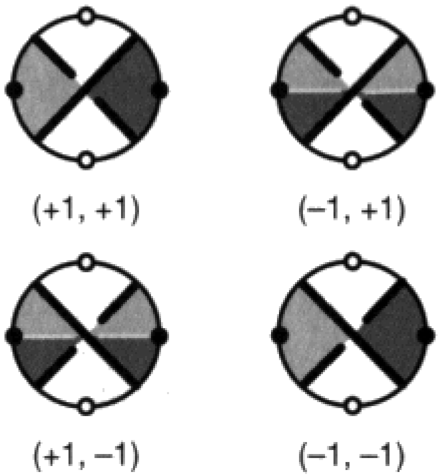


Fig. 9

These different types of crossings, now identified by a punch, have already been studied in Chapter 3.

Still using the 4-punch, we can resort to other decompositions separating a larger aggregate of crossings.

This node $_{63}$ can be divided into two 4-punctures by a cycle extracted from its T-graph.

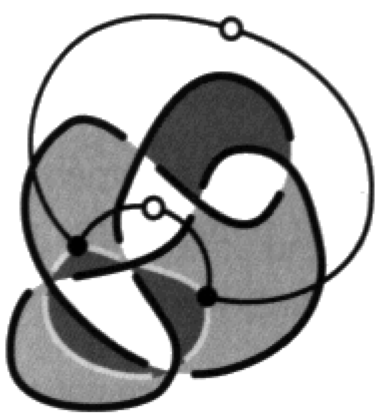


Fig. 10

Here we can clearly see that the cut and non-cut parts of the object are delimited by cycles in the Terrasson graph.

However, we can use 2n-punctures for any integer n.

2n-pins

A 2n-puncture is delimited by a cycle of the T-graph with n full vertices and n empty vertices.

Let's return to the presentation of the object we gave as an example:

Fig. 11

It can be broken down into two 6-punches containing the non-cut part and the cut part respectively:

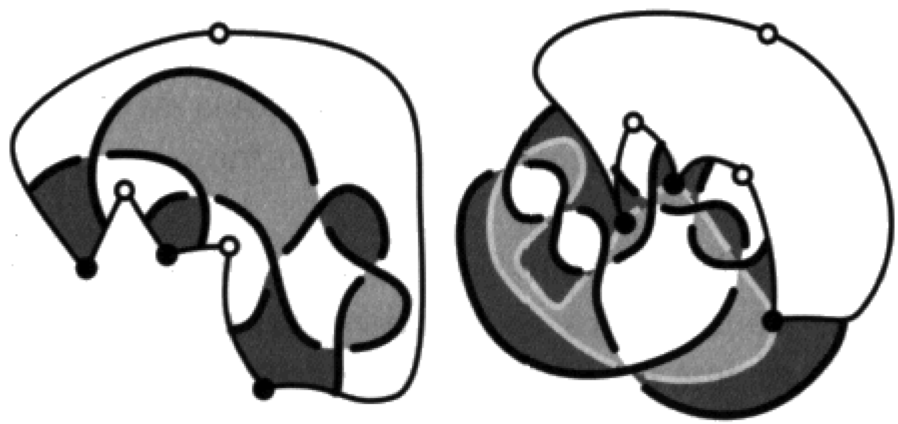


Fig. 12

The number 2n, which designates this type of object, is given by the number of vertices carried by the cycle that forms the edge of the punch. This is the definition of this type of object.

The presentation of the second punch is different from that of the first.

In the first case, the area outside the punch, delimited by its edge extracted from the T-graph, contains the point at infinity of the plane.

In the second case, this point at infinity is inside the punch.

But this difference only persists if we consider our drawings traced on the infinite plane (the interior of the disk); it disappears as a difference if we consider our drawings traced on the sphere (the completed plane), of which, of course, we are only looking at one location. This is the question we have already encountered and dealt with in connection with the duality of span surfaces and dual presentations in Chapter IV.

Through a kind of zero-order topological change of presentation on the sphere—which we will call the reversal of the punch—we find the presentation of the punch that we gave above:

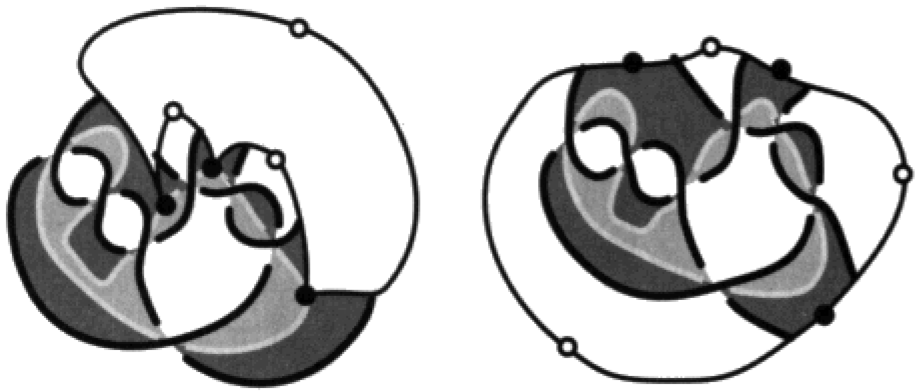


Fig. 13

This transformation, which reverses the punches, is necessary for the composition we are now going to study. Now that we have finished presenting the 2n punches, we can learn how to compose them together.

1. **Composition modes**

We already encountered a composition mode in Chapter IV when it seemed necessary to create the compound $s_1 \# s_2^{-1} \# s_2$ as $(s_1 \# s_2^{-1}) \# s_2$ or as $s_1 \# (S_2^{-1} \# s_2)$ in order to demonstrate the reciprocal implication of two binary relations.

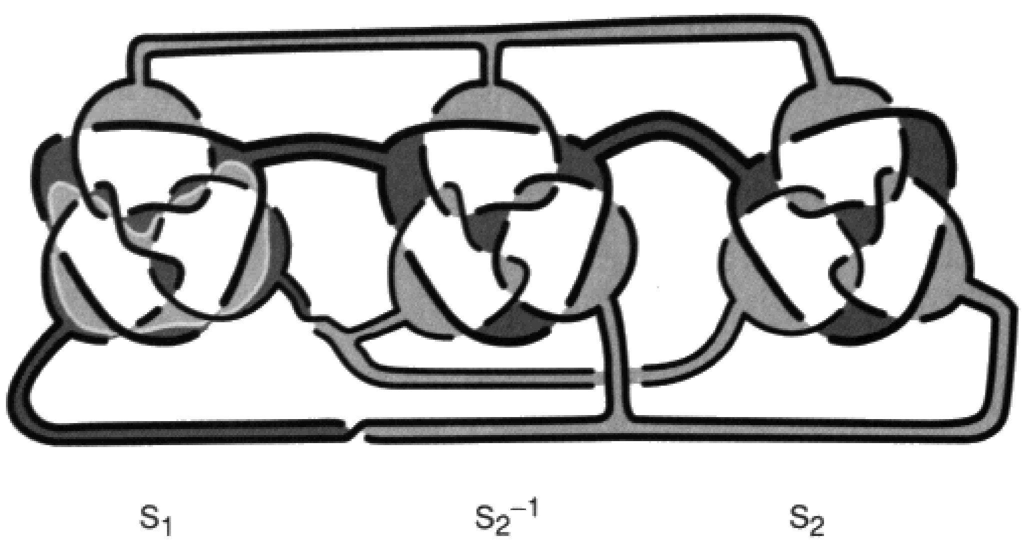


Fig. 14

Here, we want to identify, among other comparable modes, and define more rigorously, this mode of composition that connects the different rounds of two chains with ribbons. We denote this composition with the character # (hash) in this example.

We now add to this character a number $\#_n$ that specifies the degree of connection between the two objects. In our example, this is $\#_4$, $(s_1 \#_4 s_2^{-1}) \#_4 s_2$. This index corresponds to the number of ribbons used in this particular mode of composition. It will not always be possible to refer to the number of ribbons to define the degree of such a composition, so we define it in another way.

Admittedly, without claiming to obtain a well-known algebraic structure, such as a groupoid, a group, or a ring, or even a field, we extend this type of composition here because we want to specify the topological structure specific to these objects.

These standard algebraic structures come from number theory and geometry because of the prominent role played by group structure; we explained this in a previous work[2]. When we talk about topological structure, we are indeed placing ourselves in the context of these algebraic structures, in terms of categories, so to speak.

We will return to this composition of two objects by several ribbons in order to define a relationship between the objects.

We want to give a more precise definition of this mode of composition and show what more general modes it opens up.

a0 - ***Definition of composition***

We perform our assemblies by following subgraphs of the Terrasson graph of each of the objects to be composed. This involves producing composable elements based on the edges and vertices of the T-graph and performing two openings and a punch reversal from these components. Composition is always performed according to a tree taken from the T-graph of each component. A tree is a graph in which the removal of any of its edges separates it into two unconnected components. The resulting composition is always performed according to a cycle of the T-graph of the composed object. A cycle is a closed path.

Let's take an example with the composition of these two objects.

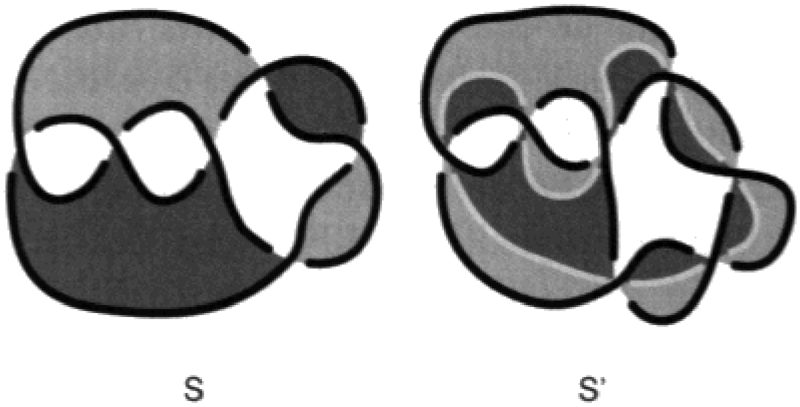


Fig. 15

Determining the characteristics of the composition

We choose a connected subgraph from the Terrasson graphs of each of the objects to be composed. This subgraph must be a tree, as we have just mentioned.

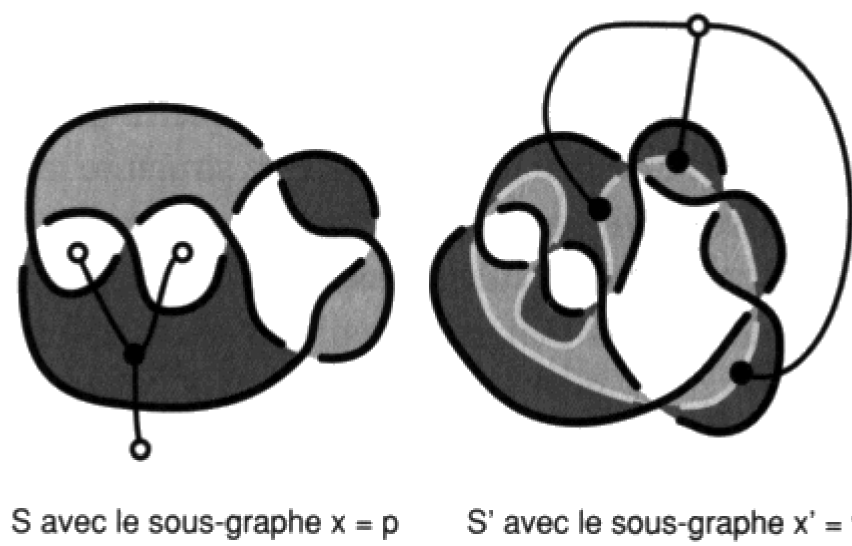


Fig. 16

In the example we have chosen here, we have two extreme cases of trees. These are star trees around a single point, which represent special cases.

These subgraphs must have the same number of edges. This number *n* defines the degree of composition. Here, *n* is equal to 3. We will therefore refer to 3-composition, and more generally to n-composition.

The respective shape of each subgraph, denoted x and x', specifies the mode of composition. We denote the node S accompanied by **x** either before or after the letter that designates the object S in question, depending on its place in the composition of the two objects:

$S_{x \#n x'} S'$

Here, the first subgraph is a star subgraph around a full vertex, which we denote *p*. The second is a star subgraph around an empty vertex, which we denote *v*. We therefore denote the composition we are now performing as:

$S_{p \#n v} S'$

Opening the punches

We open the punches by splitting the edges and multiplying the intermediate vertices. The vertices at the ends of the edges remain unchanged.

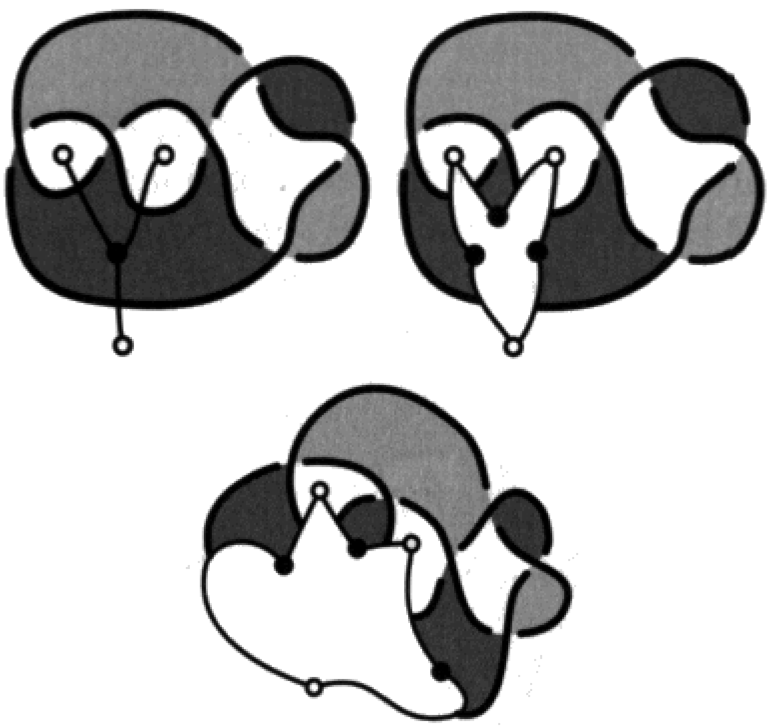


Fig.17

Let's do the same with the other object to be composed:

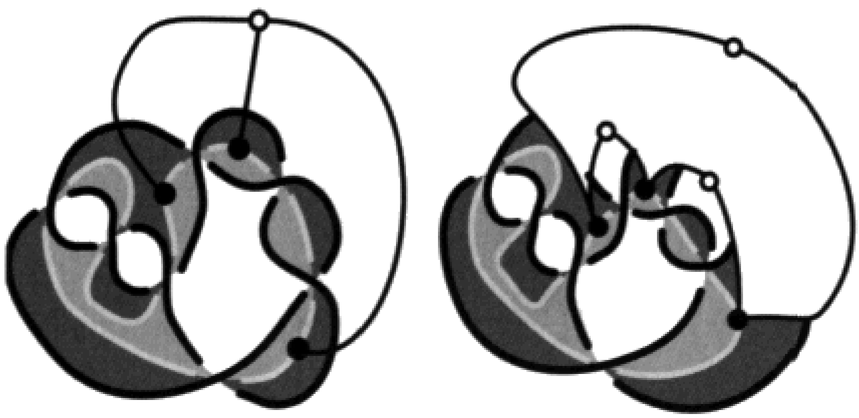


Fig. 18

We obtain two 2n-punches. Here, 6-punches.

Flipping a punch

One of these punches must be turned over by continuously deforming the sphere so that one of them is positioned in such a way that it can be placed in the open area of the other.

Fig. 19

To deal with this reversal, simply refer to Chapter IV, where we discuss the duality of presentations on the sphere[3].

Actual composition of the two punches

We can thus fit these two objects together, with a slight rotation, respecting the solids and voids and the coloring of each of them, if applicable.

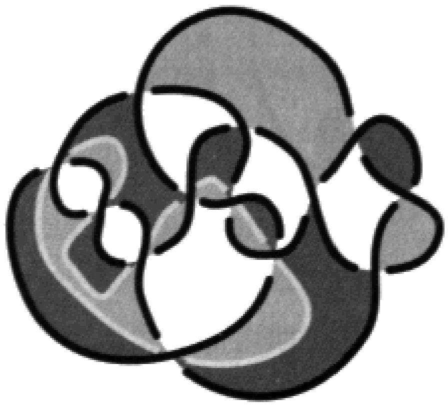


Fig. 20

This composition is done according to a cycle of the T-graph of the composite

object. Removing this cycle, we have constructed the object denoted $_{\mathbf{Sp\ \#3\ v}}$ S':

Fig. 21

This object is still a node or a chain; it is indeed a mode of composition internal to the multiplicity of these objects.

Having defined composition in its most general form, with the help of an example, let us now consider specific cases of these modes of composition in order to rediscover the intuitive montages we were already practicing. We will then specify a main assembly mode for describing all objects based on the objects that constitute the source of this description. These generating elements were presented in the previous chapter. They are cut nodes and combs, two families that are dual to each other.

a1 - ***2-composition***

2-composition is defined by two consecutive edges in the Terrasson graph of objects that are assembled together. These two consecutive edges form the simplest case of a star tree. At degree two, there is no other tree. This portion of the graph always has three vertices, and the middle vertex will be used to characterize the composition, as it is either a full vertex or an empty vertex. When the punch is opened, it gives rise to a 4-punch.

There are four modes of 2-composition

The general form of the 2-composition is:

$S_{x \#2 x'} S'$

We may also write it as:

$S_{x2} \#_{x'2} S'$

where x2 and x'2 denote pairs of connected edges, isolated in the T-graph of the objects to be composed.

There are only two types of such pairs for a given object. This explains why there are only four modes of 2-composition.

In order to clearly define these two types of connections for each object, let's illustrate their differences using the example of the cloverleaf node. If we consider two successive edges of the Terrasson graph of the cloverleaf node, there are only two possibilities:

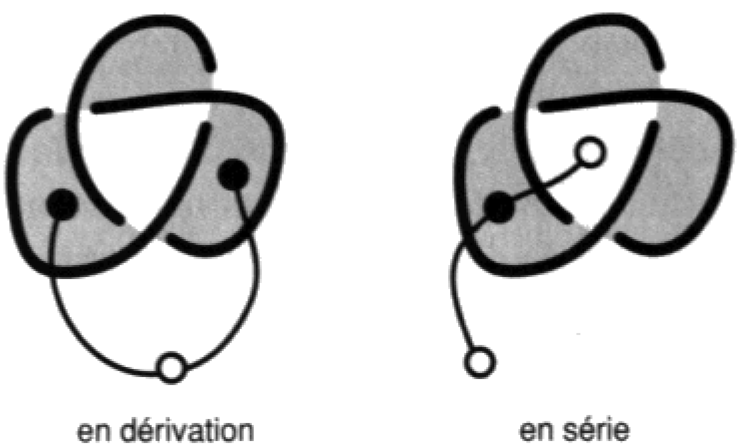


Fig. 22

In the first case, the intermediate vertex between the two edges of the graph is a void vertex. In the second case, the intermediate vertex is a full vertex.

In the case of 2-composition, if the intermediate vertex is a void vertex, we say that the composition is done on that side by derivation. We can write $\mathbf{x} = {}_{d2}$ to denote this derivation, or $\mathbf{x} = {}_{v2}$ to denote the void point.

Otherwise, the vertex is a full vertex and we will say that the composition is then done in series on the side of this object. It will be written as $\mathbf{x} =_{\mathbf{s2}}$ for the series mode or $\mathbf{x} =_{\mathbf{p2}}$ for the median full point.

It is noteworthy, from the outset, that these two modes are dual to each other; they interchange during the duality of the object produced by the composition in question.

Let us give a first example of a 2-composition, when the two composite objects are connected in parallel, i.e.:

$S_{\mathbf{d2}} \# _{\mathbf{d2}} S'$

An intuitive concept with precision

Let's choose some very basic objects such as the clover T and the simple entanglement

E. We will therefore perform the assembly:

$T_{\mathbf{d2}} \# _{\mathbf{d2}} E$

Let's start by choosing a 2-graph that corresponds to this mode of composition in each of these two objects:

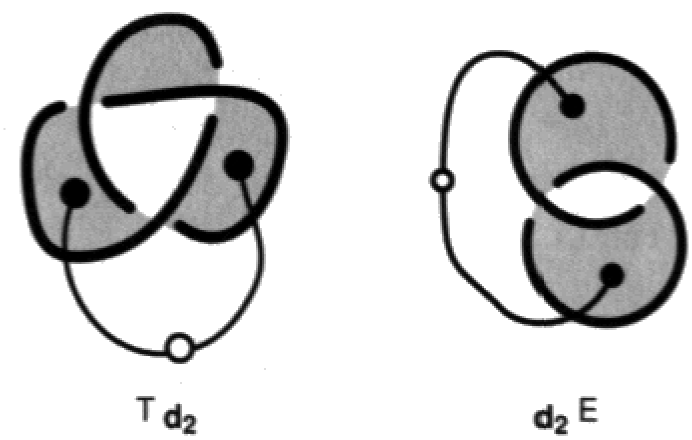


Fig. 23

Let's open the punch for the first object to be composed.

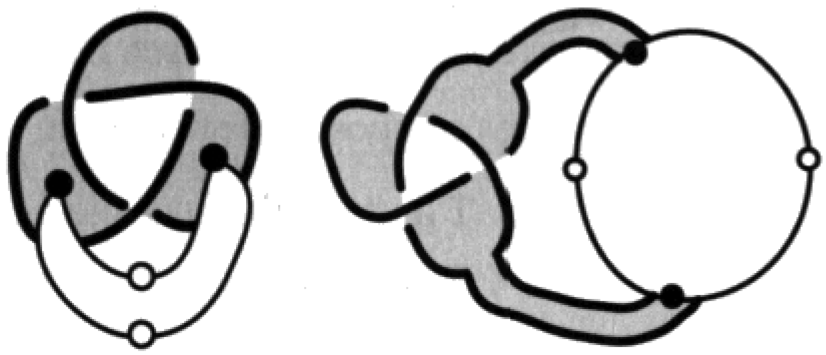


Fig. 24

We continuously deform the knot thus opened to reveal portions of ribbon by playing with the presentation, without changing anything in the knotting or the order structure of the punched object along the cycle that forms its edge.

Let's open the punch of the second object:

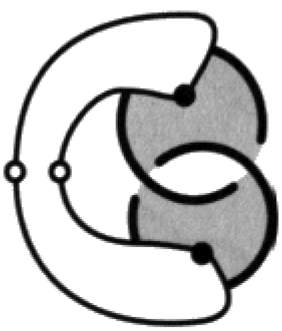


Fig. 25

Let's turn this punch over to obtain a punch that lends itself to the composition and surrounds the intertwining.

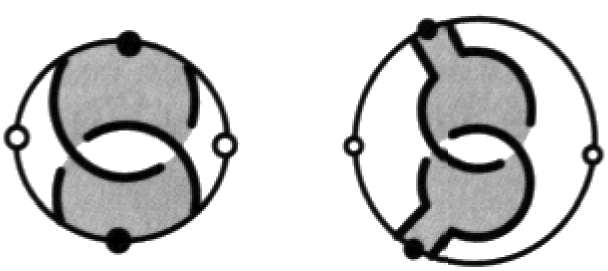


Fig. 26

We also distort the presentation of the punch to show portions of the ribbon in this type of assembly.

We now have two objects that are easy to assemble together.

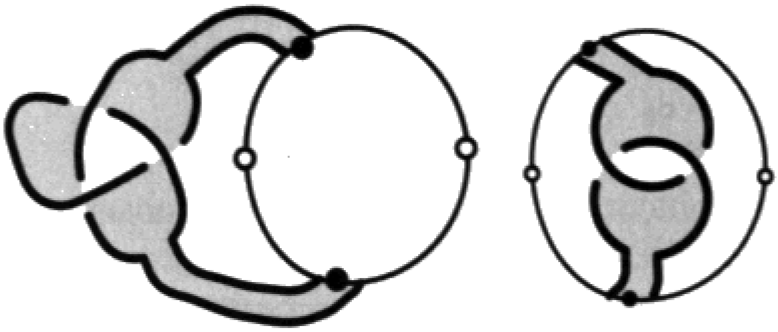


Fig. 27

All that remains is to actually assemble them.

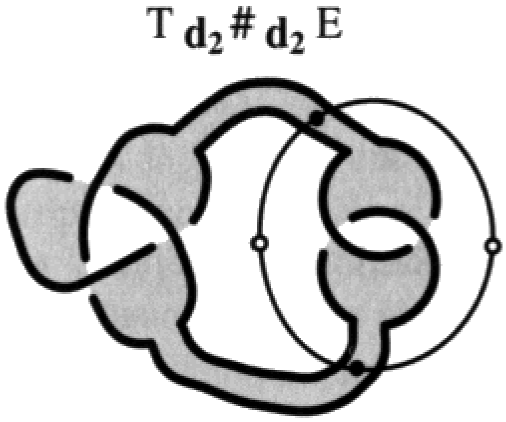


Fig. 28

to see that by carefully following the different steps that define these methods of assembly, we obtain a construction that clarifies, by defining it more precisely, the example of intuitive assembly using straps or fabric ribbons.

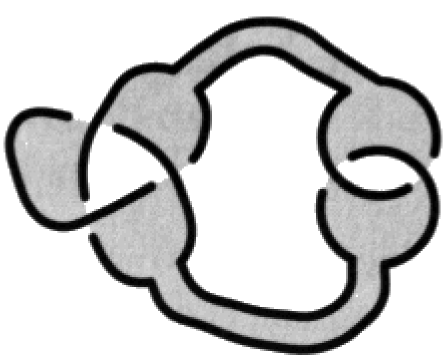


Fig. 29

With movements of zero degree, these two presentations show the same object. They are equivalent.

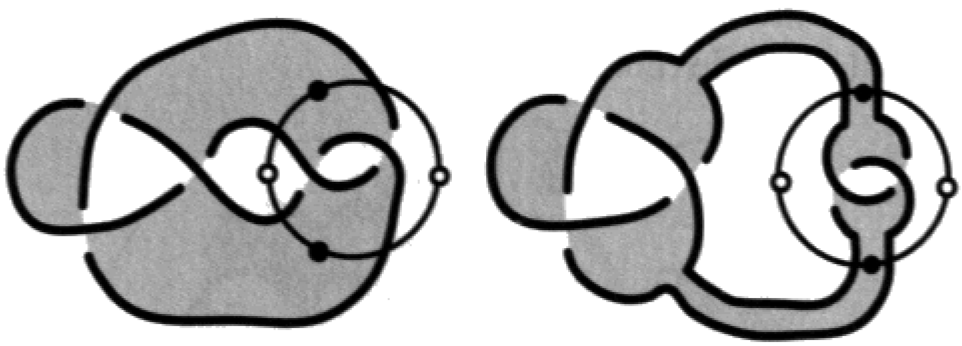


Fig. 30

It is important to note that we have presented the 2-composition so far with uncolored surfaces. If the assembly, the tracing of the portion of Terrasson's graph that we use, respects these colorings, separating the node part and the non-node part, things can become even more precise.

Let's give an example of a 2-composition ($S\ _{d2}\#_{s2}\ S'$) starting from the clover and its dual.

Second example

Let us take a cloverleaf node that we are preparing to be assembled by the 2-composition in derivation $_{Td2}$ and its dual presentation inverted with respect to the crossings T' = T*–1 that we are preparing to assemble in series $_{T's2}$:

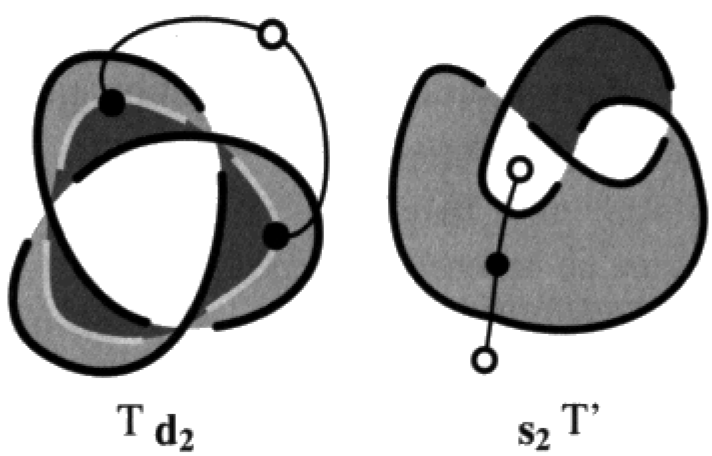


Fig. 31

By opening the punches, which are retracted here on their respective edges, producing the two edges of the Terrasson graph of the starting node, we can compose them together.

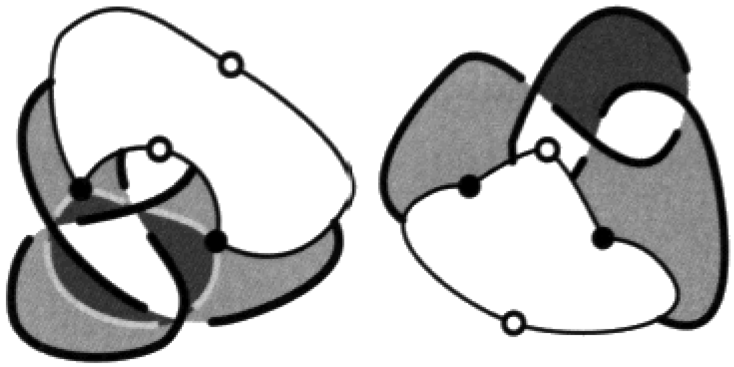


Fig. 32

To do this, we flip the punch of one of the two, here the dual of the clover assembled in series.

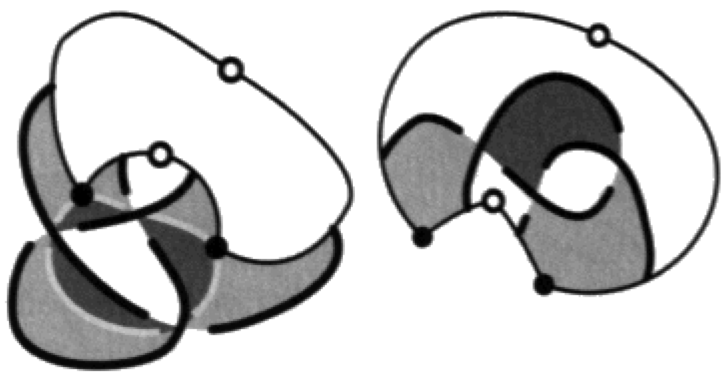


Fig. 33

Then we assemble along the valence four cycle, characteristic of 2-composition, to obtain the composition T $_{d2}$ # $_{s2}$ T':

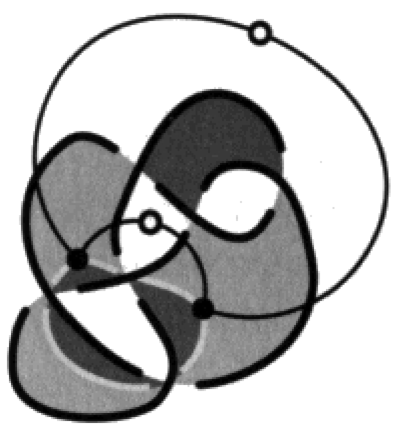


Fig. 34

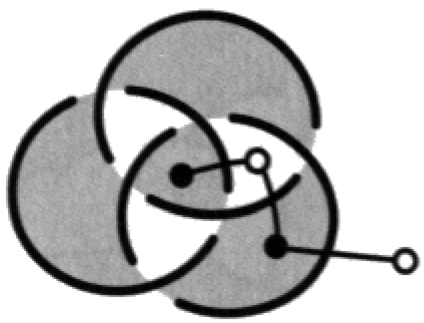
After these two examples of 2-composition, which remains the simplest form, let's return to the general case.

a2 - ***Multi-composition***

As there are 2-compositions, which have just been defined and presented with examples, there are many modes of multiple composition or multi-composition, referred to as n-compositions.

Let's consider a tree extracted from Terrasson's diagram, i.e., a subgraph that does not contain any cycles. If this tree has a number of edges equal to n, we will refer to it as an n-tree extracted from Terrasson's graph.

Here is a path of three edges:



Bo x_3

Fig. 35

This portion of the T-graph takes us from the periphery to the interior of the node in a path comparable to the movement of a knight in chess.

This is precisely the image Freud uses, starting with his studies on hysteria [1. f], when he wants to explain how work progresses in analysis. It is at this point in his text that he states the general law establishing the increase in resistance as inversely proportional to the distance separating the subject from what he calls, at the time, the pathogenic nucleus. The elements of the material gathered by analysis are, he says, like files arranged in rows around this nucleus. Through these rows, progress is made in a manner similar to the movement of a knight in chess.

We mean only one thing, as in the case of the montage of the drive, the libido, and desire, discussed in Chapter II. Why not consider, at the outset, that this presentation of spatial constraints is specific to allowing us to follow what Freud and Lacan say about this other scene? This place, it must be said, we could not find without them, and it is very difficult to establish its laws. Whether it is a model or the thing itself is another question. To answer it, we must not forget that in terms of the knot, we are still in the exploratory phase, even if this allows us to begin to formulate some constraints that apply in such a place.

If we return to our multi-composition example and open the punch defined by the duplication of the edges of this n-tree and its intermediate points, we obtain a 2n-punch.

Here, from a 3-tree, a 6-punch:

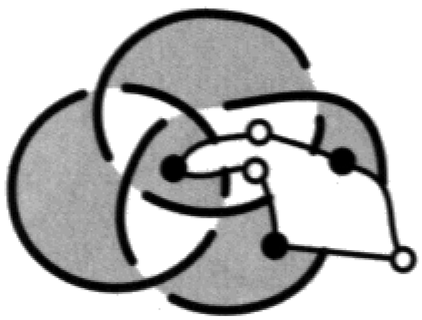


Fig. 36

This process, in the example of a 3-composition, allows us to define the multi-composition that can develop according to various tree structures, as a generalization of the 2-composition. In this case, we will refer to an n-tree of the n-composition, of which we will give other characteristic examples.

Let us open and turn such a punch:

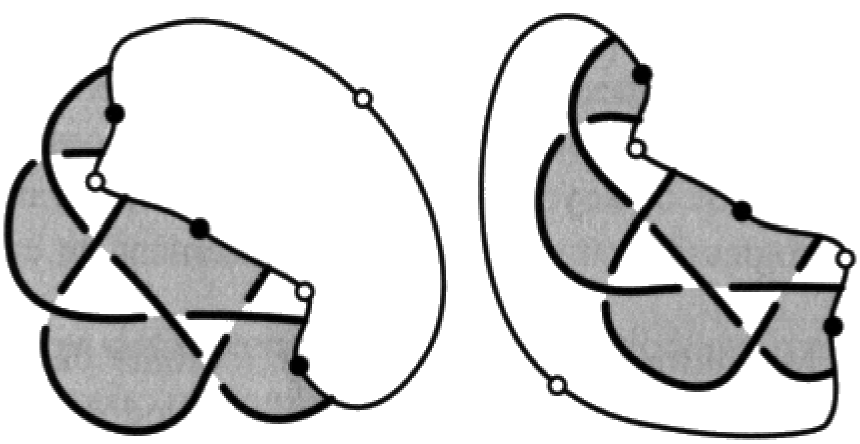


Fig. 37

By combining these two objects from the same node, the open punch and the punch turned over and then rotated by half a turn, we obtain an example of a new assembly mode
:

(Bo $\mathbf{x_3}$ # $\mathbf{x_3}$ Bo)

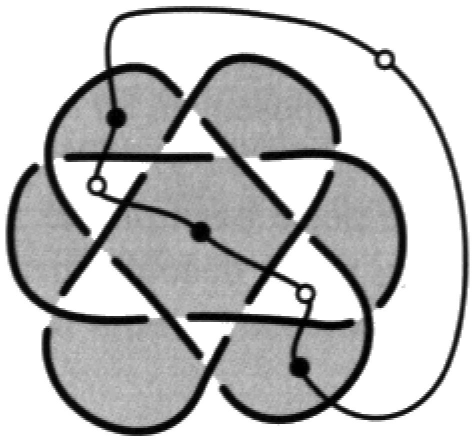


Fig. 38

There are two n-compositions that are of particular interest to us. Among the variety of n-compositions that follow the variety of trees with *n* edges, we will focus on the two extreme modes of composition that are characterized by strict star trees.

a3 - ***The two extreme multi-compositions***

At the extremes of n-composition possibilities, as with two-composition, there is the mode of derivative n-composition and the mode of serial n-composition.

The serial n-composition is characterized by an n-star tree whose central vertex is a full point. We denote it by **Ssn** or **Spn**.

Derivative n-composition is characterized by an n-star tree whose central vertex is a void point. We denote it Sdnou **Svn**.

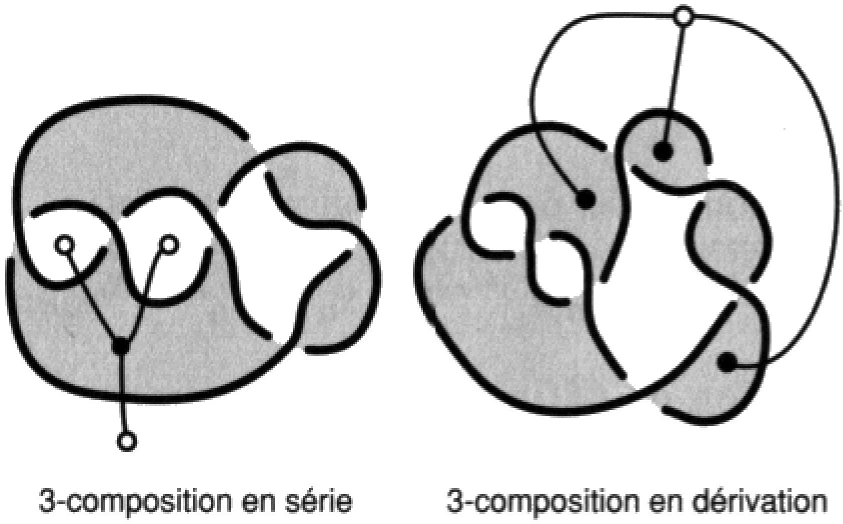


Fig. 39

The n-composition derivation mode respects the coloring of non-nodes. The n-composition series mode respects the coloring of pure nodes.

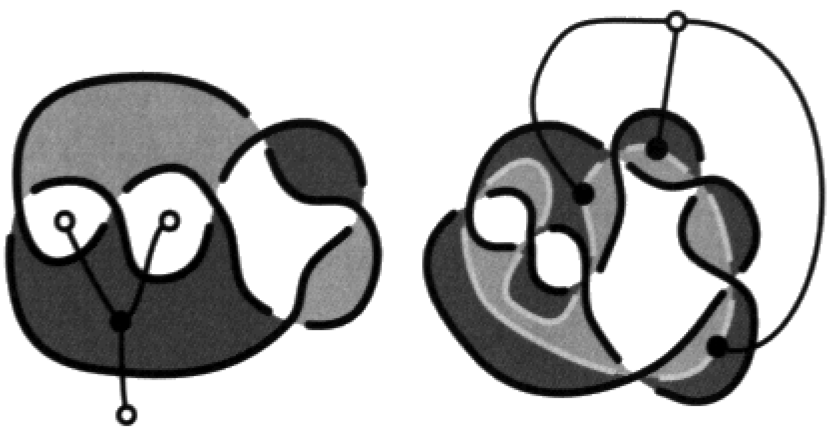


Fig. 40

We have already used these composition modes in the first example of assembly that we gave when introducing this practice. We will use them to complete the description, according to the mode of regular assemblies, of how any object in the theory is constructed.

These drawings can be continuously distorted at will. The portion of Terrasson's diagram used has a structural value of invariance in that it precisely defines and highlights, for reading, the mode of composition.

a4 - ***Composition in duality***

Let us introduce an effective result from this descriptive part.

We call the dual presentation of a given object the presentation obtained from the presentation of the given object, equipped with the dual surface and deformed in its dual presentation. Note how the dual presentation of a given object produced by these modes of composition then creates an assembly that can be deduced from the given assembly. It is written by marking each term of the initial assembly with an asterisk.

We want to talk about the following distributivity:

$(S_{xn} \# _{xn} S')^* = (S^*{}_{xn^*} \# _{xn^*} S'^*)$

Provided that a duality relationship is defined between the portions of Terrasson's graph that govern the montage. We want to say what **xn*** is, knowing $_{xn}$.

With regard to this property of duality, we will refer to the Japanese reading of their language, insofar as it gives rise to a permanent double interpretation, simplified here from a rhetorical point of view but transposed between the voice and the gaze.

Let's give some examples where the relationship between **xn*** and $_{xn}$ is easy to establish. Starting with 2-composition, there is duality between derivation composition and series composition.

$d2^* = {}_{s2}$ and $s2^* = {}_{d2}$

If we return to the example $(T_{d2} \# _{d2} E)$,

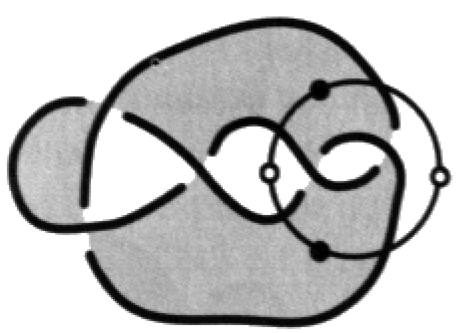


Fig. 41

It will be written, if we denote T* as the dual presentation of the clover T and E* as the dual presentation of the entanglement E:

$(T^*{}_{s2} \# _{s2} E^*)$

The dual presentation is thus obtained by duality:

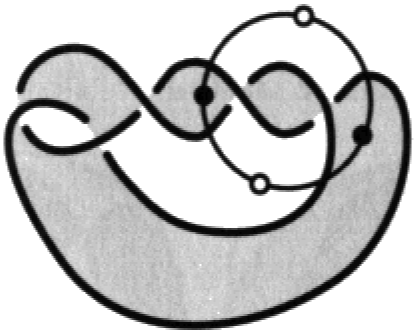


Fig. 42

And this can be shown from the definitions of these modes of composition:

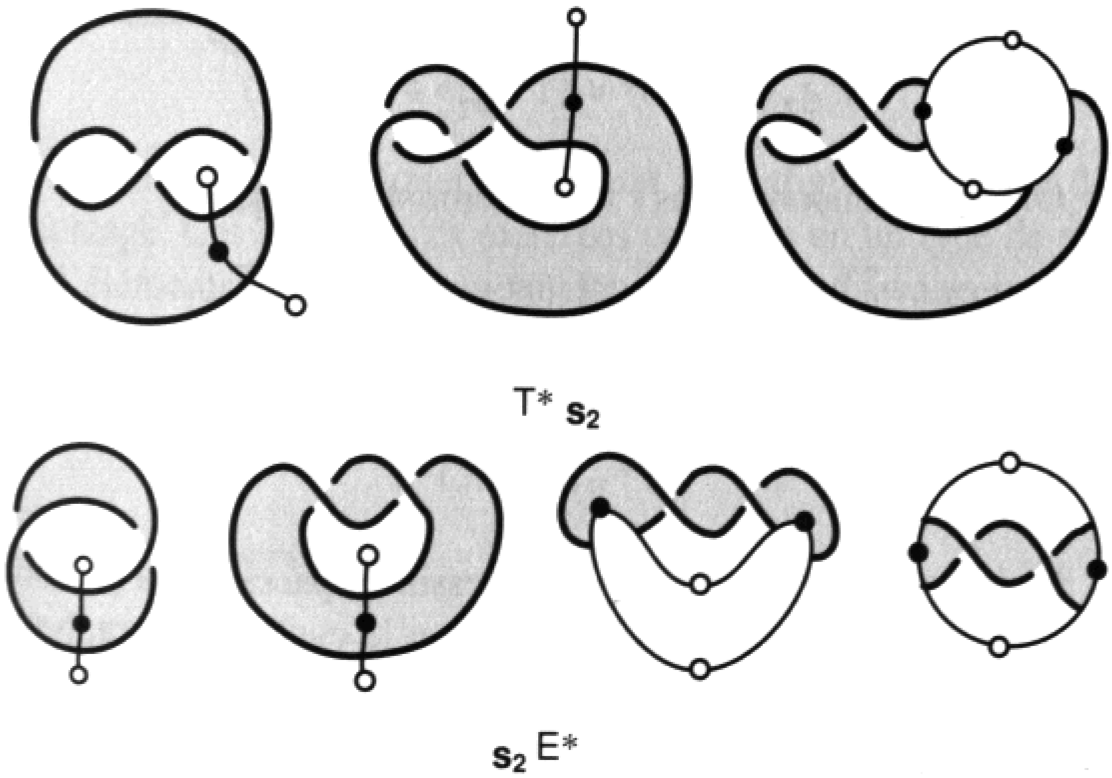


Fig. 43

And gives rise, in the figure, to a direct reading of duality, which is an excellent exercise in constant double interpretation.

The following example shows self-duality. It is the question of *the amphicherality* of certain objects.

If we return to the case presented above ($T\ {}_{d2}\#_{d2}\ T^*$), it gives node 63 in the classic table:

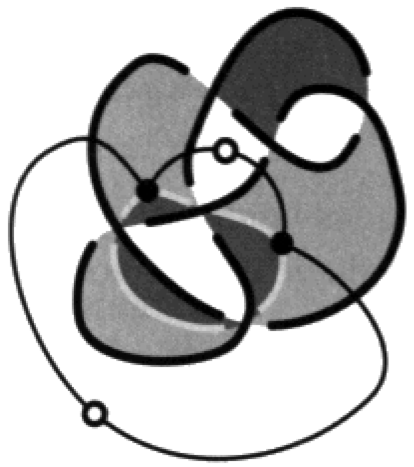


Fig. 44

which is constructed from these two components, which are two dual openings of the trefoil node:

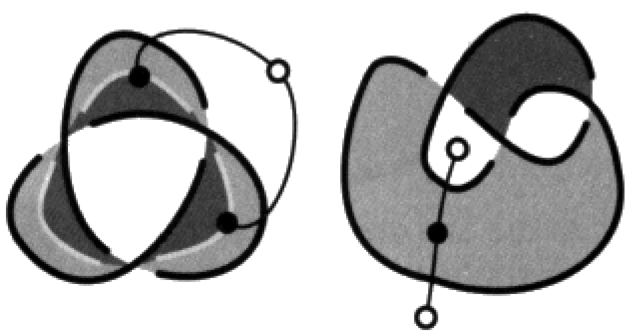


Fig. 45

We obtain:

$(T_{d2} \# _{s2} T^*)^* = (T^*_{d2^*} \# _{s2^*} T^{**})$

However, we advance, without having demonstrated it here, that:

$T^*_{d2^*} = T^*_{s2}$ and that $_{s2^*} T^{**} = {}_{d2} T$

That is, we would obtain by duality:

$(T^*_{s2} \# _{d2} T)$

The reader can verify this by switching to the dual presentation and decomposing the object according to the T-graph cycle that separates the cut and non-cut parts of the resulting object.

It should therefore be noted that, in this case, we obtain the same thing, except for duality, by means of a change in torsion.

It should be noted that the notation for the composition of two objects is

commutative: $(T^* {}_{s2}\#_{s2} T) = (T {}_{s2}\#_{s2} T^*)$

We call this situation, which will seem less and less surprising in our description, autoduality, except for a reversal of the torsion signs. This can be compared with what has been called *amphicheral knots* in the literature dealing with knots since Tait.

Let's give one last example with multi-composition, using the first example showing a 3-series composition, $_{s3}$, and a 3-derivation composition, $_{d3}$. We defined them above and called them the extreme modes of n-composition.

They are dual to each other in the general case:

$dn^* = {}_{sn}$ and $\mathbf{sn^*} = {}_{dn}$

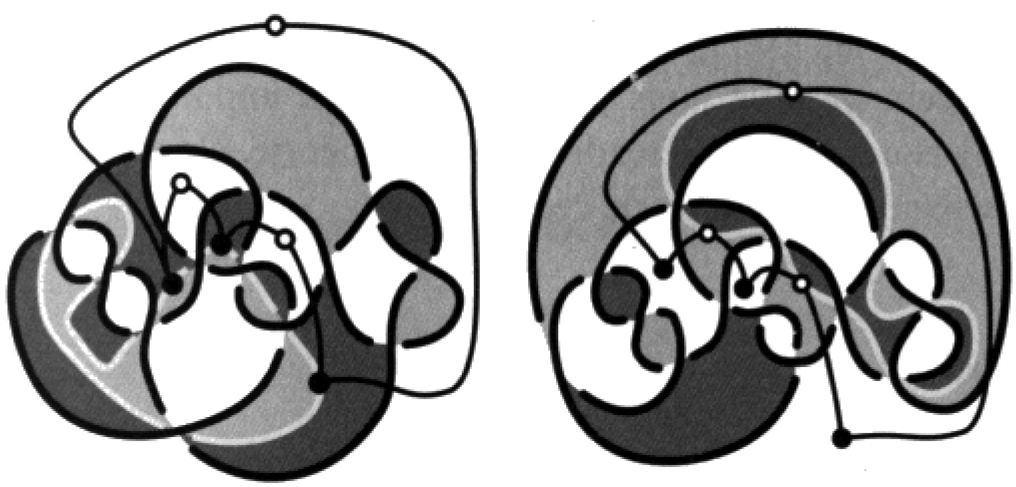


Fig. 46

and make it easy to predict the dual configuration of such an assembly. These two extreme modes, combined in a type of assembly of specific elements, will help us to advance in the graphical description of our objects. We will now deal with assemblies of any node part with any non-node part.

2. **Regular assemblies**

In the examples given above, we can be more specific.

For a given coloring, in each case, when it comes to the exact separation of the components of the cut part and the non-cut part, related to the coloring, the source material of the object forming these parts comes from pure nodes and non-nodes.

This is the case in the first example of a montage that we gave. We also recomposed node $_{63}$, from the clover and its inverted dual presentation, when we wanted to give an example of two compositions following a coloring and the determination of different parts that it produces in a given object:

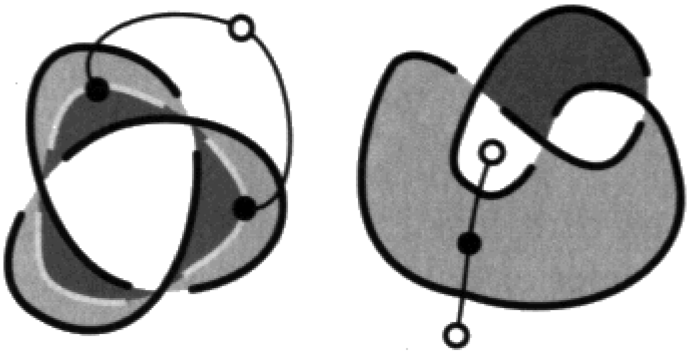


Fig. 47

These two examples are montages that display regularity. To make this apparent to the reader and formulate its definition, we will define a class of objects.

We will call objects of this type regular montages, since they are the prototypical examples of such montages. In relation to this family, we will be able to situate any of the other objects that we intend to theorize about. From now on, this will be the definition of these objects.

a1 - ***Definition***

A regular montage, as we designate an object by this name, is a compound of combs and cut nodes[4], following T-graph cycles, in such a way that the components are assembled exclusively according to the extreme modes of n-composition:

(nC $_{\mathbf{dn}}$ # $_{\mathbf{sn}}$ Pe)

— the cut nodes are composed according to the mode of n-composition in derivation. The graph is star-shaped around a void point,

— the combs are composed according to the series n-composition mode, the graph is star-shaped around a full vertex.

Node $_{63}$ is a regular assembly:

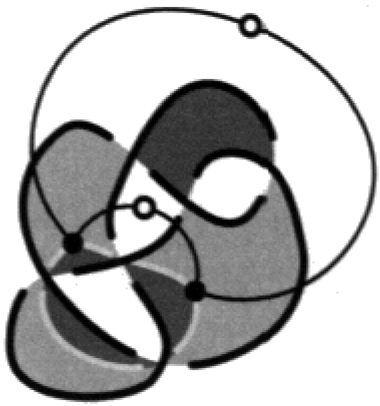


Fig. 48

The other example, the first one we gave of composition, fig. 11, although assembled in a regular manner, is not a regular assembly as an object. This means that it is not characteristic of the type of assembly on which it depends; it is not part of the class we are isolating. We will clarify further on, in fig. 80, what distinguishes it from the objects we refer to here as regular montages.

Let's give an example of such regular assemblies when there is only one component with regularly composed cut-off and non-cut-off parts.

Here is the regular circuit in question:

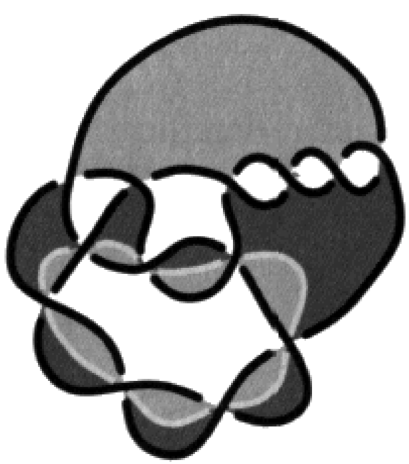


Fig. 49

Here are the components, one for each part:

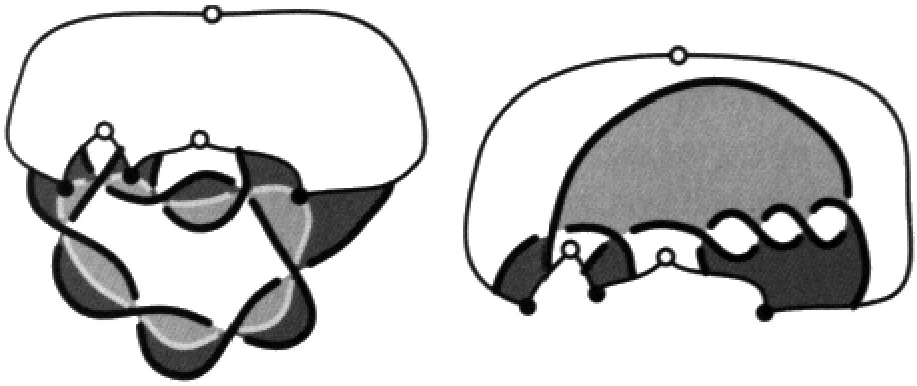


Fig. 50

It is broken down into its two parts by a cycle in its T-graph.

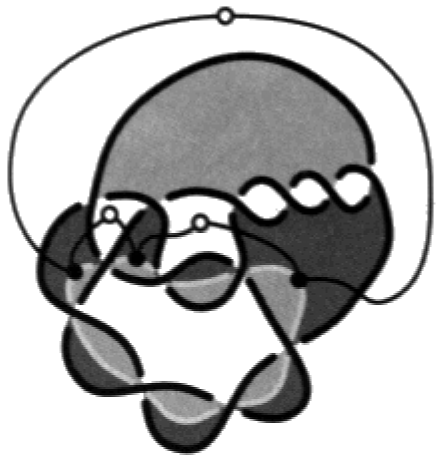


Fig. 51

A regular assembly forms an object whose node part and non-node part can be strictly isolated as coming from a comb and a cut node.

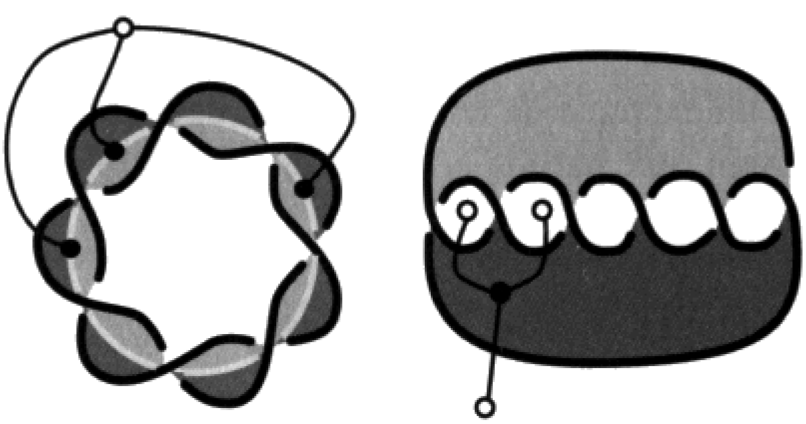


Fig. 52

To better understand this, let's detail the assembly procedure in this case. Let's open the punches placed in the two sources.

First, the bypass punch in the cut node:

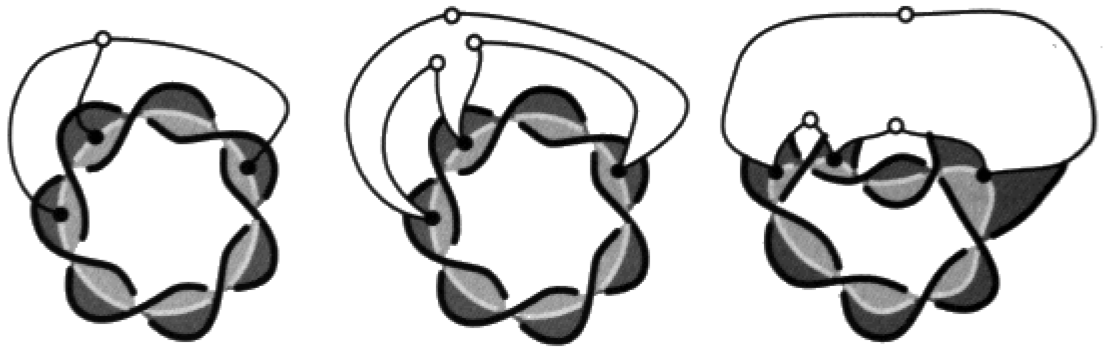


Fig. 53

Then the series punch in the comb:

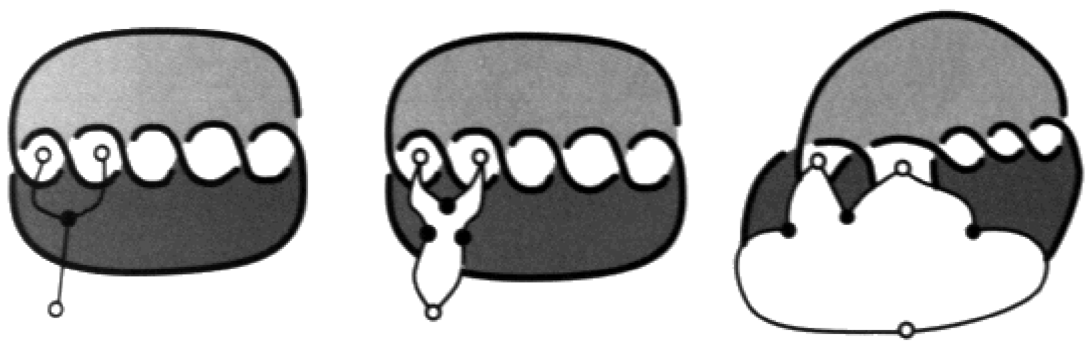


Fig. 54

We turn one of them over, the non-node in the figure above:

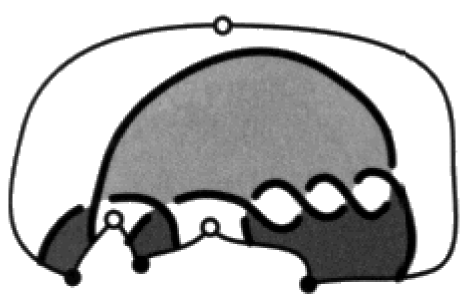


Fig. 55

We then place it in the space created in the cut knot by the opening of the punch in order to form the desired regular assembly:

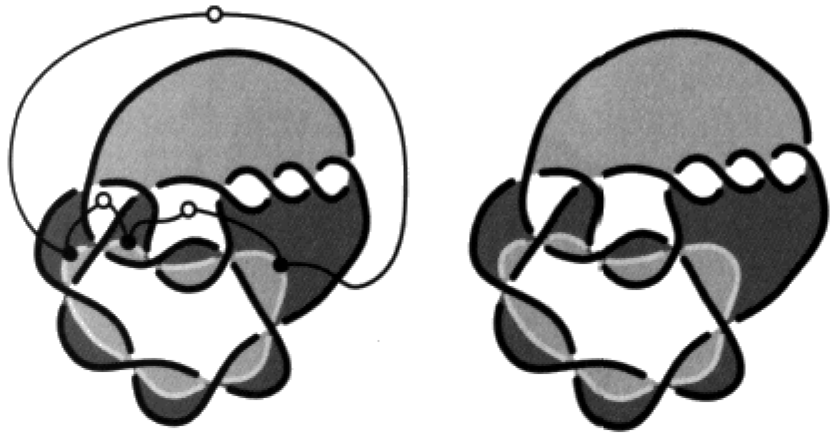


Fig. 56

This regular assembly reflects the composition of the node and non-node parts of any object. To complete this description of alternating and non-alternating chains and nodes, we still need to explain

— how regular assemblies can be compiled thanks to a remarkable property for each order corresponding to the number of components of their node and non-node parts;

— how any object is obtained from such an assembly;

— Conversely, how can we always find the regular assembly corresponding to any object colored by our algorithm?

First, let's look at two more examples of such regular montages, but this time with several components with cut and non-cut parts, assembled regularly.

Other examples

First, let's take a case where there are two components with cut parts and one component with non-cut parts.

Here are the three components:

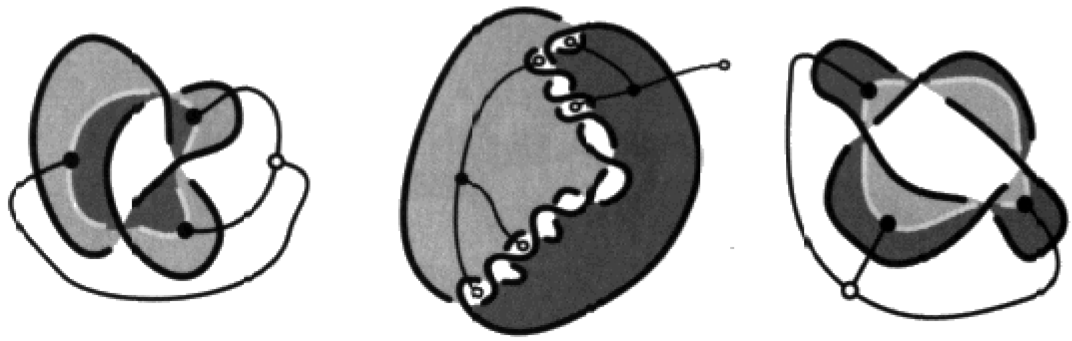


Fig. 57

Here is the regular circuit in question; it is non-alternating:

Fig. 58

A regular assembly forms an object whose node parts and non-node parts can be strictly isolated, such as combs and cut nodes. We will use it later in Fig. 88.

Third example

Let's return to our second example, where there is one cut component and two non-cut components.

Fig. 59

Let's detail the assembly procedure in this case.

We open the punches placed in parallel in the cut node, in series in the two non-nodes.

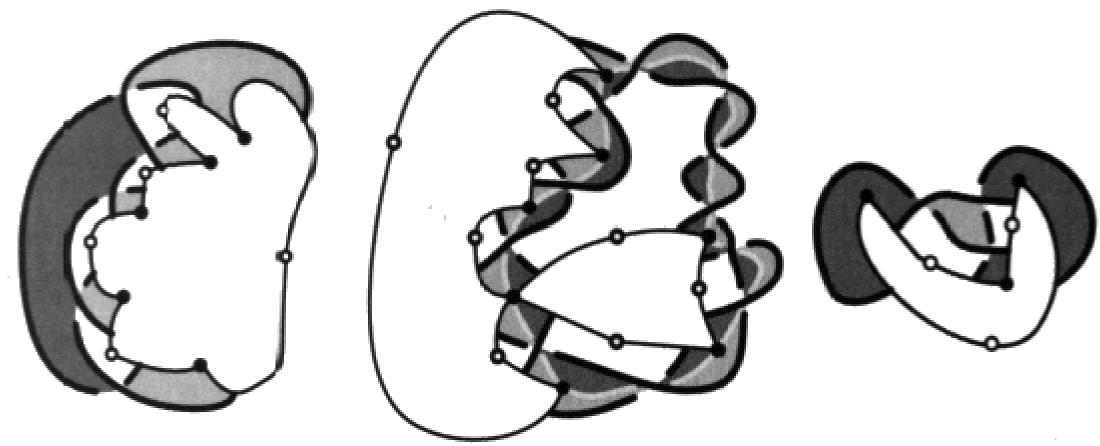


Fig. 60

We reverse two of them, the non-nodes on either side of the figures above:

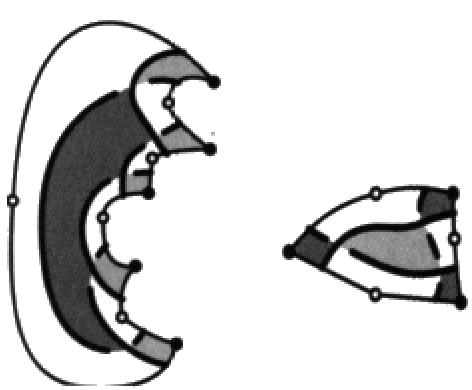


Fig. 61

We then place them in the two spaces created in the cut node by opening the punches:

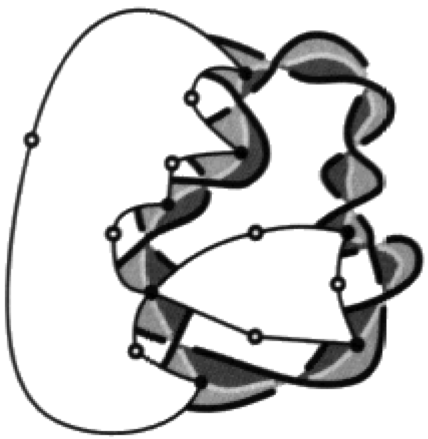


Fig. 62

This is done in order to form the desired regular non-alternating assembly.

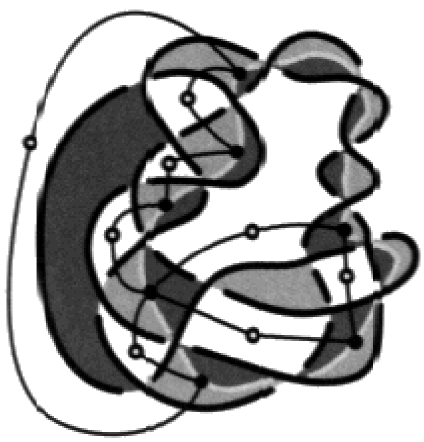


Fig. 63

These regular assemblies reflect the composition of the node parts and non-node parts of any node. We will explain this to complete this description of alternating and non-alternating chains and nodes.

But first, we give an important property of regular assemblies. a2 - ***Remarkable***

property of these assemblies

A regular assembly with two components, a node part and a non-node part, corresponds to a three-strand braid, a 3-braid.

Fig. 64

This regular assembly of two components can be represented as a skein that shows this correspondence; this skein presents it as a circular closure of a 3-braid.

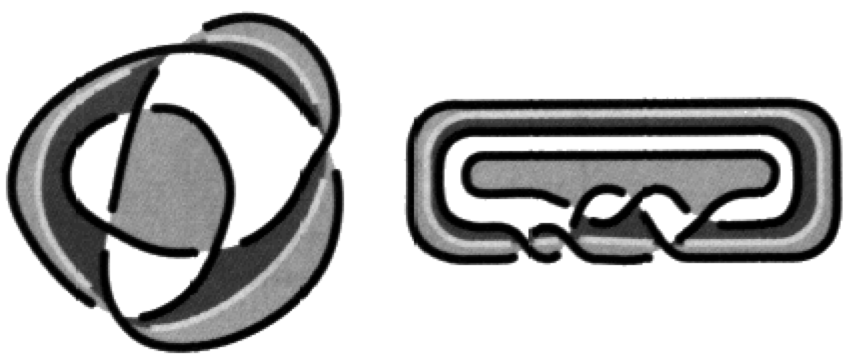


Fig. 65

We can always establish this correspondence between a regular assembly of two components and a 3-braid. It suffices to note that the definition of the regular assembly of these two components ensures the existence of two extreme points that are easy to define and a path that joins them.

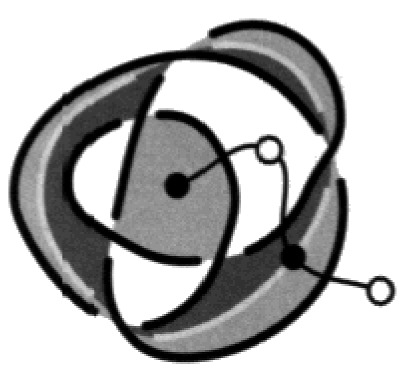


Fig. 66

There is a full point in the Terrasson graph in a full area of the non-node component, which was not used when assembling the comb; this full area is monochrome in the non-node part.

There is a void point in the Terrasson graph in an empty area of the cut node piece, which was not used during regular assembly; this empty area is surrounded by the cut in the node part.

In this case, we will call this solid area the central area, and the empty area the peripheral area.

The shortest paths connecting the point in the central filled area to the point in the peripheral area in Terrasson's graph are always the same length.

In the case of two components, it consists of three edges.

All that remains is to change the presentation of the regular assembly, using topological deformations of degree zero based on this fragment of the T-graph.

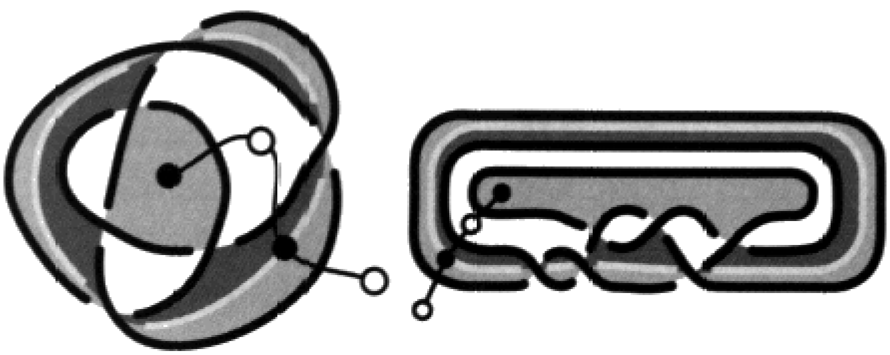


Fig. 67

If we open this subgraph to form the edge of a punch, as we have learned to do since we turned it over to form the punch, the braid is then perfectly defined, enclosed in this punch, as a regulated object.

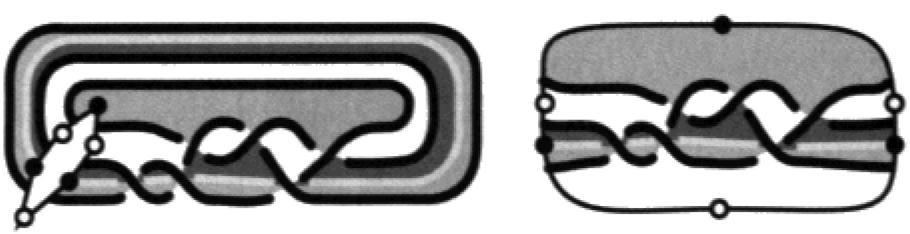


Fig. 68

Thus, in order to form the skein that reveals the braid defined by the regular assembly, it is sufficient to know how to find the central point and the peripheral point. We hope that the way in which we have described them above will be sufficient for the reader to learn how to find their way around these drawings by practicing this determination.

Conversely, any three-strand braid closed in a circular manner produces a regular pattern made up of two parts, a knotted part and a non-knotted part.

Fig. 69

This construction process does not exactly determine the number of circles in the regular assembly produced. It will consist of one, two, or three circles at most.

The closure in a skein is not so strict either; the closure of the braid can be designed differently to give other presentations of the object. But the skein is always present, with duality, on the sphere.

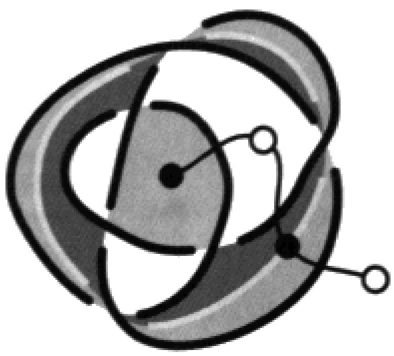


Fig. 70

This result, demonstrated by construction, which is always feasible, can be extended to give rise to a principle that matches the number of parts to the number of strands in the associated braid.

a3 - ***Extension to n parts, n + 1 braids***

Any regular assembly made up of n components corresponds to an (n + 1)-braid.

We can make the same observation when the regular assembly has more than two parts. There are always two extreme parts of the regular assembly that allow us to find the central point and the peripheral point.

Fig. 71

There are always two points (solid or empty) marking two areas of two components that have not been used by the assembly. We call these the extreme parts of this assembly. For example, a regular assembly consisting of three parts corresponds to a 4-braid.

Three parts comprising a regular assembly, if taken separately from the cut nodes and combs, represent six specific areas. In each comb there are two solids, and in each cut node there are two voids.

Four of these areas are used to regularly assemble the components in pairs, meaning that the two outer areas are not used in the assembly. They refer to the outer parts. We advise readers to check this method of counting on an actual assembly of several parts. An actual assembly consists of assembling the parts using combs and cut nodes.

All minimum paths connecting the extreme parts have the same number of edges. When there are n components, the minimum paths are of length (number of edges) n + 1.

Here is a suggestion for such a path in the chosen example:

Fig. 72

In fact, in the case of this 3-assembly, one of the shortest paths in the Terrasson graph goes from one extreme point to another, has four edges, and necessarily crosses four arcs. The same is true for the rest.

We are therefore dealing with a 4-braid here, and in the general case of an n-assembly, it will be an (n + 1)-braid.

Since regular montages of n parts alternately nodes and non-nodes are (n + 1)-braids, they can be compiled and described exhaustively for each order marked by this number n.

In the table on the following page, we give some examples of regular weaves made of two parts obtained by braiding three strands.

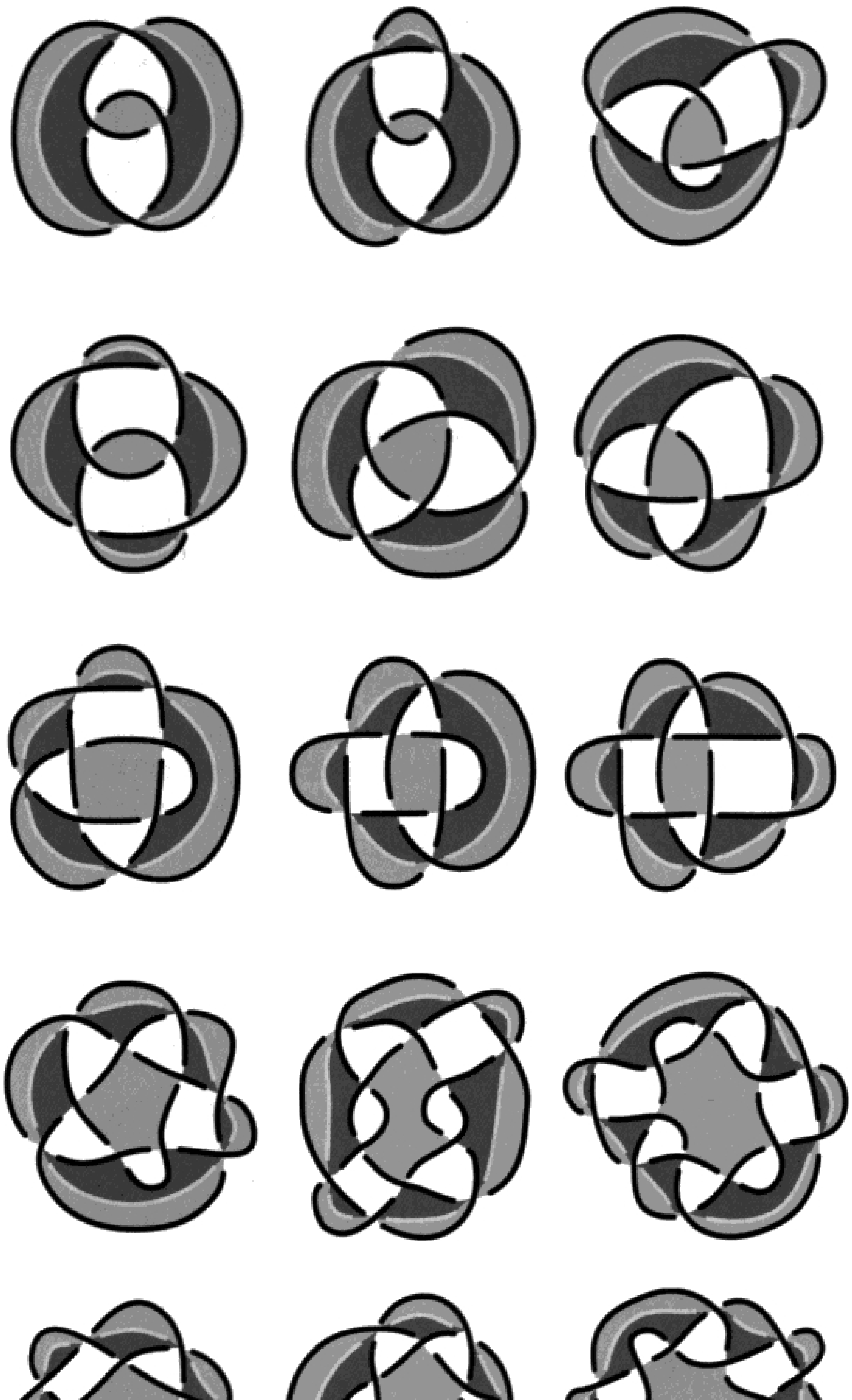
We leave the compilation of braids in the case of three, four... n strands for separate studies outside this work, but we should point out that they must be carried out based on the permutation groups of n objects, usually denoted S_n, in each case.

N-braids give m-chains, counting *m* rounds with m ≤ n.

Armed with this object, the regular assembly, characteristic of the assembly that governs the construction of any object we are theorizing about, we now turn to the description of a general and arbitrary case of an alternating chain or knot, before generalizing to non-alternating cases.

But first, let us table the first regular assemblies obtained from 3-braids

Tables of regular assemblies from 3-braids



3. **Transition from regular assemblies to arbitrary cases**

In any case, a knot or chain is an assembly of pure knots and non-knots made according to the regular assembly method we have just defined.

Pure nodes are composed in a mode known as derivation via a subgraph of the T-graph in a star configuration around a void point. Non-nodes are composed in a mode known as series via a subgraph of the T-graph in a star configuration around a solid point.

In the previous chapter, we established that any pure node comes from a cut node by adding cross-connections to it. Similarly, we established that any non-node is obtained from a comb by making cross-cuts in it.

Consequently, any node or chain can be obtained from a regular assembly on which we make cross-sections that can be isolated in its components.

a1 - ***Let's add the cross sections***

Let's take another example of a regular assembly we have already seen and make four cross sections, two for each part.

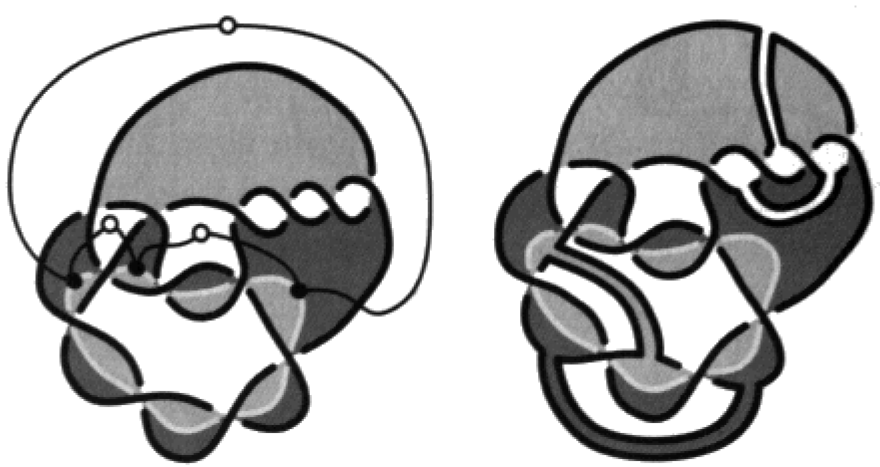


Fig. 73

We modify its presentation by zero-order movements in order to obtain a case that presents all the desired generality of any object.

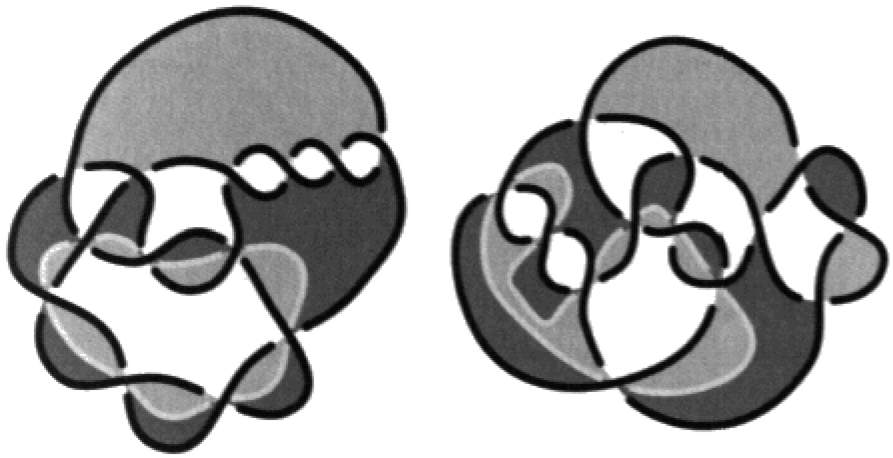


Fig. 74

There is therefore a relationship in our graphical description of objects between this regular assembly and this least specific case.

Conversely, any object can always be related to a regular assembly. This is what we want to show now, starting by clarifying this relationship.

a2 - ***Let's formalize the cross sections***

The cross sections we have been discussing since the previous chapter can be formalized graphically as transformations of the Terrasson graph.

Cross sections are openings and closings of portions of two edges of the T-graph. Or, to put it another way, they are inversions modulo a 4-punch, in the sense that we will specify, of such pairs of edges.

For a transverse cut, the midpoint between the two starting edges is, in this case, a full vertex.

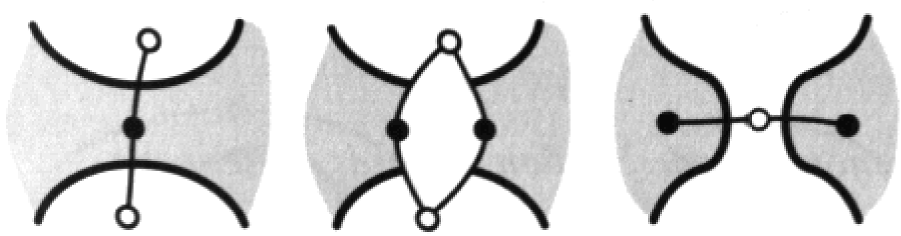


Fig. 75 The

inversion causes the midpoint of the transformation result to be a void point.

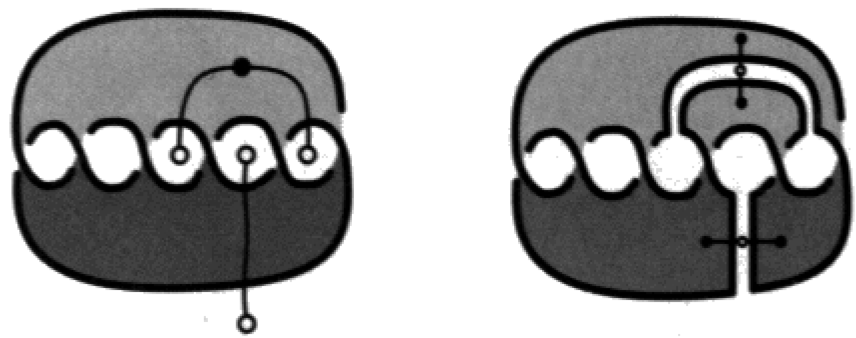


Fig. 76

This also applies to a transverse ramp in a dual manner:

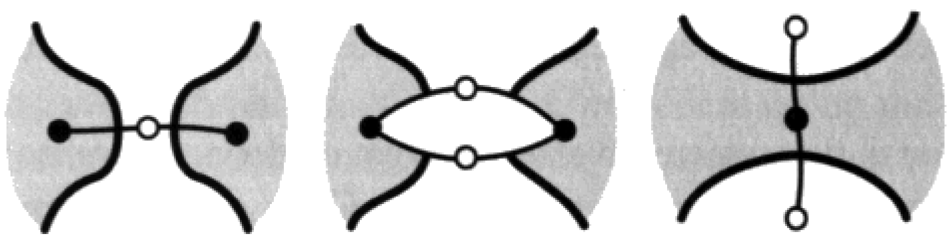


Fig. 77

The midpoint between the two starting edges is a void vertex. The inversion of the punch causes the midpoint of the transformation result to be a solid point. This is indeed the inverse transformation of the previous one.

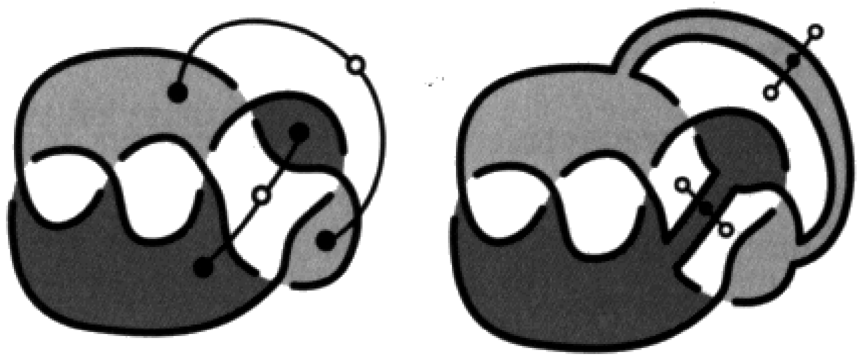


Fig. 78

It is therefore necessary to construct a category of Terrasson graphs and their transformations.

a3 - ***Let us return to the description of any case***

Consequently, any case of a node or chain will be obtained from a regular assembly of multiple components on which we invert punches.

In our previous example, we mark the portions of the T-graph made up of two edges that will give rise to the inversion of the punches.

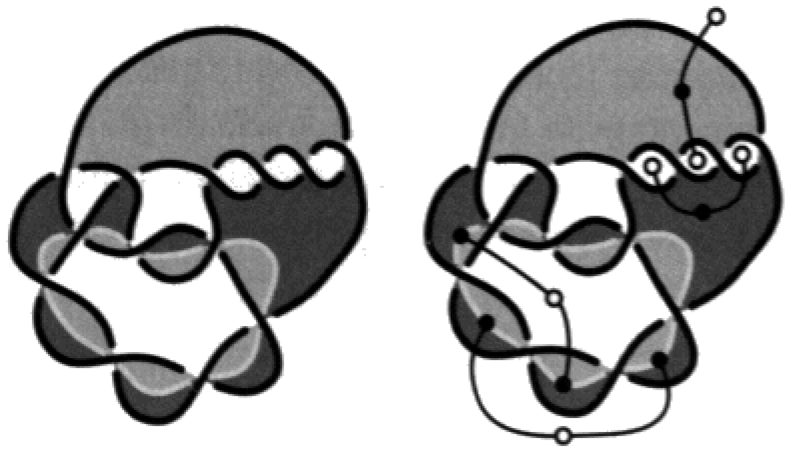


Fig. 79

Let's invert the punches as we have just shown:

Fig. 80

This gives us an arbitrary object that does not have the characteristic of regular assemblies, i.e., being composed exclusively of cut nodes and combs. Since the regular assembly shows us how the parts are composed, in the specific case of the comb and the cut node

We can assemble the cut parts, made of pure nodes, with the non-cut parts made of non-nodes, as the cut nodes and combs that make up a regular assembly are composed.

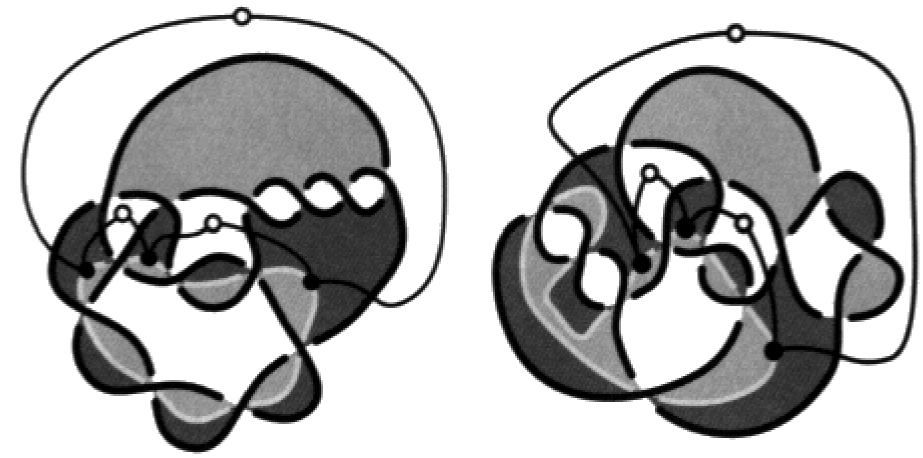


Fig. 81

These are indeed the same type of assemblies that we have called regular and which give their names to the objects that represent them. The assembly is described using the Terrasson graph cycle.

We open the punches in a similar way, on the side of the components with a cut:

Fig. 82

We do the same on the side of the components without cuts.

In addition, here we turn the edge of the punch in the same way:

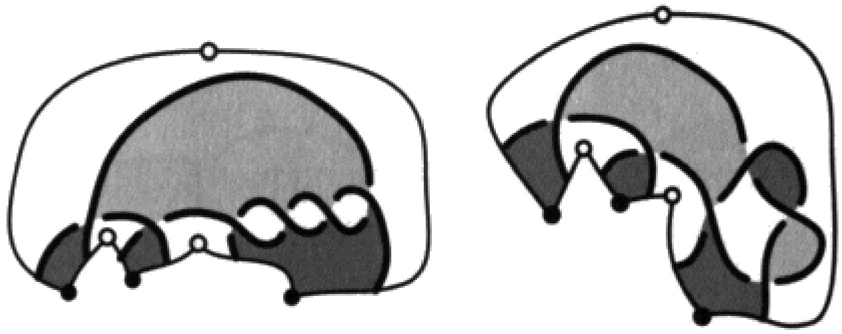


Fig. 83

We thus show how a regular assembly gives the assembly principle for any case

This being established on a single example, it is important to demonstrate that we can move from regular assembly to any case by means of transverse cuts to define the graphical relationship that supports our description. This relationship and its inverse are as follows:

Fig. 84

If the cross sections are made between the node and non-node parts of the regular assembly, i.e., they do not respect the cycles of the T-graph that separate these parts, we will refer to another assembly. This means that there may be a set of cross sections between different regular assemblies. But we know how to characterize the sections that correspond to a regular assembly and any object.

These sections do not cross the cycle that forms the boundary of the regularly assembled punches.

Conversely, this encourages us to follow the reasoning that ensures we will always find the regular assembly corresponding to any given case:

— since an object can always be broken down into node and non-node parts;

— that these parts are always pure knots and non-knots;

— that these pure nodes and non-nodes are always produced from cut nodes and combs by cross sections. This is demonstrated in the previous chapter for non-nodes and applies in a dual manner to pure nodes.

We then always obtain the regular assembly from which an object originates by applying the same transformations in reverse.

Here is an example using the same relatively simple case (only two parts), but one that is already quite general.

Fig. 85

We thus show how, from any case, we find the regular assembly that corresponds to it and how any case can always be obtained from a regular assembly.

Let's give some more examples of non-alternating cases

Let's look at some other examples of this relationship between regular assembly and any object. Here we see that several assemblies produce the same object, apart from a change in color.

In the case of a regular assembly already constructed above in example fig. 63, these will be transverse straps in the knot section:

Fig. 86

Conversely, we can reconstruct the regular assembly in question:

Fig. 87

In order to reconstruct the regular assembly, the part made up of a cut node is obtained from any pure node by making transverse cuts.

And in the case of the other regular assembly constructed above, it is the transverse cuts in the non-knot part that give the non-alternating chain, the knot of July 23, 1993, which we have already discussed in previous chapters of this book:

Fig. 88

In the other direction, the part consisting of a comb in the corresponding regular assembly is obtained from any non-knot by adding transverse straps:

Fig. 89

a4 - ***This is why there is complication***

This is why there is complexity in knots and chains. We have shown that their richness and complexity is due to the transverse cuts and ribbon straps in regular assemblies.

This complexity generally prevents any knot or chain from being presented as a tangle, as is the case with what we have called regular assemblies, where there are no additional straps or cuts.

This complexity prevents the theory of chains and knots from being directly reducible to the theory of braids.

a5 - ***Non-alternating cases***

Non-alternating cases are obtained from alternating cases by replacing crossings of the given torsion with crossings of inverse torsion. This means that they follow the frames from a graphical point of view.

They have additional nodal, plastic, and topological properties.

In conclusion, we can now answer the question about the closure of tangles that L. Kauffman asked us at our home one evening after dinner. If we respect our colorings and, under these conditions, decompose the objects according to the T-graph cycles that separate the knotted and non-knotted parts of these objects, then there is always a way to close the punches obtained, and it is unique, such that:

— the node parts turn out to be pure nodes mounted in series (closing the edge of the punch gives a star graph around a void vertex);

— the non-node parts turn out to be non-nodes mounted in series (closing the edge of the punch gives a star graph around a full vertex).

Now that this clarification has been made in the graphical description of the objects, let us return to our **T0** theory of entanglements in order to continue developing the plastic, i.e., nodal, work of our object.

This work is entirely defined by the movements of the theory, the Reidemeister movements augmented by improper and proper Gordian movements (homotopies) presented in Chapter IV as a space of deformation specific to knots and which erase them.

It gives rise to a nodal description that has an arithmetic, i.e., numerical, translation, and we want to discuss the graphic consequences of this in the reading of the drawings.

We now move on to the second part of this chapter.

II. Nodal plasticity

0. **Knots**

a1 - ***Quantification of movements***

Let us return to the set **E0**, defined in Chapter IV, of movements of different types:

TE0 = { **B1**, **B1***, **M2**, **T3**, **G**, **H**, **H*** }

in an enumeration that we arrange in a table, as shown in Table **1** below.

We define an application f of this set in the numerical set {–1, 0, +1}, that is: φ: $E_0 \rightarrow \{-1, 0, +1\}$

This application assigns a numerical value to each movement, based on the number of twists involved in the crossings. This basic calculation is summarized in Table **2** (see following pages) accompanying the previous one.

Given a change in presentation ***$*** defined by the series (x_1, x_2, …, x_i, …, x_n) with X_i□$\in E_0$, we can define the set s of numbers in the series:

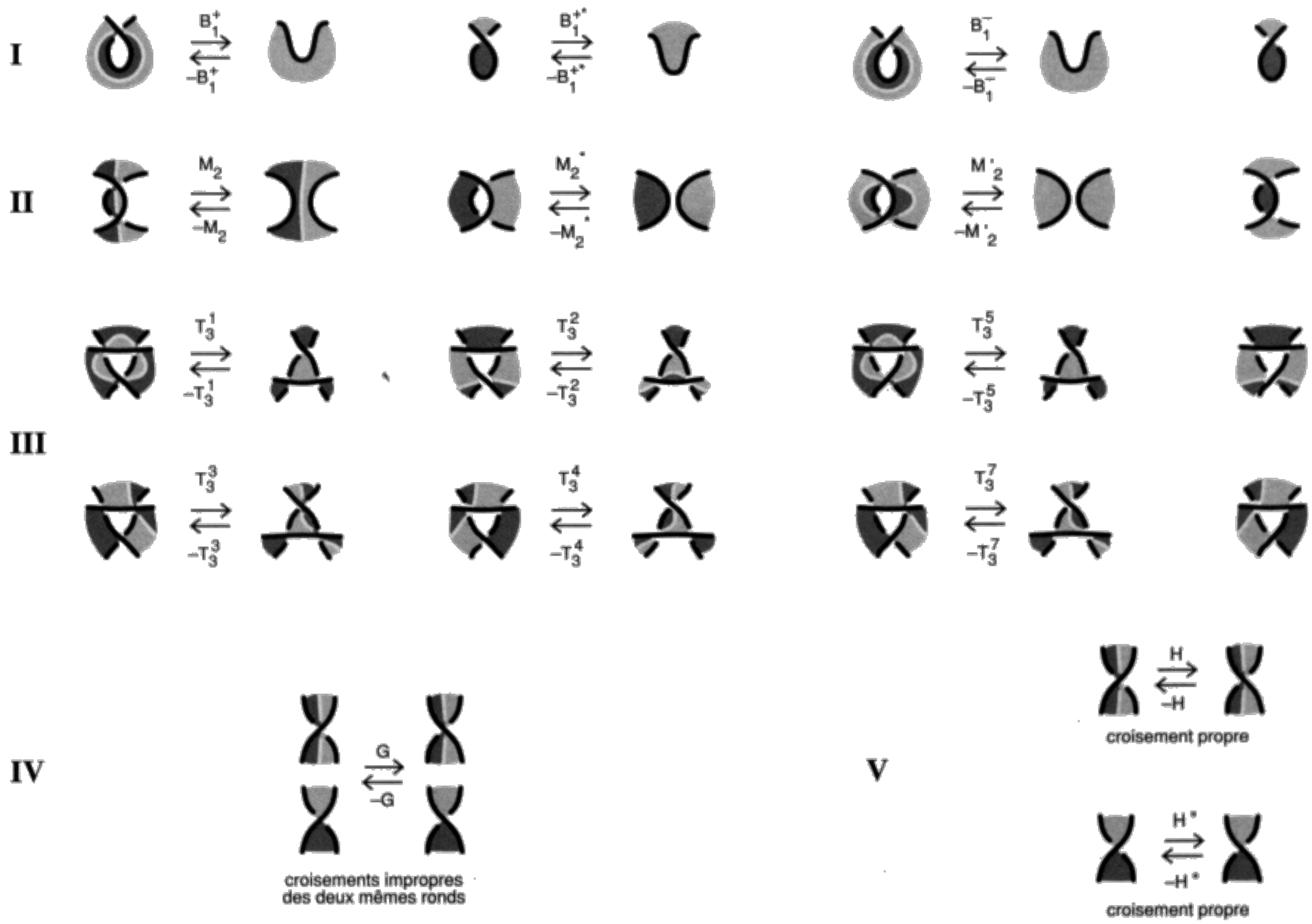
s = { b, b*, t, g, h, h*}

such that each of them is the sum of the values of the movements of each type considered in TE_0.

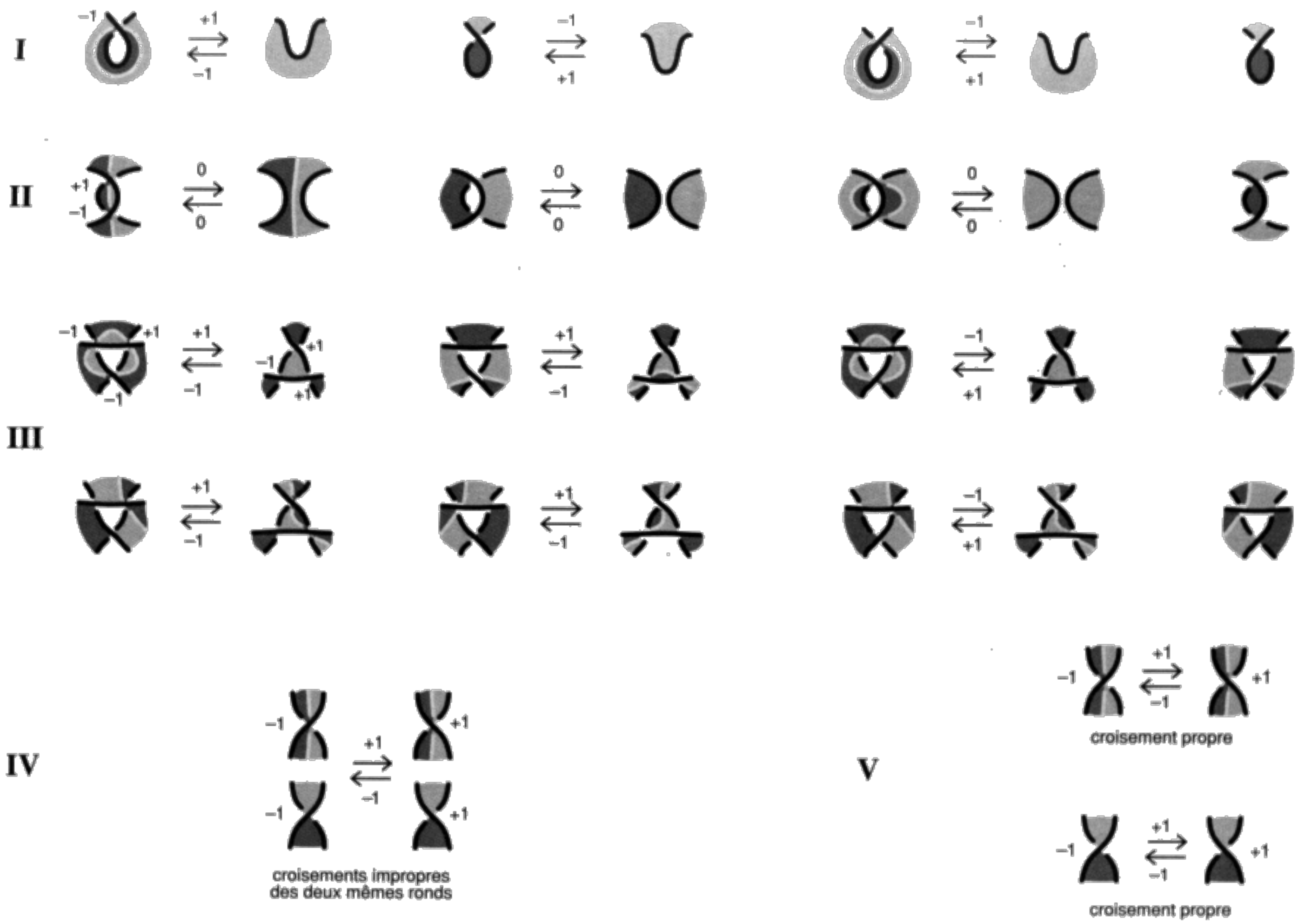
Movements **M2** correspond to a zero digit. Thus:

$$b = \sum_{X_i \in B_1} \varphi(X_i), \qquad b^* = \sum_{X_i \in B_1^*} \varphi(X_i), \qquad t = \sum_{X_i \in T_3} \varphi(X_i)$$

$$g = \sum_{X_i \in G} \varphi(X_i), \qquad h = \sum_{X_i \in H} \varphi(X_i), \qquad h^* = \sum_{X_i \in H^*} \varphi(X_i)$$



I
B_1^+
$-B_1^+$
B_1^{+*}
$-B_1^{+*}$
B_1^-
$-B_1^-$
II
M_2
$-M_2$
M_2^*
$-M_2^*$
M'_2
$-M'_2$
III
T_3^1
$-T_3^1$
T_3^2
$-T_3^2$
T_3^5
$-T_3^5$
T_3^3
$-T_3^3$
T_3^4
$-T_3^4$
T_3^7
$-T_3^7$
IV
G
–G
croisements impropres
des deux mêmes ronds
V
H
–H
croisement propre
H*
–H*
croisement propre



We thus have an application **F** that maps each change in presentation ***$*** to a set **s** of digits:

F(***$***) = s

a2 - ***The number of knots***

Given two presentations s_1 and s_2 of a knot or string, equivalent by the relation R_0, i.e., one is the transform of the other by a change of presentation ***$***, then
:

s_2 = ***$***($s_1$)

We can describe the average number of cuts Σ (s_2) of one as being the average number of cuts Σ (s_1) of the other, increased by the number of knots N_s of the change in presentation:

Σ (***$***($s_1$)) = Σ ($s_1$) + N_s

For a given span area, we have defined the dual span area obtained by exchanging solids and voids.

Thus, we describe in the same way the average dual cut number $\Sigma^*(S')$ of one in terms of the average dual cut number $\Sigma^*(S)$ of the other, and the dual knot number N^*s of the change in presentation that makes them correspond:

$$\Sigma^*(\$(s_1)) = \Sigma^*(s_1) + N^*s$$

Knot expression

Given two presentations s_1 and s_2 of a knot or link, one of which is the transform of the other by the change of presentation ***$*** of encryption s, the number of knots Ns and its dual N^*s are written as:

$$Ns = b + t + 2\,(h + g)$$

$$N^*s = -\,b^* - t - 2\,(h^* + g)$$

This proposition can be demonstrated for each elementary case, based on the definition of Σs, i.e., its expression in terms of the number of crossings of each type, the effect of elementary movements on the types of crossings, and taking into account the quantification of these
movements.

Thus:

$$\Sigma\,(\$(s_1)) = \Sigma\,(s_1) + b + t + 2\,(h + g)$$

$$\Sigma^*(\$(s_1)) = \Sigma^*(s_1) - b^* - t - 2\,(h^* + g)$$

Quantification of our example

Let us return to the two extreme figures in the series of movements already encountered as an example in Chapter IV:

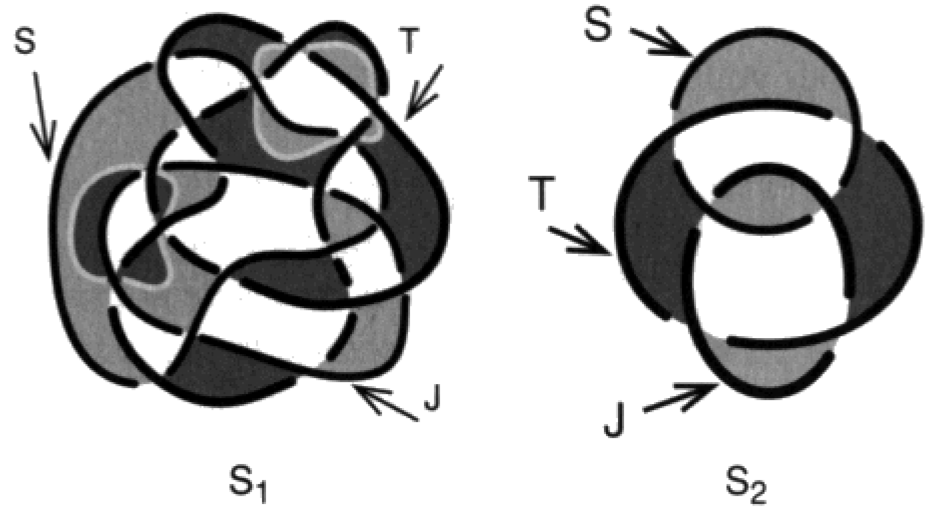


Fig. 90

This is the weighted series in plus and minus, given in Chapter IV:

(+G, + T3, M2, M2, – T3, M2, + T3, + T3, + T3,

– T3, – T3, M2, – B1, + T3, M2, + G, M2, 2 (M2),

2 (– T3), 3 (M2),4 (– T3),2 (M2), + B1, – B1)

The set of digits here is:

s = { b, b*, t, g, h, h*} = {–1, 0, –4, +2, 0, 0}

On the other hand, we know from the calculation that:

Σ ($_{S1}$) = –2, and that $_{cp}$ = –5.

Similarly, by calculating in the dual, we obtain the value of Σ* ($_{S1}$) = +3.

And we can establish $_{Ns}$ = –1 and $_{N*s}$ = 0, based on our proposition and the numbers in set s:

$_{Ns}$ = b + t + 2 (h + g) = –1 + (–4) + 2 (0 + 2) = –1 – 4 + 4 = –1

$_{N*s}$ = – b* – t – 2 (h* + g) = 0 – (– 4) – 2 (0 + 2) = + 4 – 4 = 0

However, for the Olympic chain $_{S2}$, we have the following figures:

Σ ($_{S2}$) = –3 and Σ* ($_{S2}$) = +3.

So we can verify the last two formulas:

$\Sigma\ (s_2) = \Sigma\ (s_1) + N_s -$

$3 = -2 - 1$

$\Sigma^*(s_2) = \Sigma^*(s_1) + N^*_s$

$+3 = +3 + 0$

This is what we wanted to verify in this example.

It is therefore clear that, given that the averages of the cut numbers Σs and $\Sigma^* s$, which are dual to each other and from the same presentation, are well defined, the knot numbers of two different presentation changes between the same two given presentations are equal term by term.

Invariance of the number of knots between two given presentations

Let there be two presentations s_1 and s_2 linked by two changes of presentation ***$*** and $', with respective encryptions s and s', then their respective numbers of knots are equal:

$N_s = N_{s'}\ N^*_s$

$= N^*_{s'}$

From this encryption, we conclude a very general consideration which states that a presentation s_1 of an entanglement represents a series ***$*** of movements for another presentation s_2 of the same entanglement. We summarize this formulation in the expression:

S1 → S2

—

$

and which we will use to deal with knots in their relation to the non-prototype knot of the entanglements of one to three circles that we listed in the previous chapter in the section devoted to nodal plasticity. We will then generalize these results to the theory of entanglements of four or more circles.

1. **Interpretation of Σs**

We will refer to the knot number N_s and dual knot number N^*_s of a presentation S of a knot or link, for r ≤ 3, as the dual knot numbers of each other of any change of presentation that transforms the non-knot s_0 contained in this knot or link into this presentation S:

$N_s = \Sigma_S - \Sigma 0$

$N^*_s = \Sigma^*_S - \Sigma^*0$

knowing that $\Sigma^*0 = -\Sigma 0$ as we have just recalled.

We can thus interpret the average of the numbers of the cut of a presentation and its dual number, using the formulas:

$\Sigma_S = b + 2h + t + 2g + \Sigma_0$

$\Sigma^*_S = -b^* - 2h^* - t - 2g + \Sigma^*_0$

where **s** = {b, b*, t, g, h, h*} is the encryption of any change in presentation that transforms S_0 into S.

Encryption of our example

Let's show this using the same example:

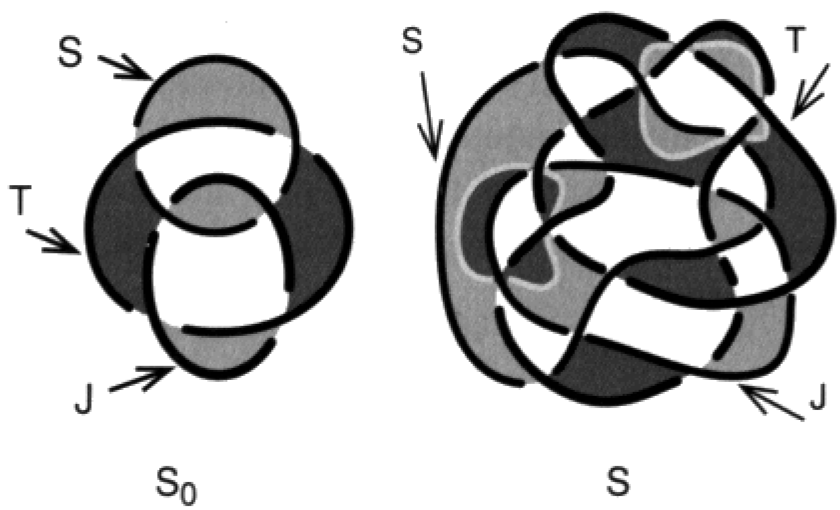


Fig. 91

If we consider the series from the previous example reversed,

\$: $S_0 \rightarrow S$

we can establish the set **s** of its digits that separate and connect these two figures, by repeating the series in reverse, or by marking the opposite signs:

s = { b, b*, t, g, h, h*} = {+1, 0, +4, –2, 0, 0}

We already know the respective averages of the numbers in the cut:

$\Sigma S = -2$, $\Sigma^* S = +3$, $\Sigma 0 = -3$, $\Sigma^* 0 = +3$

and the numbers of knots of ***$*** or S, except for the opposite signs: $N_s =$

$+1$, $N^*_s = 0$

This time, we will go from s_0 to S, unlike the previous calculation where we went from $s_1 =$ S to $s_2 = s_0$.

This reverse calculation gives the formulas for writing:

$\Sigma S = N_s + \Sigma 0 = +1 + (-3) = -2$

$\Sigma^* S = N^*_s + \Sigma^* 0 = 0 + (+3) = +3$

or, more precisely, referring to the set s of numbers in the series leading from s_0 to S, the two formulas:

$\Sigma S = b + t + 2(g + h) + \Sigma 0 = +1 + 4 + 2(-2 + 0) + (-3) = -2$

$\Sigma^* S = -b^* - t - 2(g + h^*) + \Sigma^* 0 = 0 - (+4) - 2(-2 + 0) + 3 = +3$

2. **Node part, knot part, and non-node part**

a1 - ***Knot part and non-knot part***

In any representation of a knot or oriented chain using coloring, we can associate a number of crossings that characterize them with the knot parts (parts where the cut passes) and the non-knot parts (parts where the cut does not pass).

At the knot part, the number of cuts is $k_i S$.

At the non-node part, the number of cuts in the dual $k^*_i S$.

From the main corollary of our previous chapter, we know the expression of these numbers, which accounts for the movement of the cut through the different colorings of the same presentation:

$k_i S = \Sigma S - \Sigma i$

$k^*_i S = \Sigma^* S - \Sigma i$

In our example:

Fig. 92

We have already verified these formulas for the different colorings, in the case where $\Sigma i = -3$, and we calculated: $\Sigma S = -2$, to verify a little further on that $kiS = +1$, i.e.:

$kiS = \Sigma S - \Sigma i$,

as proposed by our main corollary. We can perform the same calculation in the dual where:

$k^*iS = +6$ and $\Sigma^*S = +3$

thus verifying that: $k^*iS = \Sigma^*S - \Sigma i = +3 - (-3) = +6$.

Now we have just expressed the averages in terms of the number of knots, which are dual to each other, and the number of intertwining independent of coloring:

$\Sigma S = Ns + \Sigma 0$

$\Sigma^*S = N^*s + \Sigma^*0$

Thus, the knot part and the non-knot part can be expressed in terms of the number of knots:

$kiS = Ns + \Sigma 0 - \Sigma i$

$k^*iS = N^*s + \Sigma^*0 - \Sigma i$

However, what is verified by the number of cuts in any presentation is also verified by our standard non-nodes. That is, if we call $ki0$ and k^*i0 the dual cuts of each other of these non-nodes, for different colorings *i*:

$ki0 = \Sigma 0 - \Sigma i$

$k^*_{i0} = \Sigma^*_0 - \Sigma_i$

This therefore gives us the expression of the node part and the non-node part as a function of the knots and any cuts in the non-node content:

$k_{iS} = N_s + k_{i0}$ $k^*_{iS} =$

$N^*_s + k^*_{i0}$

Let's return to our example, in orientation $i = 1$. We calculate $k_{iS} = -3$ and $k^*_{iS} = +2$.

However, we know that $N_s = +1$ and $N^*_s = 0$,

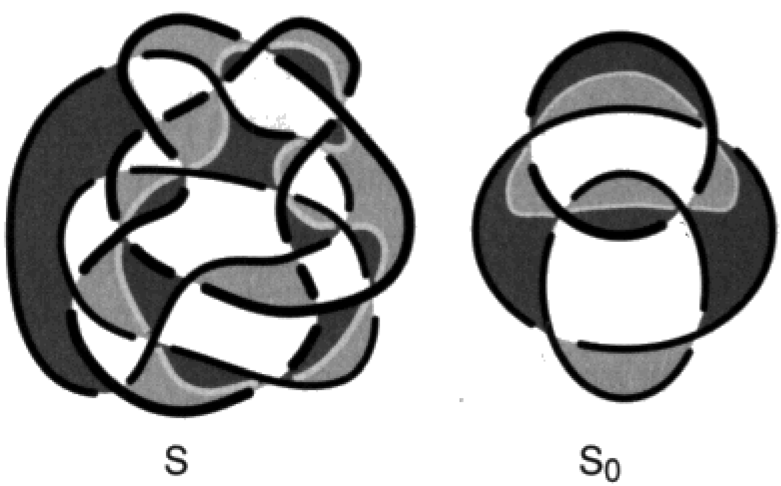


Fig. 93

$k_{iS} = N_s + k_{i0}$, or $-3 = +1 - 4$

and $k^*_{iS} = N^*_s + k^*_{i0}$, or $+2 = 0 + 2$

a2 - ***Knotted part, non-knotted part, and Lacan part***

Since we have chosen the non-knots in such a way that they have an uninterrupted presentation, there is indeed a coloring $i = 0$ for which $k_{i0} = k_{00} = 0$, and therefore $k^*00 = -{}_{c0}$.

In our example, the orientation $i = 0$ gives the cut $k_{0S} = +1$ and in the contained non-knot $k_{00} = 0$, since there is no cut:

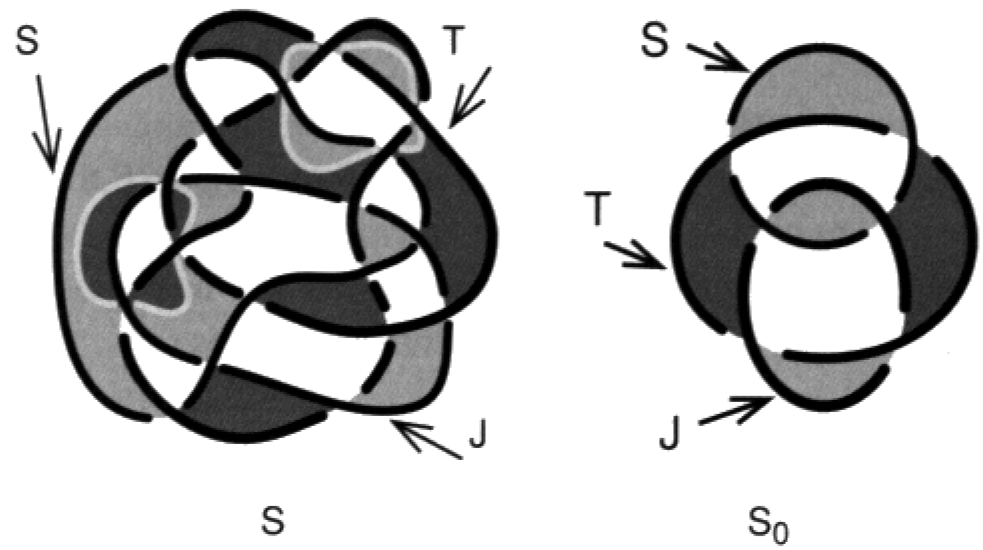


Fig. 94

Therefore, we are assured that there exists a coloring $i = 0$ such that for any presentation, its node part is:

$k_{0S} = N_s$

We will call this the knot part of the given presentation; it is its cut part in the coloring $i = 0$.

Hence the explanation for our choice to call the part where the cut passes the knot part. Our initial intuition was not wrong, since we can support it with the existence of this coloring $i = 0$, in which case it corresponds to the knot part. The point to emphasize remains that the number of Gordian movements that encode the knot is only contained in this number of knots but is not identical to it.

In our example, in fact: $k_{0S} = +1$ and $N_s = +1$.

The part where the cut does not occur, corresponding to the knotting part, is equal to:

$k^*_{0S} = N^*_s - c_0$

We will call this the non-knot part of the given presentation; it is the part where the cut does not pass in the coloring $i = 0$.

The non-knot part therefore contains a part corresponding to the dual knot number N^*_s, which we will call the Lacan part, in reference to what we have called Lacan knots, and the non-knot contained in the chain, which we will call the entanglement part.

In purely improper chain knots, the knot part and the Lacan part are opposite (b = b* = h = h* = 0).

The presence of proper crossings can make the knotting part asymmetrical to the Lacan part (b □ b* and h € h*).

In proper knots, the non-knot part is reduced to the Lacan part ($_{k0} = 0$).

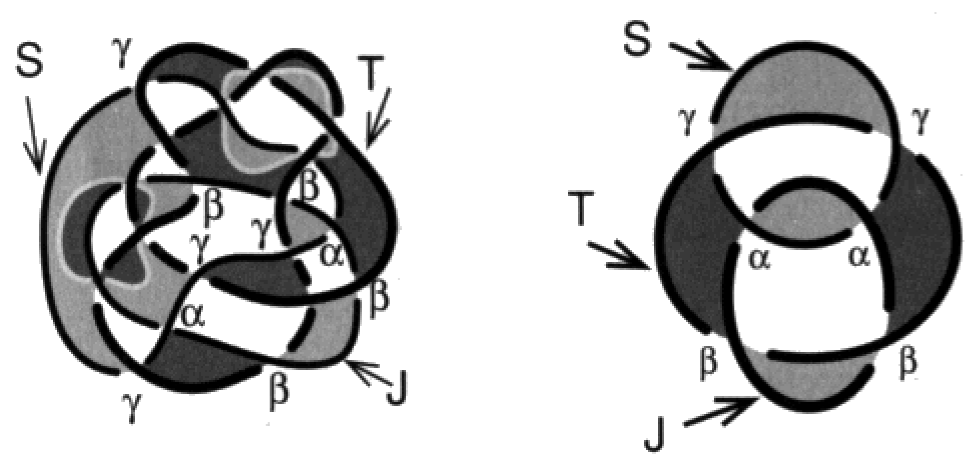
These calculations establish the terminology we have adopted when we talk about the knot part and the non-knot part of a knot or a chain, with the new precision they bring by distinguishing as the knotting part the smallest knot part, $i = 0$, that can be isolated among the colorings.

Thus, the variation in the cut depends on the entanglement due to its distributions studied previously.

Now, the movement of the cut follows the knotting through the movements studied here.

In alternating cases, non-zero knotting reveals the knot that we want to further specify, as it ensures the execution of Gordian movements and homotopies. We summarize this fact in our formula, which states that the knot is an accomplishment of cutting in our drawings, which are to be read insofar as we find traces of knotting and the non-knot contained within, particularly in alternating cases.

Let us return to the same drawing, indicating with the letters α, β, and γ the respective types of crossings of the circles JS, JT, and ST.



S S$_0$ Fig. 95

Where we see that the non-knot part of the S chain does indeed present the six respective crossings of the non-knot contained in this chain, to which are added crossings of each type constituting the Lacan part, to which we will need to return. The knotting part corresponds well to the knot part, to the cut in this minimum orientation, $_{k0S} = {}_{Ns} = -3$.

3. **Exercises**

e1 - ***Terrasson graph, node part and non-node part***

Here is a knot and its Terrasson graph:

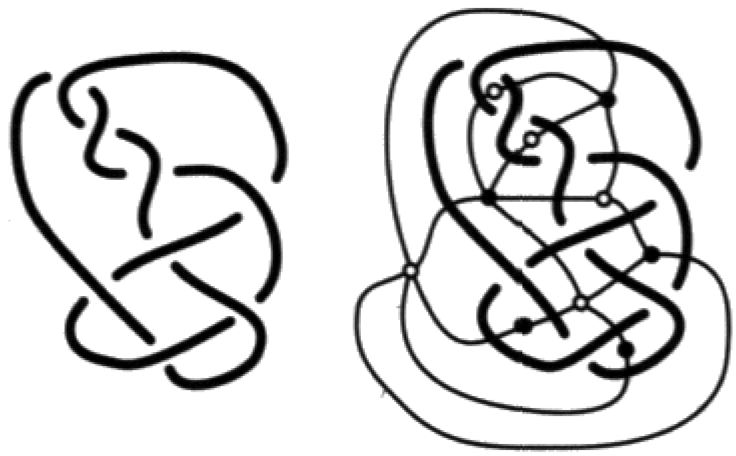


Fig. a

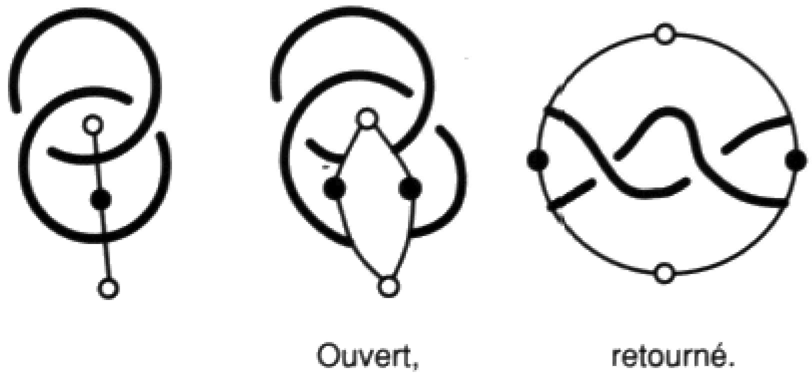
We propose retaining only the subgraph that separates the node part from the non-node part of the Terrasson graph.

Perform the same exercise for the following three node drawings:

Fig. b

e2 - ***Opening of punches, in both composition modes***

There is a transverse composition based on a portion of the Terrasson graph, whose midpoint is a solid point.



. Fig. c

There is a derivation composition based on a portion of the Terrasson graph, whose midpoint is a void point.

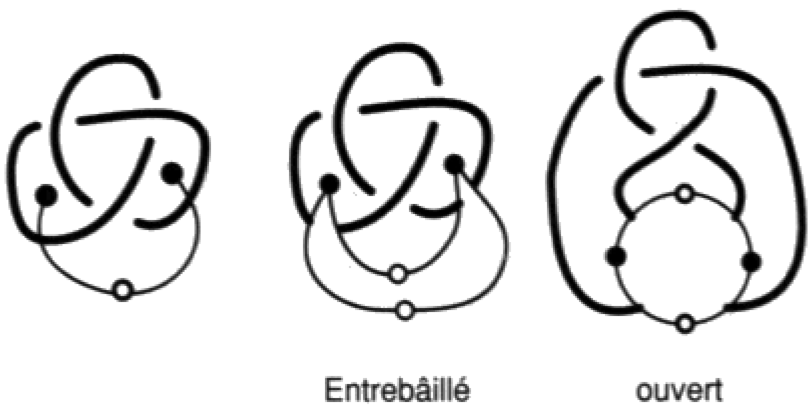


Fig. d

Let's open the following punches:

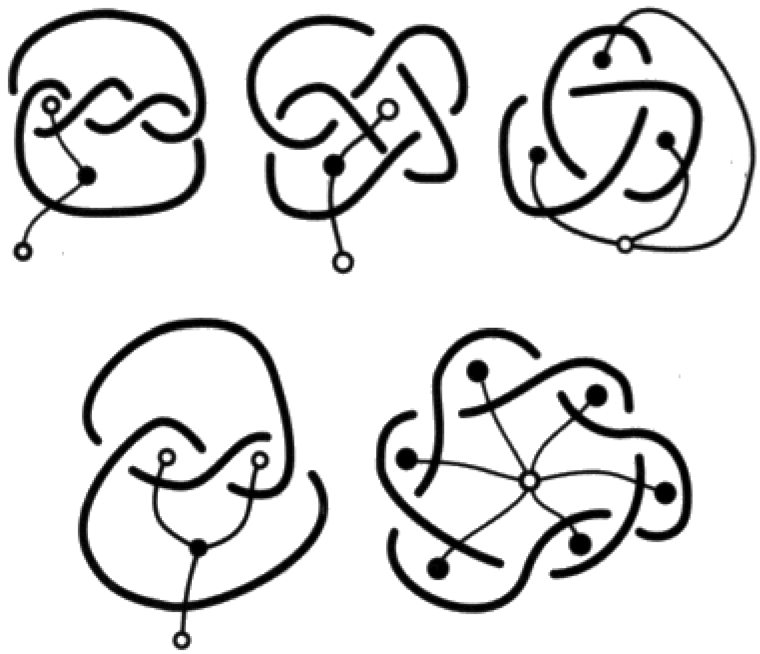


Fig. e

e3 - ***Composition of sources and node material***

We take open punches from the previous exercise to reassemble knots and chains.

What does the composition of these two parts give us?

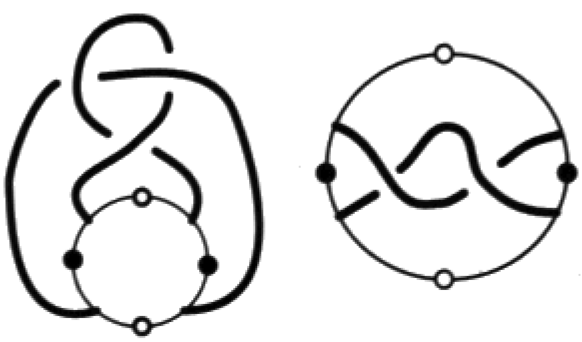


Fig. f

It is a regular composition, the parts have no cross sections. Same exercise

with these two elements, whose punches must be opened.

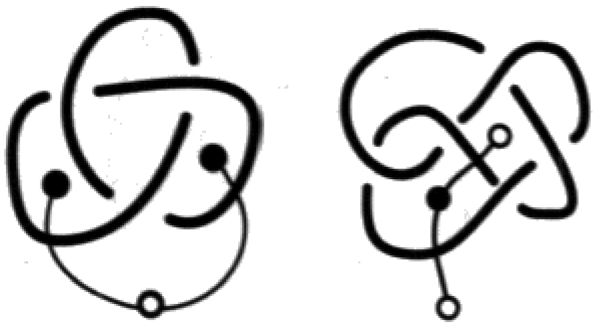


Fig. g

Note that the non-knot part is irregular; it is already a cut comb. Then, let's do the

same exercise with these two elements:

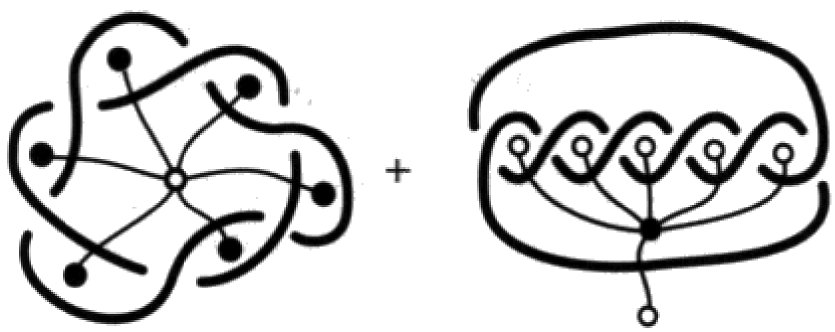


Fig. h

This is a regular assembly that produces a Slade chain.

How to compose the Borromean ring from the 3-clover. Is this a different composition from that of knot 63?

How can we compose Whitehead's chain from the 3-clover and entanglement? e4 -

The little object a in Chinese culture

We are familiar with the Chinese diagram derived from the Taoist tradition.

Fig. i

If we consider this diagram drawn on a sphere, we can complete it by showing the part hidden from view in a flattened version of the sphere with a hole in it.

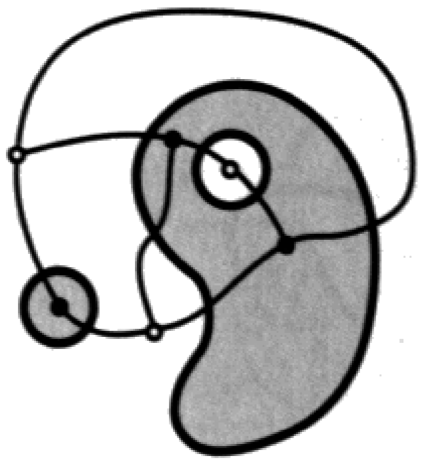


Fig. j

These are three circles placed on the plane. When, in a walk on this plane, a circle is crossed, we move from a full zone to an empty zone, or conversely from an empty zone to a full zone. It is important to change the quality of the zone when a circle is crossed. We can accumulate circles without ever making them meet. In this exercise, there are no crossings.

We have drawn the T-graph that structures this elementary configuration, as we are studying in this exercise.

Any configuration:

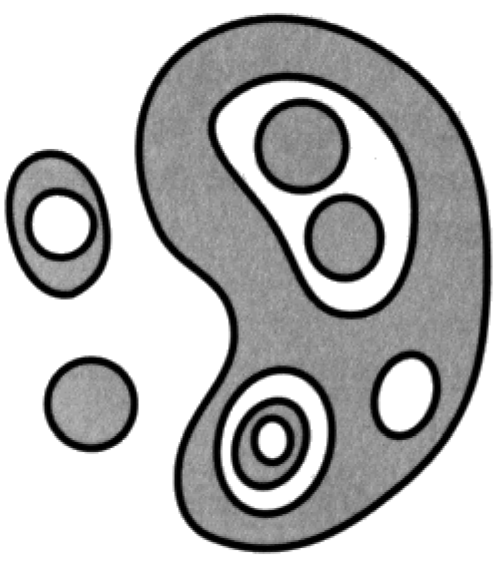


Fig. k

It can be broken down by such a graph, a Terrasson graph, into pieces of two types, and only two types.

We call them punches.

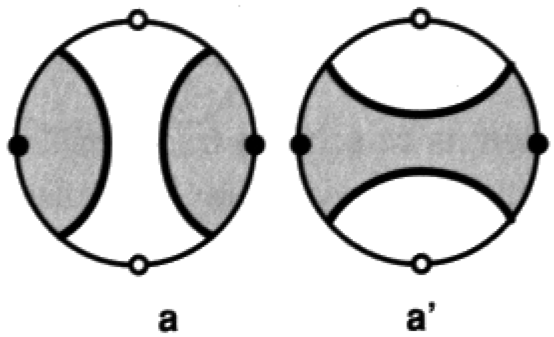


Fig. l

Punches are composed by joining two edges of their graph that form an edge:

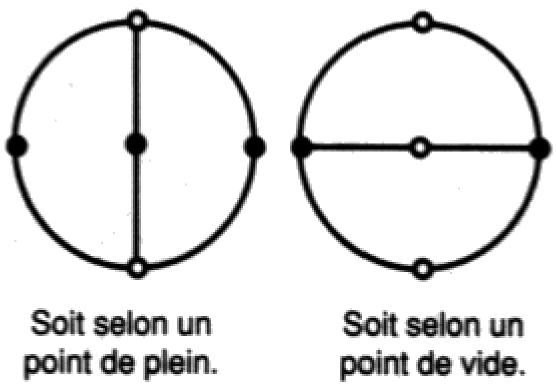


Fig. m

The combination of two punches produces a new generalized punch.

A clasp must be added to this combination of punches to obtain a configuration of the type proposed.

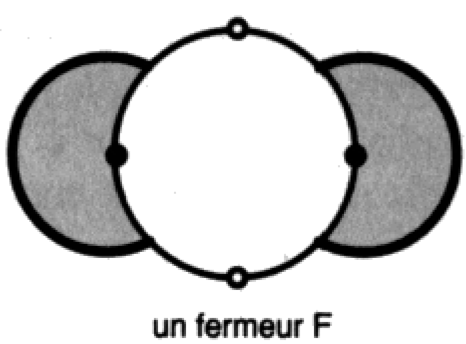


Fig. n

Question: How can we find the Terrasson graph that breaks down the given configuration in the case of any configuration of circles?

Method: Start by tackling the problem from both ends.

a — Try to break down increasingly complex configurations, starting with the simplest ones.

a' — Practice composing punches (a) and (a') in all directions of the plane.

Solution to the exercise on the small object a to the Chinese

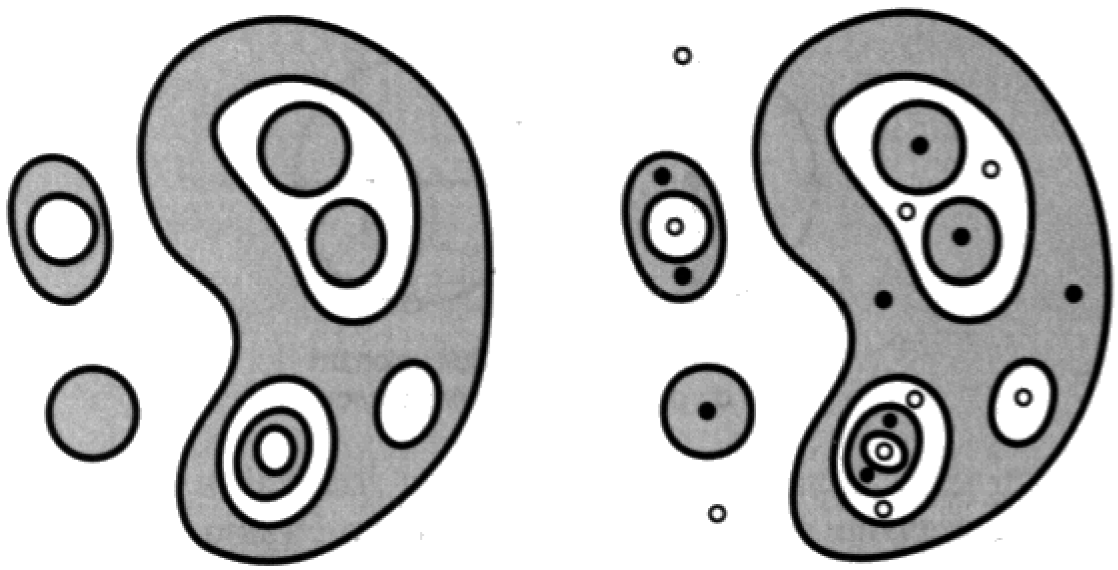


Fig. o

Simply place one dot in each area (solid or empty) that does not contain any other areas, and two dots for each area containing other areas. These solid and empty vertices are then connected by edges to form the Terrasson graph, which breaks down any configuration of circles.

Note: The composition of two punches (**a**) by solids adds a solid circle.

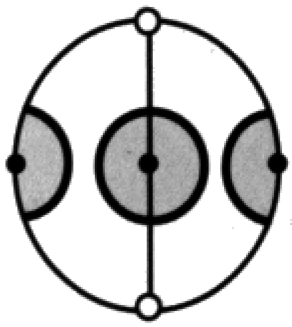


Fig. p

On the other hand, the composition of two punches by empty areas (**a'**) adds an empty circle.

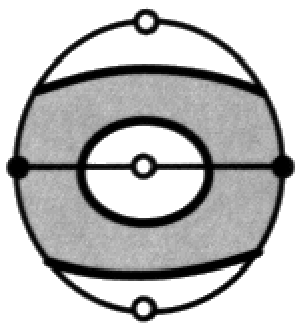


Fig. q

To solve the exercise, it is easier to draw the configuration of circles by following the two directions of the plane.

The circles in the filled areas are drawn successively from left to right, and the circles in the empty areas from top to bottom of the sheet. Similarly, following this principle, the filled points of the graph will be placed horizontally, while the empty points will be aligned vertically.

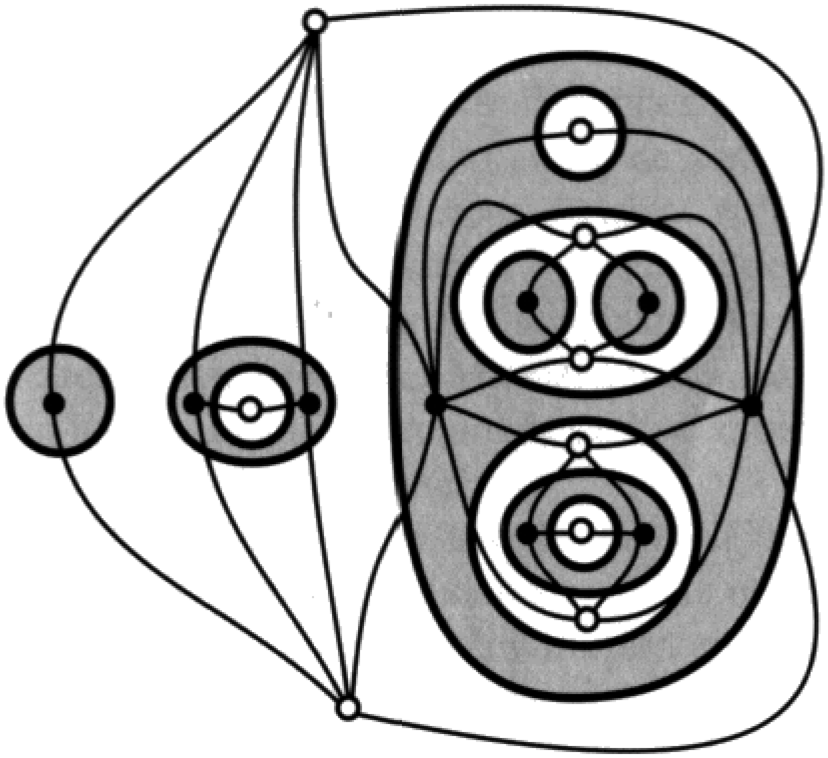


Fig. r

Based on these points, the edges are drawn in such a way as to join a solid point to an empty point that are separated only by a circle, in order to form the two types of punches (**a**) and (**a'**), with the closing punch **F** on the outside.

Chapter VII

Clinical aspects of node processes

Among the topology flashes reported from J. Lacan's seminar, there is one that deserves special attention. I have it from the publication of what anyone who heard him say it during the year of his seminar entitled R.S.I. [2 Sem XXII]. These lectures dealt with the knot formed by the three consistencies necessary for reading Freud's doctrine—the real, the symbolic, and the imaginary. At the time, I did not know its significance. Now it appears that it plays such an important role for Lacan that he hastened to carry it out in order to show how psychoanalysis works.

That is to say, to revisit the elements already presented. In terms of schemas, with the significant involution between schema **R** and schema **L**, which governs the involution between Freud's schema and Lacan's. In terms of surfaces, with the translation into the projective plane of this involution, which legislates between the torus and the Möbius strip. To manifest through these transfers an agreement, on a specific point, with his doctrine now formulated in terms of knots.

The data for this exemplary transformation are as follows. A Borromean ring with four rings can be reduced to a Borromean ring with three rings by abandoning one of its rings, which he then designates by the three expressions he equates here: psychic reality, the Oedipus complex, and the symptom [2 Sém XXII, leç 14.01.75].

We consider this passage from four to three, where we will show in this chapter the function of three, to be homologous to the signifying involution between perception and consciousness, whose model is given by Freud, with the dream of the dead child burning [1. a].

In this lesson, Lacan explains the principle behind his critical reading of Freud's doctrine. This requires a fourth term that holds the other three together. It is explicit in the chain of four. He wants to test how this discourse can support the three, where Freud's psychic reality is as if erased, having become implicit in the chain of three circles.

0. From surfaces to the knot

Let us take this opportunity to clarify once again[1] the relationships between the three chapters of Lacan's teaching, in terms of graphs, surfaces, and knots. They translate into one another. There are therefore differences among the constants. The structure of signifying involution is an invariant. We have already provided the translation of the schemas into surface problems([2]) The reader can refer to the presentation of the movement of the structure to which we arrived in terms of projective space with a hole, the Möbius strip, in the conclusion of our previous work

Very early on, Lacan deals with signifying involution—which is at the root of the rupture of semblance, causing the trickling of small letters, the erosion of the signified by this letter, prolonged by the rapture that follows if the subject counts these elements that form a series, without missing a single one—in terms of the inversion of the **R** schema by the continuous passage to the **L** schema, where Freud's psychic reality falls for a moment. This is supported by the immersion of these schemas on the projective plane.

Let us present here these two states of the structure on the immersed projective plane[3] where we can see that it is indeed through the retraction of the $\mathfrak{R}$ zone in state **R** of this structure that we move to state **L**. This state is undoubtedly instantaneous and repressed by the subject who experiences it,
outside of analysis, in the ordinary functioning of the structure, and represses it to the point of believing themselves to be ill when they become aware of it.

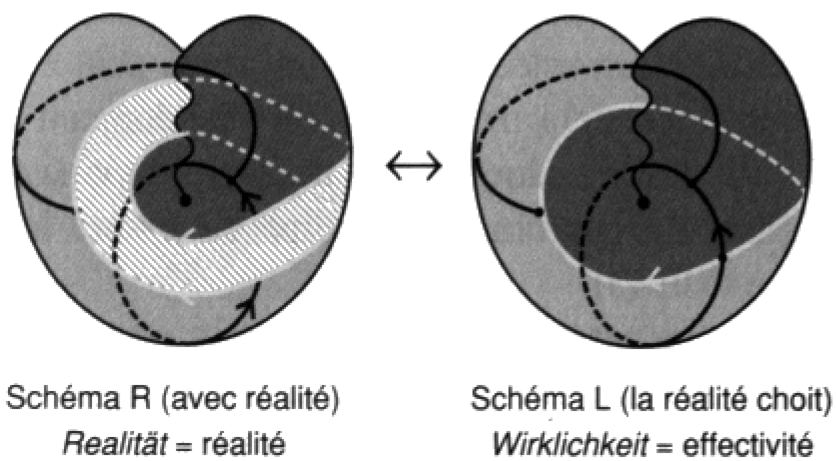


Fig. 1

This pulsation, covered by all kinds of mystifications in the culture that nevertheless maintains its symbol, but only produces it at the end of the most secret mysteries, we have constructed in logic[4] following Tarski, to demonstrate, through Hans, that it is indeed the imaginary function of the phallus revealed by Freud "as the pivot of the symbolic process" [É a 20, p. 555].

This dynamic of dispute, which can only be evoked through metaphor, becomes rigidified in industrial culture to the point of becoming a holophrase. This rigidity arises from a fear of misconduct, due to a lack of consistent ethical reflection. Conversely, the challenge to abuses is left to the most foolish demagogues. While the scoundrels claim to justify these abuses by the significant necessity of this misunderstood structure, they never explain it.

Let us return to the rupture of semblance, in terms of knots, between the four-chain and the three-chain [2 Sém XXII, leç 14.01.75]. We compare this with the formulation we have just mentioned in terms of an immersed surface. The figures correspond term by term in this translation.

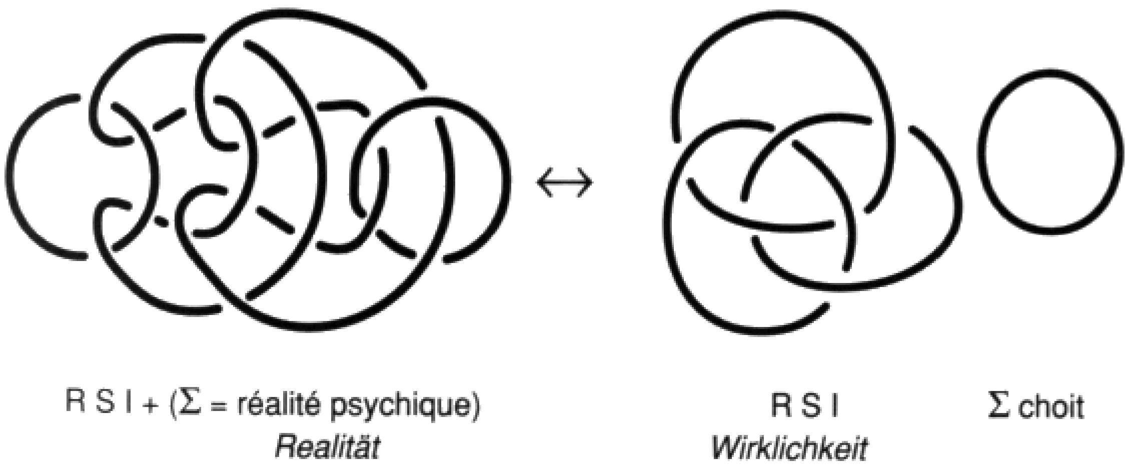


Fig. 2

The same pulsation of the structure occurs between these two states of the chain. But here, in this nodal version, we will see that it is one of the terms of the transformation that acts within it.

This happens, as we said at the beginning, in the dream of the father who sees his dead child burning while addressing him, when in fact the little boy's corpse has caught fire in the next room. The more appropriate nature of the material in this formalization provides a first reason for this new translation.

To clarify the second reason for this translation, in the context of the previous version, we can recall Lacan's response to the preliminary question that it would be desirable to ask at the outset of any possible treatment of psychic causality, the psy-thing.

Indeed, his reading leads Lacan to spread out the terms of President Schreber's delirium on a projective plane pierced three times.

However, the dysfunction of the structure that stands in the way of the subject of psychosis has been formulated since Freud, as Lacan points out, by a foreclosure, like a hole in the Other, our structure, where the signifier is no longer welcomed, which means that signification, our pulsation of structure, no longer takes place.

Foreclosed is a term used in grammar and law, meaning null and void, obsolete. The subject believes himself to be "above that," that it is "good for others," primitives, savages, ancients, foreigners, barbarians, provincials...

Some people act on this foreclosure by saying "it's outdated," and there is no question of returning to it. They express the phrase "I don't want to know anything about it" by using, for example, the interjection "Whatever!".

This failure of the function of invoking speech in a place of discourse—this speech no longer resonates with what is said—the subject no longer hears what is said, no longer wants to hear it in this place, beyond the fact that he usually forgets it. He excludes this function, which is certainly imperative, from the signifier, which is thereby reinforced. Disbelief presides over delusional belief. This function is usually recognized by the subject without thinking about it. Bordering on the impossible, which gives it its proven value, if it is encountered on occasion, it becomes an event. Unable to metaphorize the event, through the assumed pulsation of the structure
— it is not a matter of consciousness but of recognition — the subject will experience it in their passion, in their delirium, in a flow of action without resolution.

For it must pass, whatever the cost, contrary to what evolutionary thinking believes, and since it cannot take the path of writing in speech, the erosion of small letters, it will pass through the test, less masked to the subject himself, of the structure in question. He experiences himself as xenopathic. Lacan renders this situation through holes in the structure.

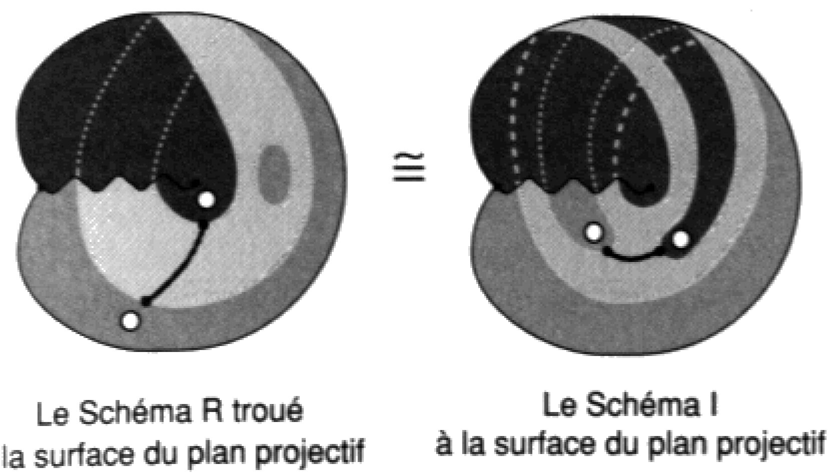


Fig. 3

It is therefore like a hole (**a, a'**) in zone $\mathfrak{R}$, which prevents it from closing in order to open, correlated with the two holes $\mathbf{P_0}$ and $\mathbf{\Phi_0}$ respectively in zones *S* and *I*, that Lacan, in 1956,
This hole prevents the momentary erasure, by retraction, of the zone $\mathfrak{R}$, the fall of $\mathbf{\Sigma}$.

Let us show that this situation is indeed that of schema **I**, since it is obtained from the deformation—Lacan speaks of caricature [É a 20, pp. 563–575]—of schema **R** thus perforated.

We establish this correspondence through the series of figures placed outside the text on the following two pages.

We can then ask ourselves the question. Does what we will call, with Lacan, *scientific subjectivity* change anything in terms of what psychoanalysis must respond to?

It is not the subjectivity of the scientist.

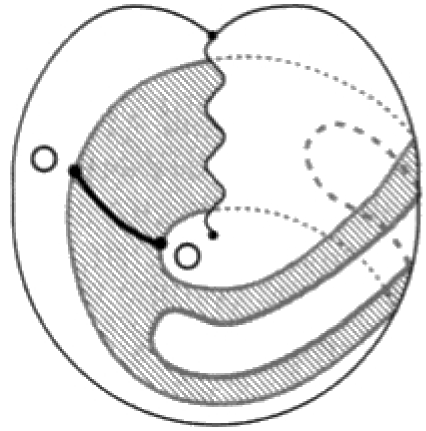
Indeed, at the end of his preliminary remarks, Lacan defines this scientific subjectivity as "that which the scientist at work in science shares with the man of civilization who supports it."

Rather, it is the attitude already noted by G. Bachelard in his *new scientific spirit* when he remarks that an atomist scientist can perfectly reconcile, in his subjective division, the animistic or religious beliefs of his maternal culture with his scientific theories. Here we see the state of mental degradation reached by certain scientists today, who believe themselves obliged to reject their culture in the name of the Vienna Circle Manifesto. We know that this circular argument claims to reject psychoanalysis and only accepts behaviorism. It is therefore not surprising that autism is on the rise (F. Dolto) and that politics is deteriorating into legalism and fundamentalism.

Caricature of reality

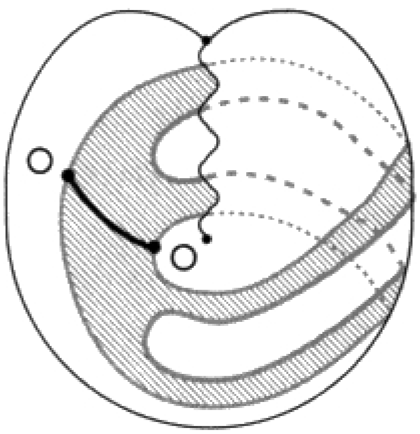
according to President

Schreber

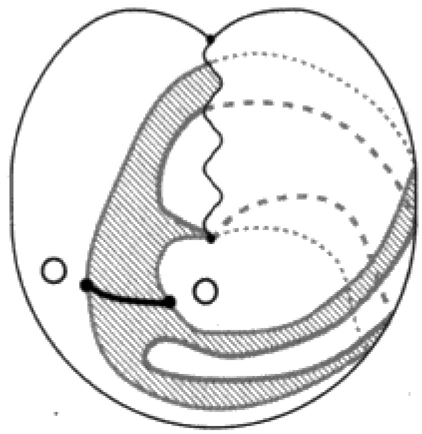


The hole aa' is located in the
@ (hatched) area.

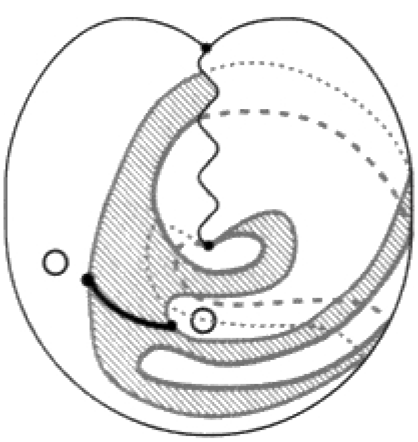
We extend this hole aa',



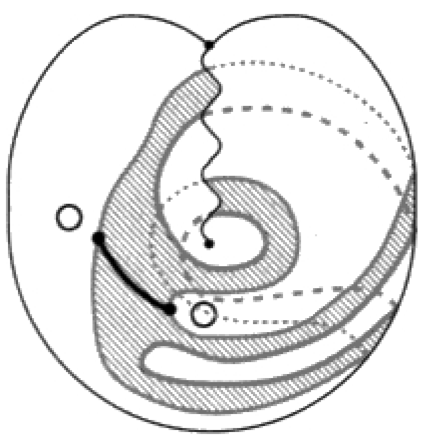
until it reappears on the other side of the immersion line.



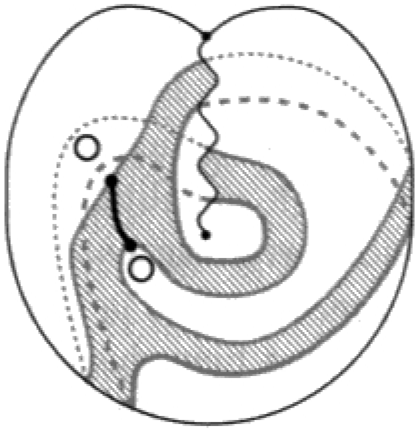
We retract a strap at the point of immersion.



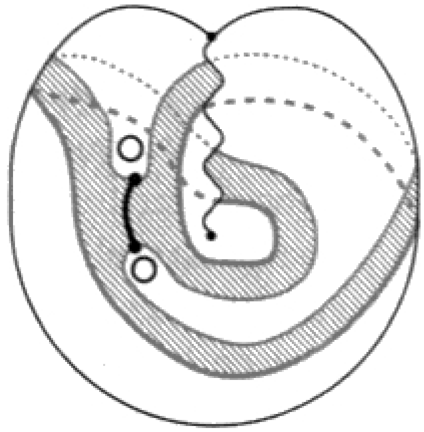
We are constantly turning continuously.



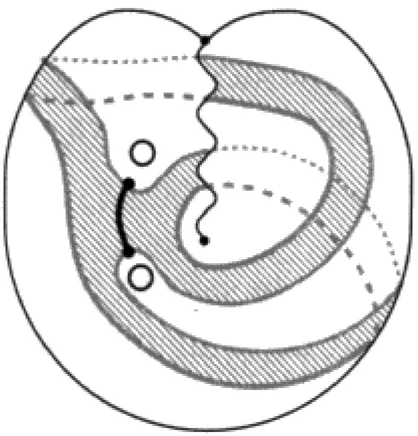
We distract the strap upwards.



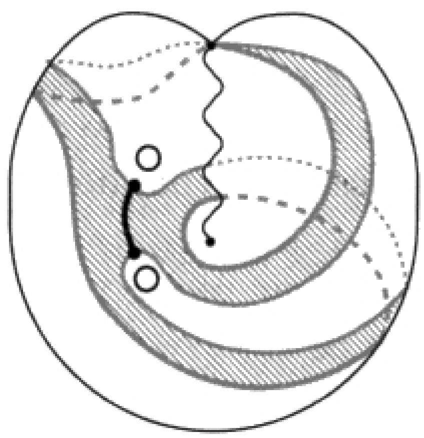
Nous déformons continûment



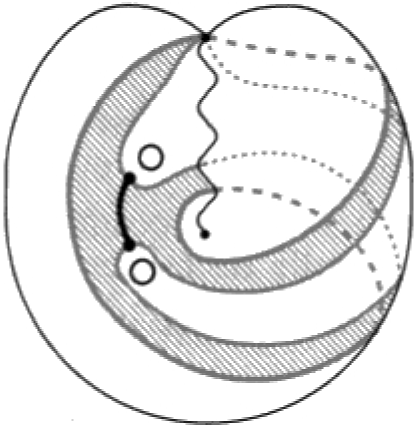
What is the d8jdacar on the left?



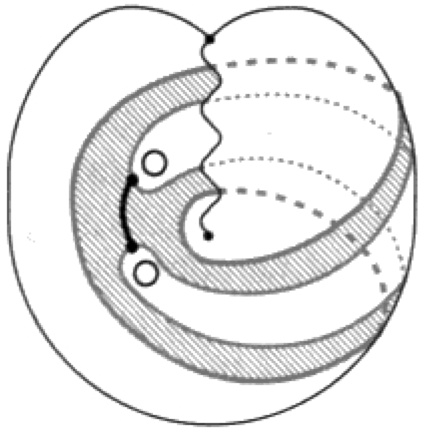
Nous faisons se traverser les deux bretelles le long de la ligne of insertion.



We draw the second line at the upper point of branching.



bretelle autour de ce point.



la bretelle vers le bas to obtain diagram I.

Lacan then defines this subjective position by the three coordinates of a "discourse on freedom," a "concept of the real," and a "belief in Santa Claus," in which we recognize the interlocutor in B. Pascal's debate on the void.

This "discourse on freedom" is described as delusional, and we see daily evidence of this in what is published and disseminated by people who are considered serious. In this context, it is not surprising that soapbox orators try to compete in demagoguery with all those involved in politics.

The "concept of reality" is an alibi to which scholars are complicit, hostages to a pact that is dedicated to ignoring the question of the subject, the subject of science, let's be clear, even in commerce. It has already happened that blood contaminated with a virus has been sold under the pretext of good management.

Social psychosis, combined with a belief in Father Christmas, established by Pascal and continued by Newton, Marx, Einstein, Lenin, Freud, Lacan, and many others, provides the coordinates of what only the discourse of analysis can respond to when psychoanalysis has surrendered in the face of the malaise in civilization.

This is not a matter of psychologizing the scientist. We must welcome this subject responsible for the error of laboratory manipulation, without falling into rudeness, as in the case of the artist. For here too, the psychoanalyst has only to follow.

An author such as K. Popper thus very aptly places the question of desire that drives researchers when they devote themselves to a field of investigation outside the scope of his concerns.

"*It so happens that the arguments presented in this book are completely independent of this problem*."

But he makes one mistake, that of rejecting this question of logic and classifying it as a matter of psychology, referring to Einstein.

However, I believe that the logical method should not be confused with coming up with new ideas or logically reconstructing this process. I can express my view by saying that every discovery contains an 'irrational element' or 'creative intuition', in the Bergsonian sense of these terms. This is how Einstein speaks of the 'search for those highly universal laws by which it is possible to obtain a picture of the world by pure deduction. There is no logical path," he says, "that leads to these laws. They can only be reached through intuition based on a kind of intellectual love ("Einfühlung") for the objects of experience" [56, p. 28].

However, following Lacan and Spinoza, we have shown that this *amor intellectualis* of the object is part of a classical logic modified into a topology of the subject. At the same time, Popper misses the point made by Freud, whom he criticizes for the verificationism he believes he detects in his approach, which is wrong, because Freud constantly refutes his doctrine, which always proves to be irrefutable and therefore relevant to the topology of the subject. Thus Popper misses the mark regarding psychoanalysis, even though he provides its coordinates here.

Psychoanalysis is thus situated, for us, in these categories, in relation to scientific research. It is not where he believes it should be placed.

It is not an experimental discipline, where competing theories falsify each other (the Popperian version of science).

Psychoanalysis is based on a logic that is invalid but irrefutable, our topology. This means that it is not a science, but neither is it metaphysics or ideology. This is where Popper's political thinking and free enterprise fail in the city, complicit in the crime of good management because it believes it regulates, as these people say, the question of the subject through competition rather than reason. Another policy is called for here, meaning that the problem of virtue has not been solved by the supposedly ideology-free managers of consciousness.

The overflow of techniques, derived from science, itself a product of the reversal of theology, particularly Catholic theology, into anthropology, as evidenced by Descartes, sees scientific subjectivity, defined by the civilization of the West, grow and spread, in pure ignorance, throughout the city and the empire. It took J.R. Oppenheimer to build the bomb to begin to worry about desire in science in relation to common sense. This new science, predicted by Galileo, is more endured by its scholars, and the civilization that supports them, than decided by them. The relationship between experts and politicians is thus reduced to its real dimension.

Following several decades of psychoanalytic practice, Lacan's observation of the difficulties, even obstacles, encountered in practice leads us to consider that the defect perceived in Schreber's psychosis is much more fundamental to scientific subjectivity, to the subject who comes into analysis.

In a speech delivered at the end of a conference in Deauville in the 1970s, in which he did not reveal everything, Lacan went so far as to wonder whether all the people who came to see him were psychotic. This was a psychiatrist's irony directed at those concerned.

“Why would anyone ask an analyst about the nature of their symptoms? Everyone has them, given that everyone is neurotic. That's why we call the symptom neurotic, and when it's not neurotic, people are wise enough not to ask an analyst to deal with it. Which proves that only someone who is psychotic would go so far as to ask the analyst to fix it."

J. Lacan, free quotation collected at the Deauville conference, January 1978, not taken from a written work.

We are not saying, as some would have us say, that "in scientific civilization, everyone is psychotic," and we do not write this. Neither does Lacan, for in his Écrits, he more readily uses the expressions psychosis and psychotic process, and so we have identified this category of scientific subjectivity.

For our part, today, we must be more precise, using the term social psychosis to clearly indicate what this process is about, not wanting to encourage the rudeness we mentioned above, which is so widespread today, to the point of impotence. We are referring to mental means, since Lacan's death.

The fact is that we are more inclined to take our clinical and methodological cues from R. Jakobson than from M. Foucault[5]. Jakobson's study of aphasia [47], based on his poetics, remains a model of its kind in structural clinical practice, apart from the two founders of psychoanalysis. This is different from the history of madness in the classical age, which does not even distinguish between madness and psychic causality. To understand this, we recommend reading B. Ogilvie [54], which definitively clarifies what can be expected from a clinical exhaustion such as Lacan attempted in his medical thesis. It is to realize the necessity of structure between innate and acquired materials. Without this, it is impossible to untangle, but of course the average reader does not even notice this. Not for a long time, because you have to follow the reasons, without missing a single one, before it closes in on the impossible([6].

There is a drama in science, exemplified by the one that plays out in analysis, but Lacan raises the question of whether it always relates to the Oedipus complex, unless it is called into question [É a 32, p. 870].

What happens in analytical discourse is therefore exemplary but not exempt from it. This social bond is part of the question, and it is not a good response to avoid questioning it, or to put only poor little people in the hot seat for the purposes of intimidation and power, as we see today, for lack of an appropriate response that involves everyone.

Let us begin by saying how much we value those who should not be called our colleagues, as the term is not appropriate in this field, but our peers. We are addressing here the analysands of Freud and Lacan, as well as the analysands of those two. This community, bound by a unique experience in this century, has always deserved our fraternal consideration.

But this observation should not be shared with the proponents of psychoanalysis who remain speechless in the face of what we must recognize in transference. The crudest ignorance on the point of recognizing desire in the twists and turns of its act.

We should not tell them, so as not to alarm them, so that they can continue to believe that they are practicing a therapeutic technique and that they are saints, which is necessary. That they can still do something for someone, or that they have the power to do so, a belief that they protect, thinking that people will turn to them.

The most adventurous among them believe that one day they will be able to cure psychosis, when they cannot even see that everyone will see that they do not see it developing in the noise and fury around them. We are playing it safe while the massacre continues around us, just a little further away.

Shared cynicism is content to demand and respond to social demand, where we draw on various social workers for a model of practice, instead of importing another.

Of course, the narcissism of the group must be spared. But this is not so that they can continue to believe that they are sufficient to fulfill their task, which consists of occupying this necessary place of pretense for others.

Would we remain within Freud's field, ignoring the existence of psychoanalysis and pretending that we should be content to maintain its current form in its true semblance? This is what certain clever minds have seen fit to implement.

Or we would have to hide from our contemporaries this passion for ignorance that inhabits them and about which they want to know nothing, this diagnosis and how to respond to it.

Or we must acknowledge that the Cartesian subject, the subject of psychoanalysis, is well defined by a fetishization of being.

Or we should also recognize, in the transition from clinical practice to the practice of analysis, that if there are structural gaps for everyone in this civilization, we may be moving away from a simple reduction to the Oedipus complex, but the edges of these gaps are knotted.

Now, the edges of these gaps are circles whose arrangement requires a substitution—some have noticed this, but have done nothing more than add another holophrase, reserving this substitution for psychosis—when the subject fails to tie them in the manner that is required, in the purest sense.

Lacan responded with circles, circles... to the ever-preliminary question that scientific subjectivity asks itself in this context of social psychosis, meaning political economism as well. And these circles form a knot.

But this is to emphasize the pulsation that must always be assumed by each subject, which requires them to endure this instantaneous shift in psychic reality according to Freud, also known as Oedipus, since it is the principle of love for the father. This is the structure of the symptom. This jouissance that truth finds in opposing knowledge [É a' 13, p. 58].

Its erasure is only momentary; like the phoenix, it always rises from its ashes, and even immediately. This is why this key to the structure will never be touristy [É a 30, p. 838].

Speaking of the Borromean knot, Lacan specifies:

"That's where... that's where the problem lies... that it's **a mistake...** to think that... it's the norm... for the relationship between three functions... that don't exist... in relation to each other... in their exercise... **except** in beings... who, because of this... believe themselves to be HUMAN..."

It is indeed an error, and denouncing it has nothing to do with seeking totality. As for the absolute, it is about detachment.

Absolute means separate, Lacan continues:

"*... **It is not...** that the Symbolic, the Imaginary, and the Real... are broken... that defines perversion... it is that they are distinct... and... that we must suppose a fourth... which is the sinthome on this occasion... which we must suppose **to be** tetradic...*"

By defining perversion in relation to love for the father as never before.

From where we started with Hans and Tarski from his failure in the phallic function.

"*What connects Borromean... (is) that perversion... (does) not mean version... towards the father... and that, in short... **the father is a** symptom or a sinthome... as you know... **The existence** of **the symptom...** is what is implied... by the **position** MÊME... the one that supposes this link... between the Imaginary, the Symbolic, and the Real... enigmatic.*"

J. Lacan [2. The sinthome].

We are now aware of what we are starting from, the function fulfilled by this fourth term and its topology, which is nothing like a broken chain. Only a version towards the father, always failed, but how? We will now show what is failed, the failure of the effectiveness of the three by the four. But we must know where to go.

This transformation from a chain of four to a chain of three, since that is how we now formulate it, as Lacan showed on the board [2 Sém XXII, January 14, 1975], must be specified, to be well defined, as a Gordian movement.

You need to color the 4-chain knot:

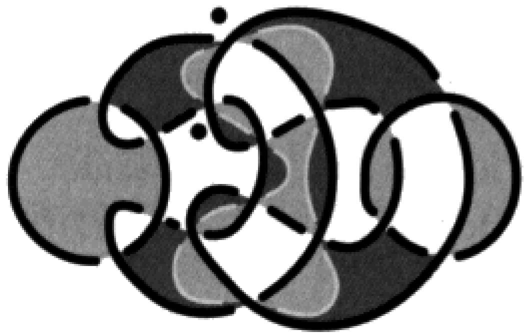


Fig. 4

Then perform the Gordian knot on two crossings. This involves reversing these two crossings, here between circles R and S,

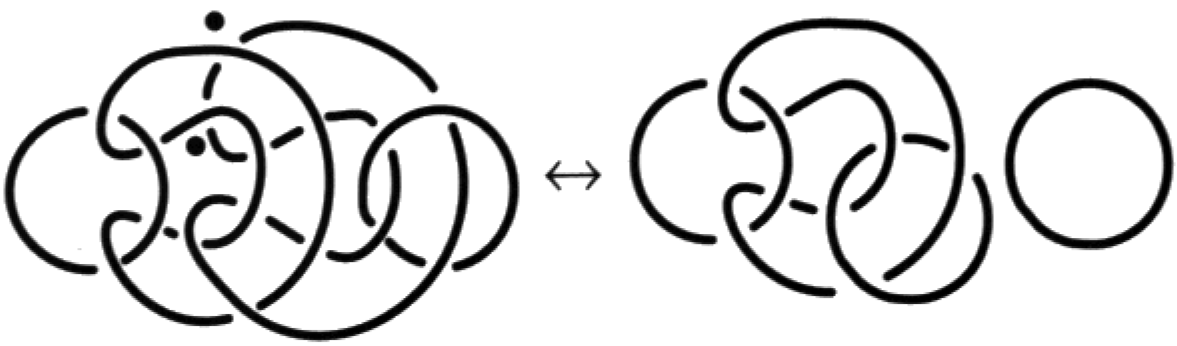


Fig. 5

in order to slide the circle **Σ**, sinthome, Oedipus, psychic reality, love towards the father, jouissance, which no longer lasts for a moment.

It is in the context of this question that we intend to deal with the object in psychosis, insofar as we want to show how and in what capacity the clover knot becomes the counterpart of schema **I**, i.e., schema **R** with three holes:

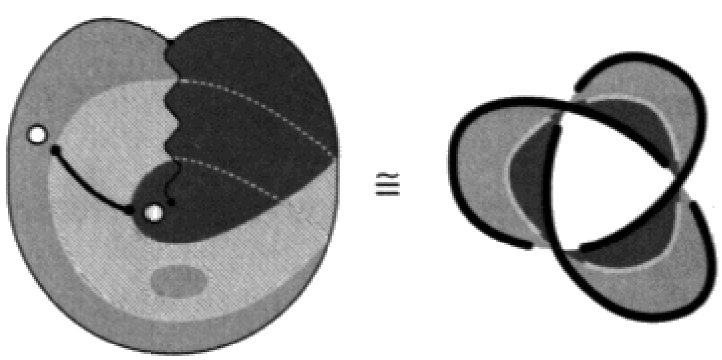


Fig. 6

in order to account for paranoiac psychosis [2 Sém XXIII] in this chapter of Lacan's teaching.

The clover knot is a knot homologous to the Borromean chain modulo two, connected by cross sections. Our final result will concern this homology independently of the number of circles in the objects.

Let us now show, with the main result of this work, the knot movement, which can provide the means of defining the number of knots contained in an object, that the 3-Borromean chain is indeed the means of implicitly formulating the 4-chain, but at the same time it also condenses this involution between chains.

We can revisit the **T0** theory of non-knots in terms of knots, with the **T00** theory of intrinsic chains, and introduce a new movement.

1. **Intrinsic chain theory and knot theory**

By defining a new movement, which we will call the knot movement, we can formulate a new invariant in classical knot theory in order to clarify our knot theory.

This refers to another theory of entanglements, reducible to non-knots where knot movement has been substituted for Gordian knots and homotopies. We will call this new theory **T00**, the theory of intrinsic chains[7].

In relation to this theory of intrinsic chains, we will first construct a theory of knots in order to define the number of knots, then we will reduce this to a theory of knots.

a1 - ***The knot movement***

Let's start by defining this new **N3** movement, which is performed on alternating triskels:

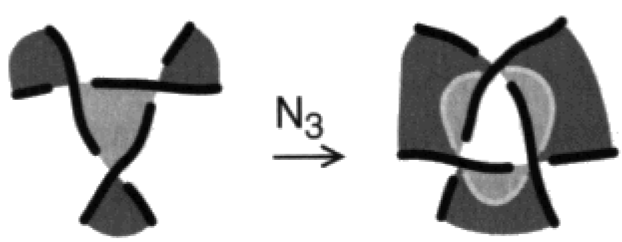


Fig. 7

This movement is equivalent to adding a Borromean knot chain to an alternating triskelion.

Fig. 8

This presentation of the knot movement justifies our calling it the Borromean chain knot movement or Borromean movement. The role it will play in knot theory establishes the structural function assigned to it in the Freudian field by Lacan.

This necessary stop, never produced before, finally allows us to see how it is by resorting to the Borromean chain knot that we can operate on the clover, which is the first primary knot[8].

Fig. 9

We will explain how to use it in general cases.

This knot movement is a combination of elementary movements in our previous theory of intertwining. The series that represents it always contains a **T3** movement and a Gordian knot, among loops.

In this way:

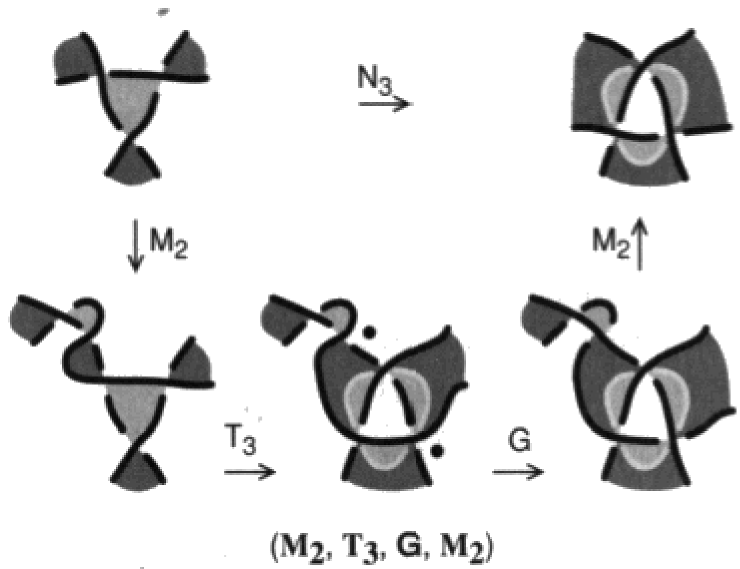


Fig. 10

Or this one:

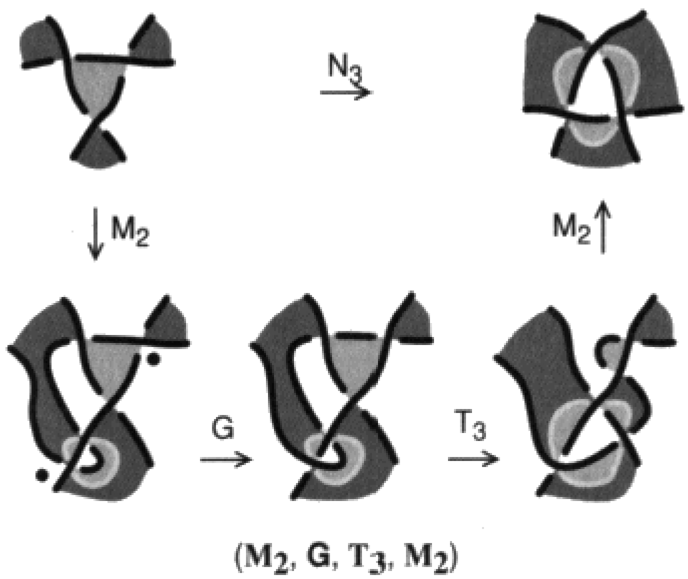


Fig. 11

We number these new movements so that their number, *a*, can be expressed in the number of knots by the formula $3a = 2g + t$.

It should be noted that if a knot movement replaces a specific Gordian knot, accompanied by a **T3** movement, any Gordian knot is not necessarily likely to be rendered by a single knot movement, but it is certainly rendered by several movements, as we will now show.

a2 - ***From the theory of extrinsic knots to the theory of intrinsic chains***

In order to write a **TN** theory of knots and construct the knot number, for any knot or chain, we will untangle (erase) this object in a **T00** theory of intrinsic chains, where positive knot moves replace Gordian moves and homotopies from the previous entanglement theory, and where we maintain the use of Reidemeister moves.

We then define, thanks to these changes in presentations, a relation **RN** (s_1, s_2) on the set of presentations of knots or chains.

That is: **RN** (s_1, s_2) ⇔ ∃ *$* (***$*** (s_1) = s_2)

This relation is an equivalence relation. We will sometimes write it as $s_1 \approx^{\mathbf{N}} s_2$.

We will call the equivalence classes defined by this relation chains. These classes of presentations constitute the objects of **T00** theory.

This **T00** theory of intrinsic chains is the theory of these equivalence classes, and it is easy to show by calculating the crossing signs of any orientation (see Fig. 27), when dealing with improper crossings, that these movements respect the distribution of chain numbers and, consequently, the **RΣ** relation.

Identical objects in this theory have the same linkings

For two presentations of chains or knots s_1 and s_2:

RN (s_1, s_2) ⇒ **RΣ** (s_1, s_2)

We now provide proof of the reciprocal implication that two chains with the same distribution of their respective chain numbers, RΣ-equivalent, would be identical in chain theory.

This amounts to asking whether, for two chains s_1 and s_2 that are RΣ-equivalent, there always exists a series ***$*** of movements in our new chain theory such that:

s_2 = ***$*** (s_1)

Answering this question in the affirmative proves the following theorem.

Two equivalent non-nodes are two identical objects

If two chains s_1 and s_2 are RΣ-equivalent, then they are RN-equivalent: **RΣ** (s_1, s_2) ⇒ **RN** (s_1, s_2)

We already know, from the R0 relation in place of RN, having encountered it with the use of composition # in our theory of entanglements, in terms of Gordian knots and homotopies, that the proof of this theorem requires demonstrating two things, which we group here into a single statement.

Chain knots and proper knots are unlinked.

A chain knot (or proper knot) can always be reduced, through a series of moves from the T00 theory of intrinsic chains, to a trivial chain (or trivial circle).

Let us give a proof of this theorem, which is remarkable in that it suffices to consider chain knots and proper knots.

Using the **M2**, **T3**, and **N3** moves, we can extract any loop from a chain knot. The **N3** move allows us to pass the obstacles represented by the alternating triskels.

By performing these same types of moves, we can extract each of the circles from a chain knot one by one, since it does not contain any intertwining.

The theorem is proven by the fact that the **M2** and **T3** moves apply to non-alternating meshes and triskels, and that we can use the **N3** move for alternating triskels. There is no alternating mesh without another alternating mesh with the opposite twist to compensate for it, and we do not encounter any other cases.

Let's illustrate this with an example; we will use the K_{2-1} # S_2 assembly already discussed in Chapter IV.

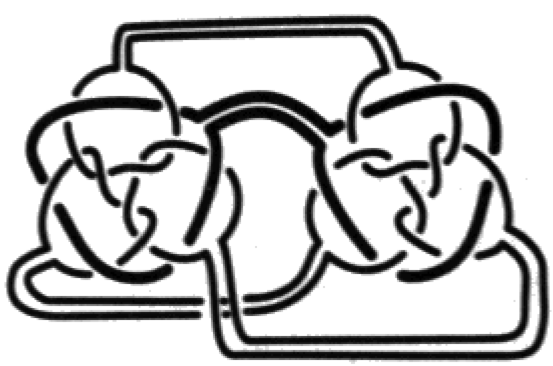
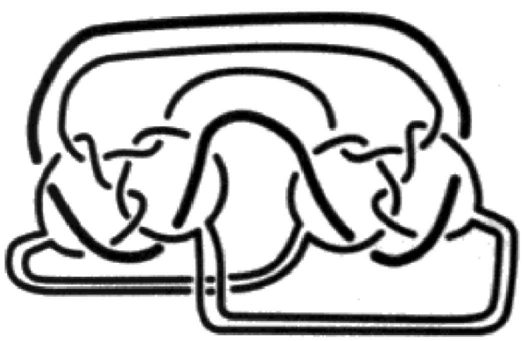
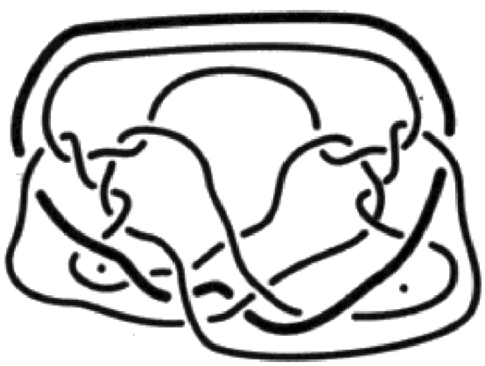


Fig. 12

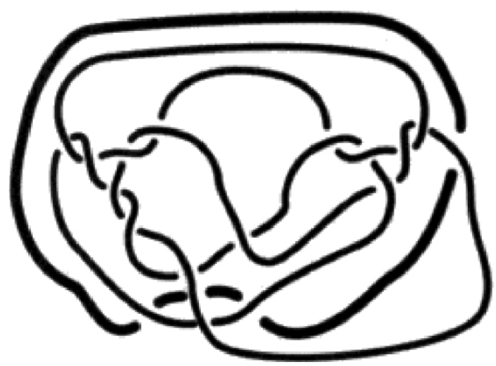


We successively remove two stitches at the top of the *'sure.

We perform a $T_P$movement and prepare to perform an Nb movement.



With the N3 movement performed downward, we are now ready to undo two new stitches.



By performing a Tt and removing a stitch...

All that remains is to delay one last stitch to have cleared a round with thicker grease.

Fig. 13

In this chain theory, let's number all of our movements arranged in Table **3**. This basic numbering is summarized in Table **4** opposite the previous one. Given a change in presentation ***$*** defined by the series (X_1... x_n) with x_i belonging to the set of our movements, we can define the set **s** of numbers in the series:

s = {b, b*, t, a}

It is then easy to calculate the following proposition.

Relationship between the averages of the number of cuts

If two chains s_1 and s_2 are linked by the series of **digit** movements ***$*** such that : ***$*** (s_1) = s_2, then the averages of the number of cuts in these two chains satisfy:

Σ (s_2) = b + t + 3a + Σ (s_1)

Σ*(s_2) = – b* – t – 3a + Σ*(s_1)

if **s** = {b, b*, t, a}.

And to take note of the new definition of knot numbers.

Definition

We will call the two expressions dual knotting numbers of each other: N_s = b + t +

3a

N^*_s = – b* – t – 3a

With these definitions established, we can now discuss TN knot theory.

Our TN theory of knots is therefore extrinsic to this T00 theory of chains. Its movements are the Reidemeister movements of classical knot theory. Its objects are the equivalence classes produced by these movements. We thus return to classical TC knot theory, where we define a new invariant through calculation thanks to this connection with the T00 theory of intrinsic chains.

The calculation consists of untying the presentations intrinsically to the T00 theory of chains in order to obtain an analysis of their number of knots, from which we retain only the number of knot movements noted *a*, in order to calculate the number of knots.

This number is invariant for ambient isotopies (Reidemeister moves), since it does not depend on **B1**, **M2**, or **T3**, as these moves only vary the number of knots but not the number of knots.

a3 - ***The number of nodes***

We begin by determining, in any node or chain S, the distribution of chain numbers Σ_i using the entanglement numbers, which are easy to calculate using colorings.

Proper nodes have a chain number of zero, and the non-node s_0 they contain is trivial.

In the case of chains of two and three circles, this allows us to determine which non-node s_0 is contained in the chain.

In the case of chains of four or more circles, the distribution of chain numbers Si allows us to determine the minimum alternating presentation s_0 contained in the chain. This minimum alternating presentation may be a non-knot, or a chain with a cut, as explained in chapter five.

The number of nodes will be determined in relation to the example case chosen to represent the entanglement state.

If there are different series of moves ***$*** that transform s_0 into S, which we formalize with the expression:

$S \rightarrow s_0$

—

$

which states that presentation S represents movement ***$*** for presentation s_0. Movement whose dual knot numbers are expressed in this new theory of intrinsic chains by:

$N_s = b_s + t_s + 3a_s \quad N^*_s = -$

$b^*_s - t_s - 3a_s$

Definition

We will call the number of knots in a given presentation the minimum number, denoted by a, of the numbers of knot moves obtained for all series ***$*** that lead from the distribution s_0 of the linkages contained in this presentation to this presentation S itself:

$a = \min. \; a_s$

s

such that: ***$*** $(s_0) = S$

with $N_s = b_s + t_s + 3a_s$ and $N^*_s = - b^*_s - t^*_s - 3a_s$

The reader can verify that this number is invariant for ambient isotopies, since it does not depend on the **B1** and **T3** moves and is indifferent to the **M2** moves.

Let's look at an example of such a calculation in a non-alternating case:

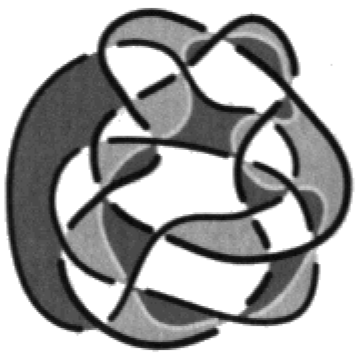


Fig. 14

The calculation of the distribution of chain numbers Si for the various orientations gives the following results[9]:

$\Sigma_{\{\emptyset\}} = +1$, $\Sigma_{\{S\}} = -3$, $\Sigma_{\{T\}} = +1$, $\Sigma_{\{J\}} = +1$

where we recognize the distribution of the first Olympic chain of negative torsion, thus determining S0:

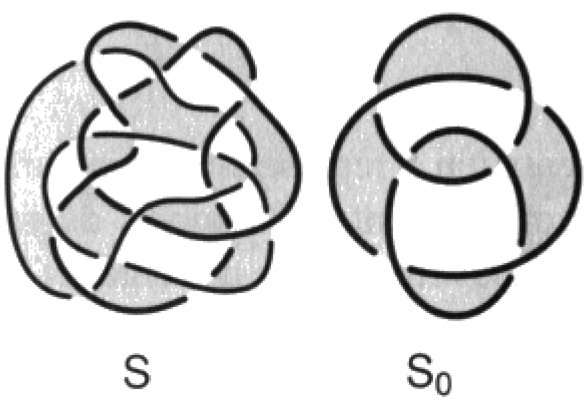


Fig. 15

We must determine the series ***$*** of movements that transforms s_0 into S. To do this, we unravel S into s_0 through a series of movements, switching to the dual presentation when necessary to perform **N3** movements.

From this alternating chain
X

We perform movement Ną. movement T,

Two stitches Mt are a déłont e‹ »•us effec-knot We perform a

We undo gncDre two vai!!es by My.

Then a T_3 towards the bottom of the figure and two M_2 . one on each side

We move on to the dual pre-mentation to undo a loop by $Ø_1$

We still need to perform a node Nt movement that alters the result.

to obtain the altered presentation of the contained non-node.

Fig. 16

We have just untied S in s_0 through a series of movements; let's call it $\$^{-1}$. It is easy to determine the series $\$$ of movements that transforms s_0 into S.

Simply reproduce this series of movements retroactively, taking care to switch to the dual when it is necessary to perform a movement **N3**. We can thus verify that the number of knots here is certainly less than or equal to two, i.e., a ≤ 2.

There is still the same uncertainty regarding its minimal nature.

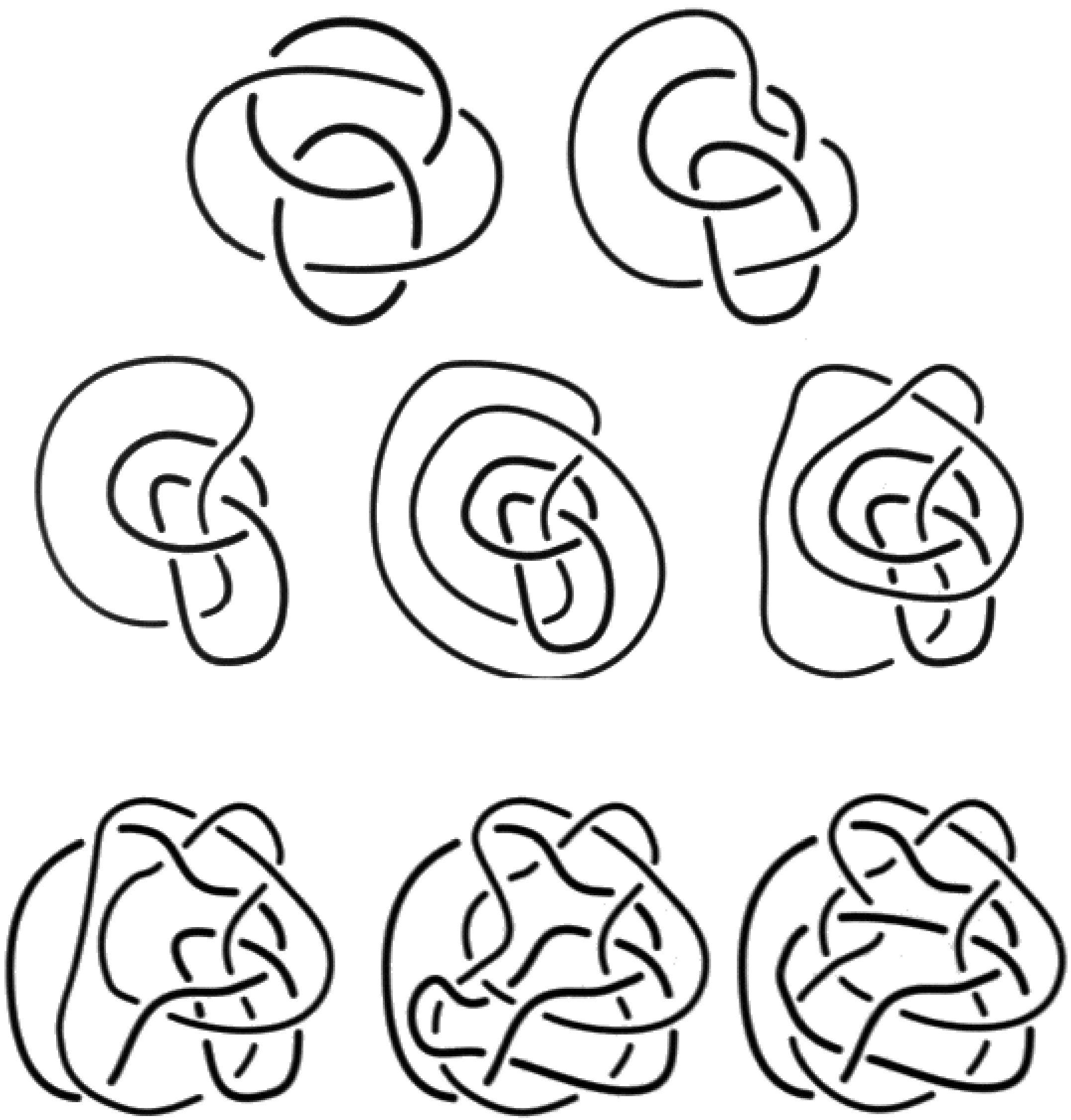


Fig. 17

Since we always calculate the dual knot count using the knot count, these different transitions to the dual representation are easy to check by calculation, as they simply transpose these two numbers.

The only question that remains unanswered is whether this is the minimum number of knot moves, which has not been decided, but which does not prevent the number of knots from existing.

a4 - ***At stake in this new invariant***

The calculation of the number of knots by drawing depends on this uncertainty, and in mathematics we can propose to search for a knot polynomial, similar to *skein calculus*, whose degree of a factor could produce this number from any presentation. Otherwise, the proof that such a polynomial is impossible to construct is also a result that is of great interest in order to provide, by a contrary reason, some clarification in this nodal field.

When calculating the number of knots, we see the importance of duality in choosing the preferred presentation of alternating cases, and consequently of non-alternating cases.

Thus, the fact that S represents a movement ***$*** for s_0 produces the number of nodes *a*, which we express as:

S → s_0

— —

$ a

But it is trivial that this use of the node number can be generalized to any pair of presentations s_1 and s_2 belonging to the same intrinsic chain for the **T_{00}** theory of intrinsic chains. A presentation s_1 representing a series of movements ***$*** for another presentation s_2 produces a node number a:

s_1 → s_2

— —

$ a

If a = 0, we will say very classically that S1 and S2 are two presentations of the same object in knot theory **Tc**.

a5 - ***A knot theory***

If we refer to a chain theory that consists of performing positive knot movements, numbered in Table **4** (see above), and their inverses, numbered negatively, we can calculate relative knot numbers, but the condition that requires choosing the series with the minimum number of knots produces a brutal modulo two quotient of this knot number.

The knot number is reduced to 0 or 1.

In the related knot theory, in a proper knot or any chain, there is either one knot or no knot. In this theory, there is only one knot.

a6 - ***Decomposition and T3 by G2 and N3***

To conclude this section on definitions, we return to recent mathematical research on classical knot theory by clarifying the analysis of Reidemeister's third move.

Indeed, when applied to a triskelion made up of three distinct string elements, it decomposes into a series containing a Gordian move from entanglement theory and a Borromean chain move from the chain theory presented above.

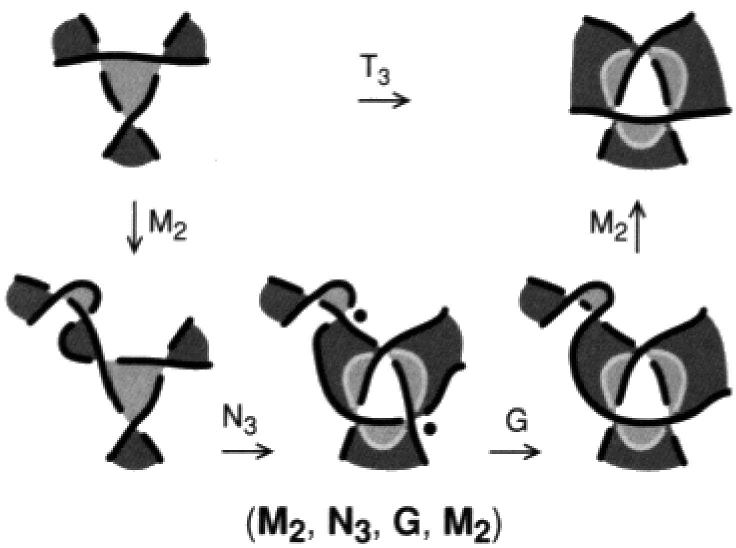


Fig. 18

We propose this result for consideration by mathematicians, in light of their existing treatment of the **T3** movement in terms of the algebra of solutions to Yang-Baxter equations [18. c].

a7 - ***Three is the reason for the ratio of three to four rounds***

It will then suffice for us to show how the reduction of the 4-chain to the three-chain, already achieved in the above by a Gordian movement, also occurs thanks to a knot movement.

We choose a triskelion in a presentation of the four-chain and perform the knot movement.

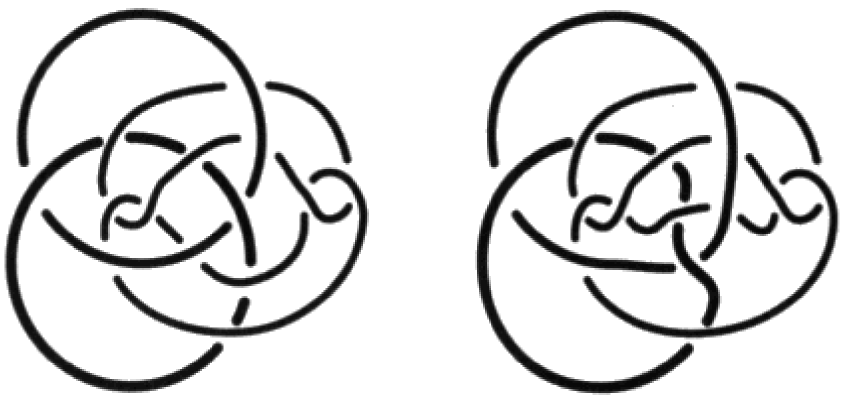


Fig. 19

This allows us to perform an **M2** movement in this drawing. Two **T3** movements would be enough to release this mesh, but we will not do so. However, the reader can verify this.

Fig. 20

The fourth round is detached by a series of **T3s**, leaving only a 3-chain, which was obtained from the 4-chain due to the action of the 3-chain through the knot movement.

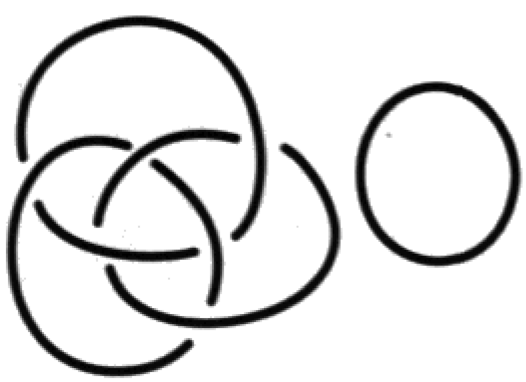


Fig. 21

The reader will notice that we have chosen different presentations of the four-chain to show how to transform it into a three-chain using either the Gordian movement or the knot movement; but this is not necessary.

2. **Differentiation of three types of knot movements**

We can now refine our analysis of the knot by filtering the multiplicity of our objects through the three distinct types of knot movements. We now define them with their prototypical objects.

The knot movement we have just constructed to define the number of knots can be differentiated according to the number of distinct loops used in its execution.

We will call the knot movement *improper*, denoted **Ni**, when it is performed on a triskelion composed of three string elements belonging to three different circles.

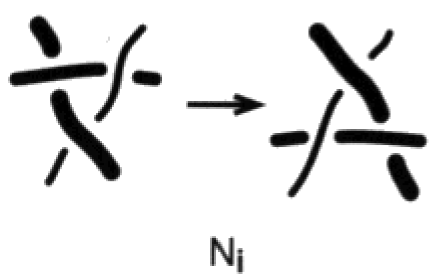


Fig. 22

This type of movement only produces improper crossings and therefore eliminates three-ring Borromean knots, whether they are Borromean or generalized Borromean:

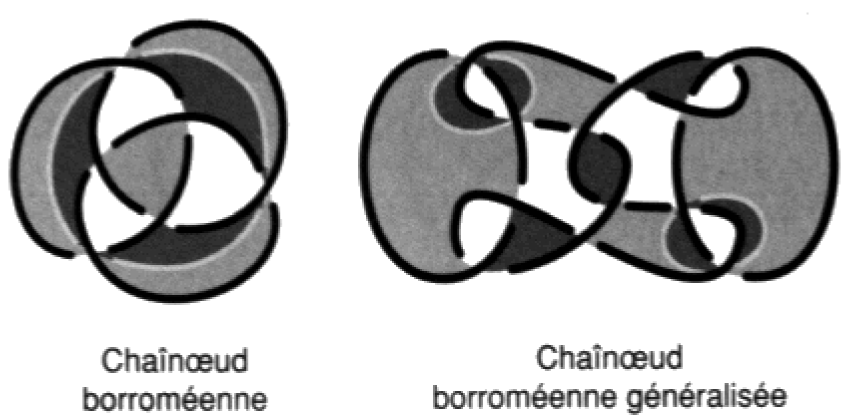


Fig. 23

It is the entry into scientific subjectivity through the absence of metaphor that is the locus of paranoid delusion.

We will call the knot movement a *hybrid* knot, denoted **Nh**, when the triskelion is composed of three string elements belonging to two different circles.

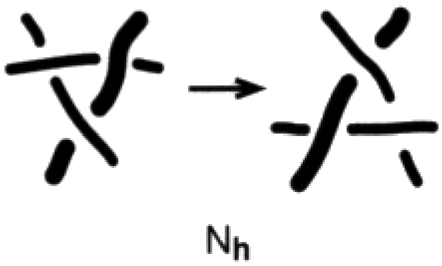


Fig. 24

This type of movement has two improper crossings and one proper crossing. It eliminates the generalized chain knots of two and three circles.

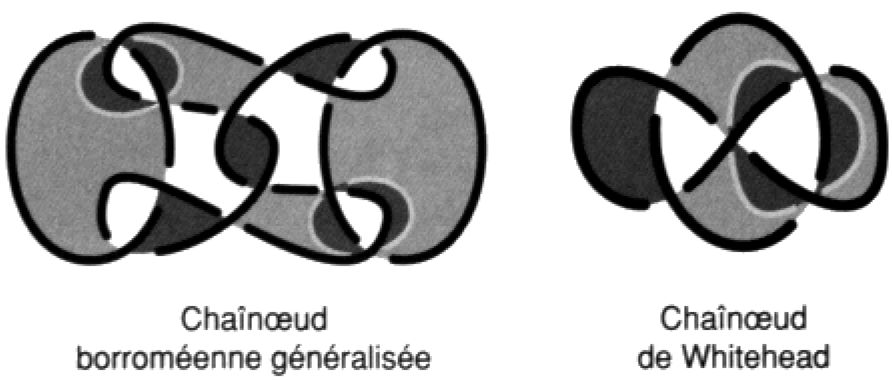


Fig. 25

Generalized chain knots of three circles are a modality of four among objects made of three circles.

It is in the effectiveness of our theory of objects made of one to three circles that the trace or remnant of the fourth, known as reality, Oedipus, the sinthome, or even love for the father, allows us to situate the act or event between reality and effectiveness.

For the same reasons, since they are also obtained by a transverse section, Whitehead's knots made of two circles are a modality of the three among objects made of two circles.

We will call *the knot movement proper*, denoted **Np**, the knot movement when the triskelion is composed of three string elements belonging to the same circle.

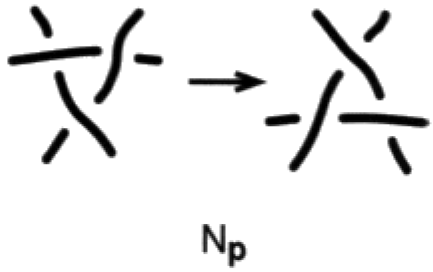


Fig. 26

All three of its crossings are proper crossings. It removes proper knots:

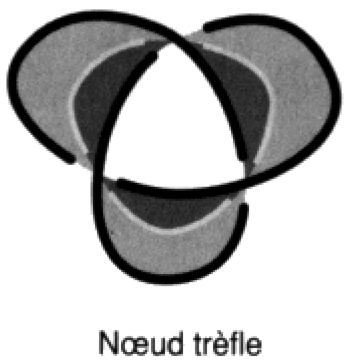


Fig. 27

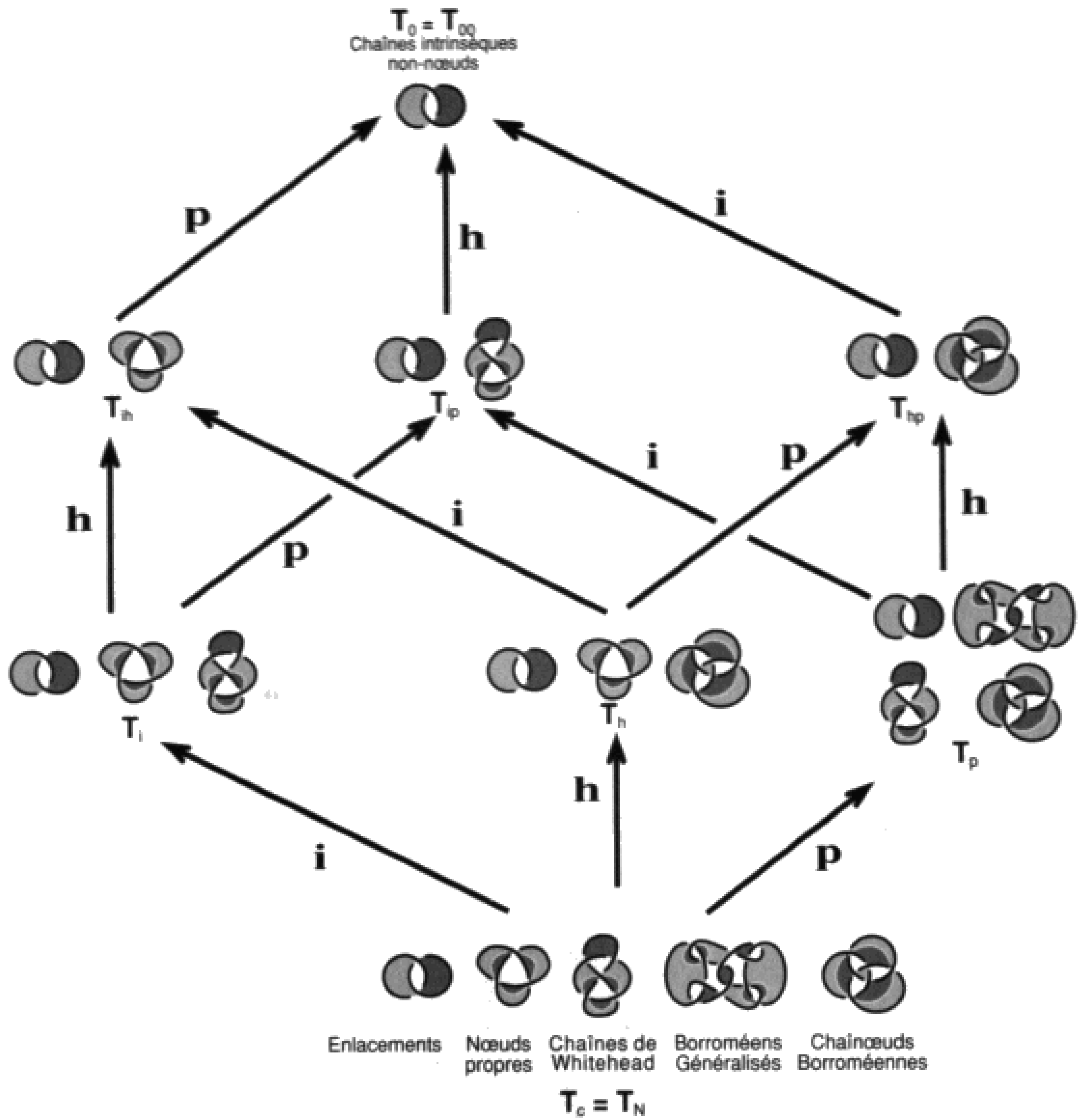
This is the renunciation of delirium. Delirium forgets, more or less partially, the non-knot part of chain knots.

a1 - ***The lattice of knot theories***

We can thus define eight different theories that will enable us to analyze the nodal space in sufficient detail.

Armed with these definitions, we adopt the principle of using these different movements in a differentiated manner. The eight theories differ depending on whether we use only one of these movements, or two or three, as we did in the theory of intrinsic chains.

Each theory will have its own theorems.



Treillis des huit théories du nœud

Together, they will enable us to consider how to remove the entanglements in order to obtain only knot theories.

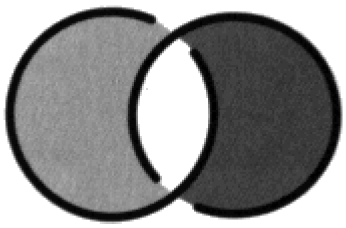


Fig. 28

We will only undertake this analysis in the works[10] that follow this one. The removal of entanglements will allow us to formulate the practice of analysis as a rejection of the neurotic torus and, consequently, of madness.

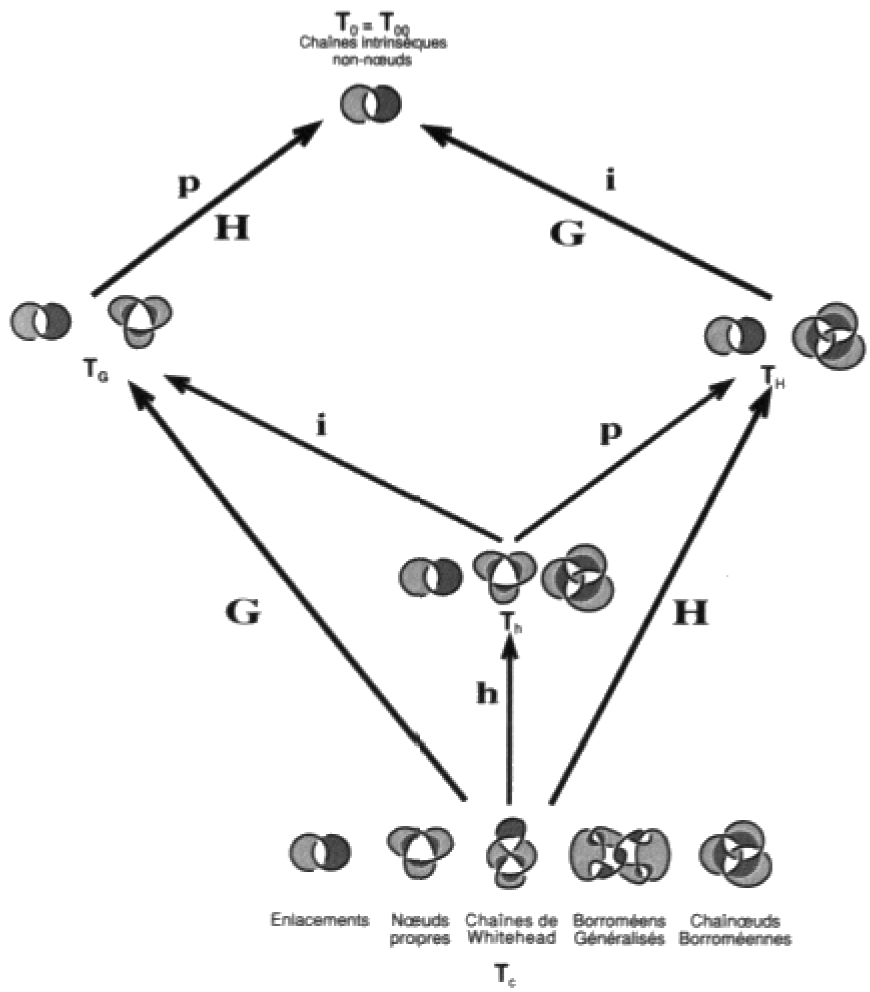
While waiting to undertake these separate studies and bring them together in this structure, the latticework of these different theories will serve to diffract the Freudian structures of psychoanalytic clinical practice.

a2 - ***Clarification in the debate between Lacan and Soury***

We now have the operator that removes the complications introduced by Whitehead's chain and the generalized Borromean knot. These complications are invisible from the proper or improper Gordian theories that distinguish the respective advances of Lacan and Soury.

This is the hybrid knot movement.

If we return to the graph we used in Chapter VI to present them, we can index it by the three types of knot movement that are thus arranged in relation to proper and improper Gordian movements.



Portion of the Trellis to locate Lacan-Soury-Vappereau

In our theories of one to three circles, the generalized Borromean:

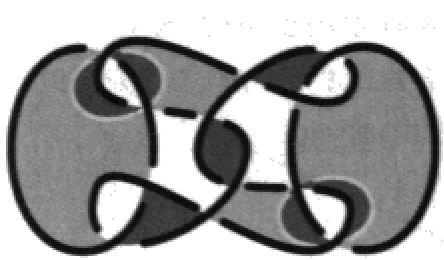


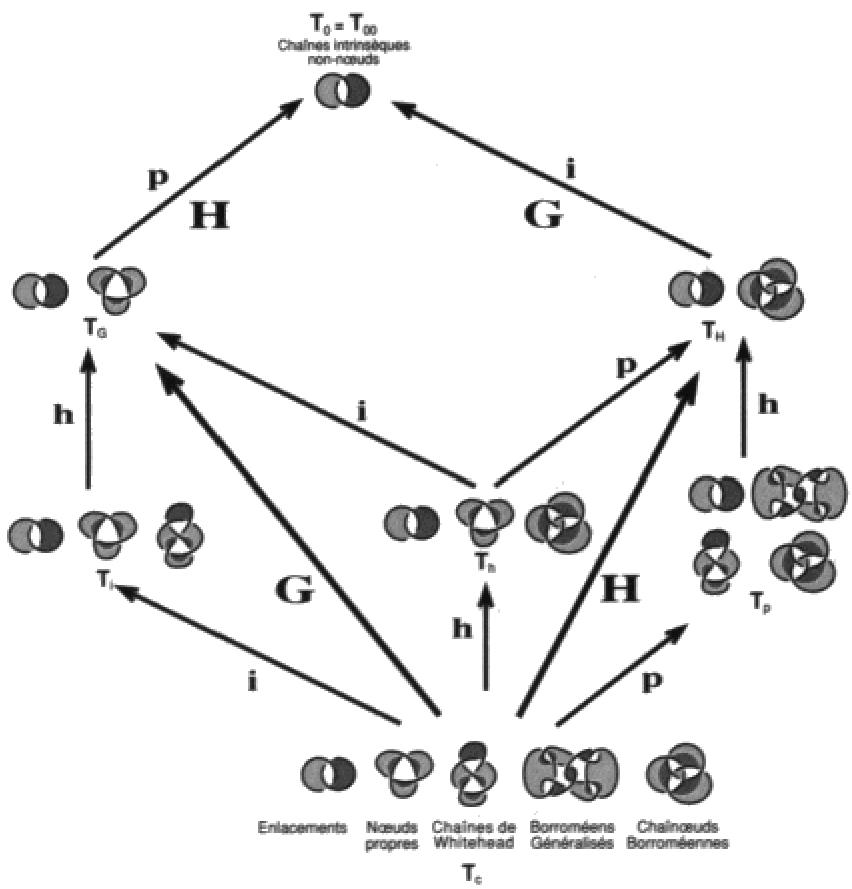
Fig. 29

represents the four in the three, or the sinthome Σ. This is clarified by the study of cross sections, which remains to be developed. There is something of the four, and therefore of the two, in this three that is different from the three of the Borromean knot.

Consequently, the hybrid movement will be more precisely, in this context of objects made of one to three circles, the operator of the involution between the four and the three.

First question raised by our presentation

A first question then arises. It concerns the permutations between these types of knot movements. In other words, is the following diagram commutative?



Portion du Treillis pour la question de la commutation

The answer to this question is easy to give. Only Whitehead-type and generalized Borromean chains pose problems. These chains break down differently in the sense that generalized Borromean chains can be broken down by both hybrid movements and improper movements, which is not the case for Whitehead-type chains, which can only be trivialized by hybrid movements.

We can now present the problem of psychoanalytic clinical practice and Freudian structures.

We will then add a final topological definition, that of homology, in order to give a complete formulation to the dialogue established between Lacan and Soury. This debate still has a few surprises in store for us, as Soury was undoubtedly more interested in Freudian clinical practice in terms of neurosis and perversion than the audience at the time suspected.

perversion. Lacan was alone in taking up the cudgels against psychosis. This homology refers to a relationship of equivalence defined between objects that may have a different number of circles. Objects with one, two, or three circles are homologous from this point of view.

Here again, we can differentiate between more or less subtle relationships of homology depending on what we consider to be knotted or unknotted.

We can vary the analysis:

— by considering them as unknotted, i.e., neutral with respect to the knot, either the free loops between them in the trivial knot, or the linked loops of the non-knots in our theory of intrinsic chains;

— by identifying objects according to the choice of one of our different theories.

This approach will lead to the question of the heterology of the knot, completing the formulation of the difference between proper and improper, with entanglements being present or absent.

Thus, the deployment and gathering of this clinic, the consequences of the exchanges between J. Lacan and P. Soury despite the great difficulties encountered and the heterology of the knot, will be the focus of the last two works in this series, given the results obtained through this continuation of our various works dealing with the knot.

3. **Freudian structures of the clinic in knot topology**

Let us now implement a number of points of overlap between nodal structures and Freudian structures.

There are two major difficulties encountered by beginners when approaching psychoanalysis through the nodal space. The study of these objects and their function remains problematic. But the first difficulty has two aspects. There are the results themselves, but there is also the use or practice of this quest, which is linked to the Freudian structures of neurosis and perversion on the one hand, and psychosis on the other. These are styles of investigation. The second difficulty is therefore linked to the first.

1 — Beyond the first aspect, to which some would like to reduce our work, we have begun to respond to the second aspect by talking about reading.

If, for Lacan, the goal is to write this clinical practice of psychoanalysis as an orography independent of the old psychiatric nosography, this initial response needs to be supported more rigorously, because we are not there yet and no one invents a writing style in isolation.

Our aim remains to practice reading and to respond to the legibility of what Lacan has stated. For the thesis that represents the existence of psychoanalysis in the world, which is not a hypothesis despite the fact that its proponents are still intimidated by it, is that there is something legible even before or without writing being constituted as such.

We now take it up again by dealing with translations, transpositions, transcriptions, in short: the capiton points proposed by Lacan between the discourse of analysis and this topology of the knot. And we already respond to the second difficulty.

This padding is done according to the structure of the poetic metaphor radicalized here into signifying condensation. This means that there is nothing to understand, there is only to follow Lacan or not to follow him, and to explain if we choose to follow him, in the sense of unfolding, of explicating the paradigm thus constructed in an absolute manner.

Learning is produced by the coordinated practice of the two texts subject to translation, as in Japanese writing. The study of knots, as we rigorously apply ourselves to it by means of string and drawing. The retroactive reading of Lacan and Freud to situate what is at stake, and of a few others to grasp what should not be done.

For we expect renewed clinical presentations that have a different purpose than to make us believe in the pseudo-medical guarantees of those who have practiced it until now. That said politely, we always set Freud and Lacan apart.

2 — As for the second difficulty, the function that is not meaning but produces it here risks remaining masked by what remains our main aim, namely, the updating of the main structure of the Freudian field on which depends, from the signifier to the symptom, any formation of the Ics and of fantasy to the subjective split. Here we add a few additional elements to the chain of reasons we have woven, for our own use, in order to express the link, confirmed by those who reject it, between rhetoric and logic. We want to establish this practice of saying by formulating the rhetorical or sophistical condition in the continuation of the condition of use formulated by Tarski of the truth predicate. Either establish it on the real side, in a less idiotic practice of narcissism that no longer resorts to material reduced to bodily fragments but to its chord, around what insists on being impossible, between intrinsic and extrinsic, or what never ceases not to be written.

We began this weaving from the very first work in this series, with a most precise definition of the function of the phallus, the dimension of pretense, starting from the fetish. This means that our progress is contingent, due to love and the chance of encounter, since in this series a piece of reality ceases not to be written.

Freudian clinical structures have their topological transcriptions in the discourse of analysis. The aim is to situate neurosis, perversion, psychosis, and psychoanalysis itself in practice. Care must be taken to distinguish madness despite its proximity to neurosis in terms of the superego, and to specify the psychic causality in its presentation through perversion in terms of fetishism.

This introduction to the Ics and sexuality, as Freud understood them, through the function of the phallus and the functioning of semblance, leads us to clarify the difference between improper metaphor and the delirium specific to paranoid psychosis.

The structure of fantasy is thus framed, and we can situate the question of the constructible object in psychoanalysis.

a0 - ***Madness and psychic causality***

To correctly situate the analytical discourse, it is necessary to distinguish between madness and psychic causality. Without this reminder at the entrance to the Freudian field, there is no chance of finding one's way around the structure.

We will present the terms after recalling this axiom, which determines the fundamental evidence of analytical discourse. It is so obvious that it remains forgotten, even unnoticed by practitioners themselves, thus falling under the principle of causal structure, which will be discussed below.

We therefore begin this chapter with a reminder of a structure that may seem ethical—and indeed it is—since our aesthetic is an ethic. It is a matter of resolution.

Resolution of desire, in the sense of resolving desire as one resolves an equation by finding the solution that satisfies it, in the sense of a resolved desire from which nothing can divert the subject. This is what neurosis abhors.

Let us return to the various definitions of madness proposed by Lacan [É a 8].

Particular madness...

The human mammal is xenopathic because of its actual prematurity. Its condition requires it to incorporate itself into discourse as if into an iron lung. It is therefore fundamentally spoken by the Other, and is thus a speaking being.

But this necessity does not oblige him to believe that he is being spoken to by another, the first definition of madness, in reality. This differs from being spoken to by the Other, a place of truthful fiction, real, more real than reality. This belief can only be established through resignation.

This renunciation has a first name, ignorance, the main definition of madness. It is not not knowing, it is refusing to recognize what he knows very well, even too well.

The result of this evasion through repentance is the self, this envelope of the abject $\mathbf{m \approx i\,(a)}$ in which to revel; the self is a crazy structure.

Thus the subject can believe in it, the second definition of madness, or believe in himself, as they say in the south of France:

"Who does he think he is?"

we say. It suits him well. This statement comes from a question:

"Who does he think he is?"

Indeed: "Who does he think he is?"

This failure to recognize leads the subject to reject their own responsibility onto the other, onto others. This is the policy of the beautiful neurotic soul, the third definition of madness, which ties in with the first.

Since the subject is xenopathic, entirely determined by the Other, that is the unconscious, the place "where it was," as Freud tells us, which I radically reject. It is very difficult for me to say "I," to recognize myself as responsible, to become a subject who says "I." But "I must become," adds Freud.

We prolong the difficulty by adding that this recognition takes place after the fact, which does not provide mitigating circumstances.

This is a difficult, even impossible ethic to bear without the notion of truth as fiction covered by fiction, that is, the Other reduced to the other. We must have an insight into the phallic function that covers everything that is not there.

If this is not the case, it is madness, ignorance with its repercussions of unconscious guilt, as if the subject were being told by this Other:

"Oh yes, it's not you! You say you had nothing to

do with it! You say you have extenuating

circumstances!

Well, you'll see if it's not you!"

An abject and totalitarian response from the Other. A structural necessity slowly discovered by Freud, he gave it the name superego, finally formulated by Lacan.

This formula is as real as that of universal gravitation.

... and collective madness

Freud proposes to describe the malaise in society based on the observation that civilization not only requires the subject to submit to the law of society and renounce his desire. Is it his desire? Is it his impulses? Freud adds that the more the subject gives in to his desire, the more they submit to the law by renouncing the Law, the greater the ever-increasing demand imposed on them by the Law, which is thus integrated.

Only petty moralists spread Freud's teaching as neurotic, as it suits them in their own way of satisfying what they are subjected to, seeking to evade it. Creon by name, call him to good people, sacred name of no, no of name.

Lacan extends it [2 Sem 3], because there is worse, which, contrary to the adage, we can be sure of as we are of the father.

Madness runs rampant in the streets. There is no social order that is not based on neurosis. The subject's self-harm ensures social stability.

Harming oneself is the aspect of the superego in neurosis, the neurotic harm resulting from ignorance.

And Lacan adds, harming others with a few cutting words, the law allows us the full extent of this.

Alternative to asylum

Psychoanalysis only begins when the subject renounces madness, having sufficiently debated the politics of the beautiful soul [É a 21].

Non-madness defines the position of the analysand; this is how psychoanalysis treats and cures neurosis. Those who say otherwise or are skeptical are madmen who have never met Freud or Lacan or any of their students who have persevered without understanding anything.

All that remains now is to carry out this analysis, to complete it in its regularity, to find its legitimate fulfillment. It is the study, to its rigorous conclusion for the subject himself, of mental causality, his own, in order to make it something other than mitigating circumstances.

Given this prerequisite, before undertaking a clinical practice of psychoanalysis from which madness is rejected, we propose here a clinical practice of preliminary interviews, that is, a clinical practice of the asylum, the world in which we live, dominated by entanglement, madness.

Clinic of the asylum

Our structural reference points lead us to revise preconceived ideas and elaborate on their objects, as it seems preferable to refer to the logical constraints of the structure, such as the texts of Freud and Lacan that have not yet been read in this regard, rather than relying on vague impressions that we are led to believe are based on experience.

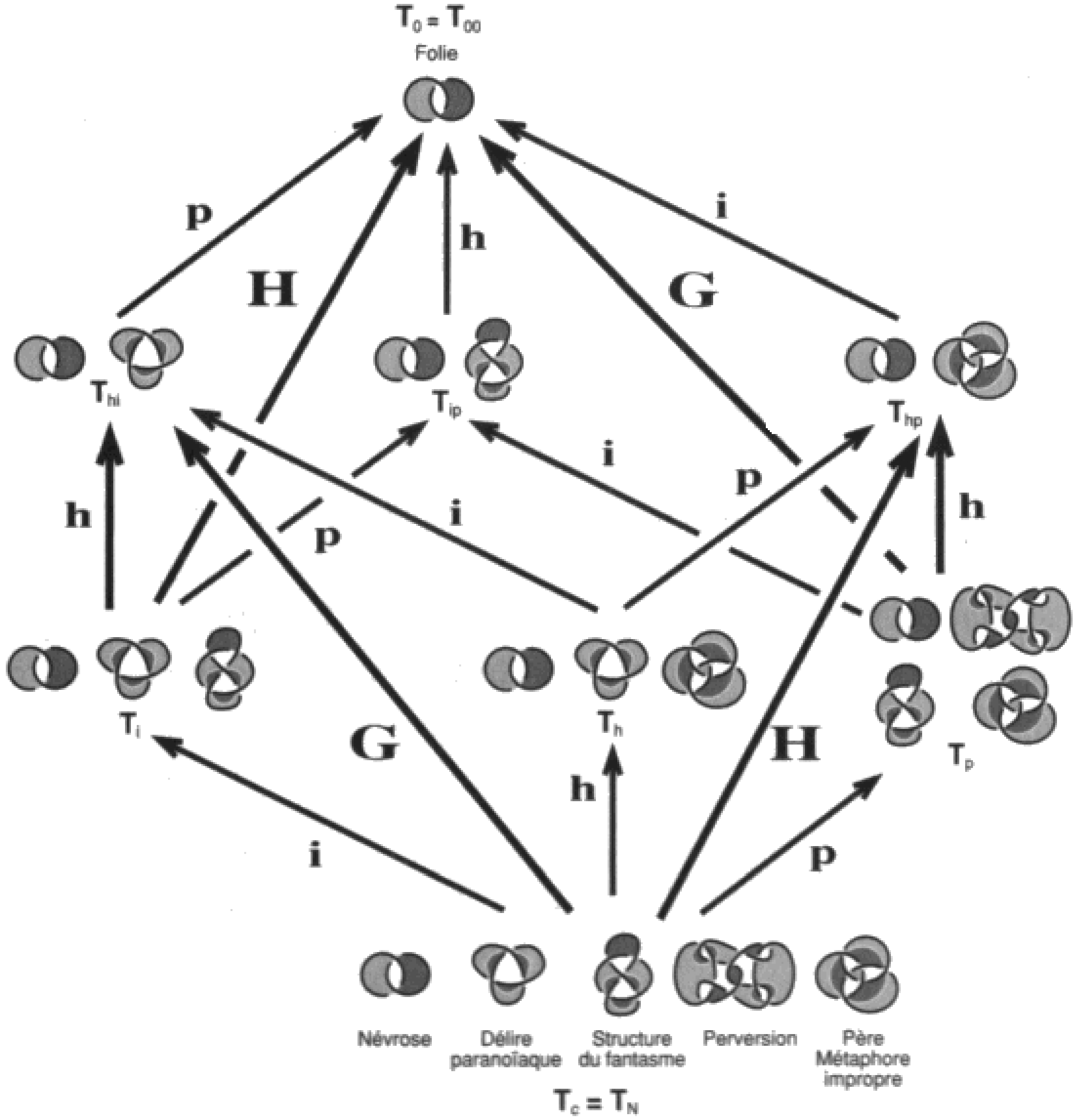
In most cases, everyone's experience merely follows what rumor has it. Twenty years of practice, in a position to provide psychoanalytic consultation, have confirmed this for us.

This crowd rumor confirms the structure by seeking to contradict it, it makes the sharp edge of the test appear blurred and shocks the personal experience of those who have had a somewhat advanced or demanding one.

Anyone who does not have Freud's clinical intuition. It is already difficult to follow Lacan's reasoning and arguments. He wanted it this way so that one could not simulate without it being seen. And it can be seen, or heard. Yet despite this, Lacan finds and places the Freudian arrows in their rightful place.

We will start again from the framework of the eight theories already isolated. They divide up the clinical structures of the asylum where we locate the Gordian movements. The proper Gordian movement is denoted by **H** and the improper Gordian movement by **G**.

The reader will note that the non-knots, in terms of entanglement, occupy all the positions in this lattice, which is why we refer to the asylum in this context.



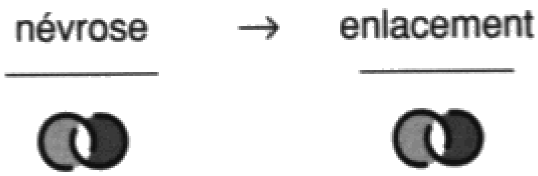
Treillis des huit théories dans l'asile

a1 - ***Psychic causality with metaphor***

Let us begin with the positions that psychoanalysis has always considered treatable. These are the styles in which the subject's capacity for metaphor is not completely destroyed, and a certain credulity persists with regard to pretense. We are referring to neurosis and perversion, which are opposed to each other in structure, as are repulsion and attraction.

The neurotic torus

The structure of neurosis is the structure of the torus. We propose to write it as follows.



How should we read this algorithm? Lacan presents neurosis in terms of desire and demand on the torus [2 a' 26]. The subject repeats the turns of his demand without realizing that he is also going through a turn of desire. This is the duality of meridian turns and longitude turns for a subject who forgets, because of his intrinsic position, that he is subject to the torus, failing to move to the extrinsic to consider the torus as an object([11].

In neurosis, the drive:

(***$*** ◊ **D**)

comes in place of the fantasy, as can be deduced from the calculation of the mathème.

Where, in fact, **D**, the demand, **Φ ≅□D** and **D** replaces **a**, the object causing desire, in the formula of fantasy.

(***$*** ◊ **a**)

The subject of this making of the torus, through this misunderstanding of desire in demand. Submission involves the subject's abdication of responsibility, his madness.

Thus, in the neurotic, the term **−□φ**, negation of the penis as a return effect of the negativity covered by the phallus, is fixed under the matheme of the subject ***$*** in its oscillation between the two terms of the fantasy.

$

—

– **φ**

This sheds light on the clinical fact that neurotics hate their own names. It is a narcissistic defect called repression. This creates some concerns for the subject regarding the assertion that he can no longer distinguish between affirmation and negation. As a result, he believes he does not believe in Santa Claus when in fact he does, a pretense that lasts.

Lacan then turns the matter around, ending up saying the same thing in reverse [É a' 26, p. 42].

Now, the toric structure in nodal terms is entanglement. Let us mark this structure with an icon.

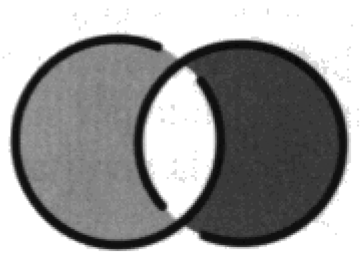


Fig. 30

The variety of entanglement is homeotopic to the toric surface. This means that regardless of their dimension, these spaces have the same fundamental group. We can traverse the same classes of trajectories in them.

From this we can deduce our scription. The structure of neurosis is the structure of unrecognized entanglement.

The structure of neurosis is to do oneself harm. The process of the superego as a response of the unconscious. The subject makes itself a torus by omitting the dimension of the Other.

The forgotten Other reminds the subject of itself through symbolic guilt, take the boat that deserves to be meditated upon, of the debt of the same name.

The forcing of the Other gives the same result in the reproaches that the subject addresses to him or the causality that he puts in him to evade responsibility.

Masturbation has the same effects. Outside of very strict asceticism, it does a disservice to the organ because of the impasse created with the Other, the neurotic subject believing that they will achieve their ends more quickly.

We will therefore note that neurosis has the same character as madness, as a consequence of the politics of the beautiful soul. The ego is a mad authority, we have said, and neurosis is a malaise of the strong, irresponsible ego.

Super-ego madness, which Freud's analytical treatment came up against with the negative therapeutic reaction that put his analysands up against the wall of the phallic function. To overcome this impasse, a more elaborate analytical discourse is needed among its proponents. Even if they are few in number, they must go so far as to accept the axioms of this discourse.

We will talk about neurotic madness, the hysterical neurosis that the analysand must leave at the door of analysis. This is why psychoanalysis cures neurosis, but it is still necessary to commit to it, and there must still be analytical discourse.

We will talk about the neurotic madness of entanglement.

We will thematize this madness through the following metaphorical and scriptural formula that dominates civilization reduced to the rank of asylum.

Behind the super-egoic madness, a voice adds: "(I) hear myself well," always the recovery.

This is how the structure manifests itself in history, in trickle-down effects. We will therefore discuss historical neurosis, whose prototype is phobia, anxiety neurosis, the hub of all neuroses. The two major neuroses, hysterical and obsessive, can be deduced from this structure thanks to the breakdown of the fantasy formula in a third calculation. Respectively:

— on the side of object **a**,

the hysterical plot, which attempts to reveal the sexuation of the object of desire, responds to the jouissance that has been forbidden;

— on the side of the subject ***$***,

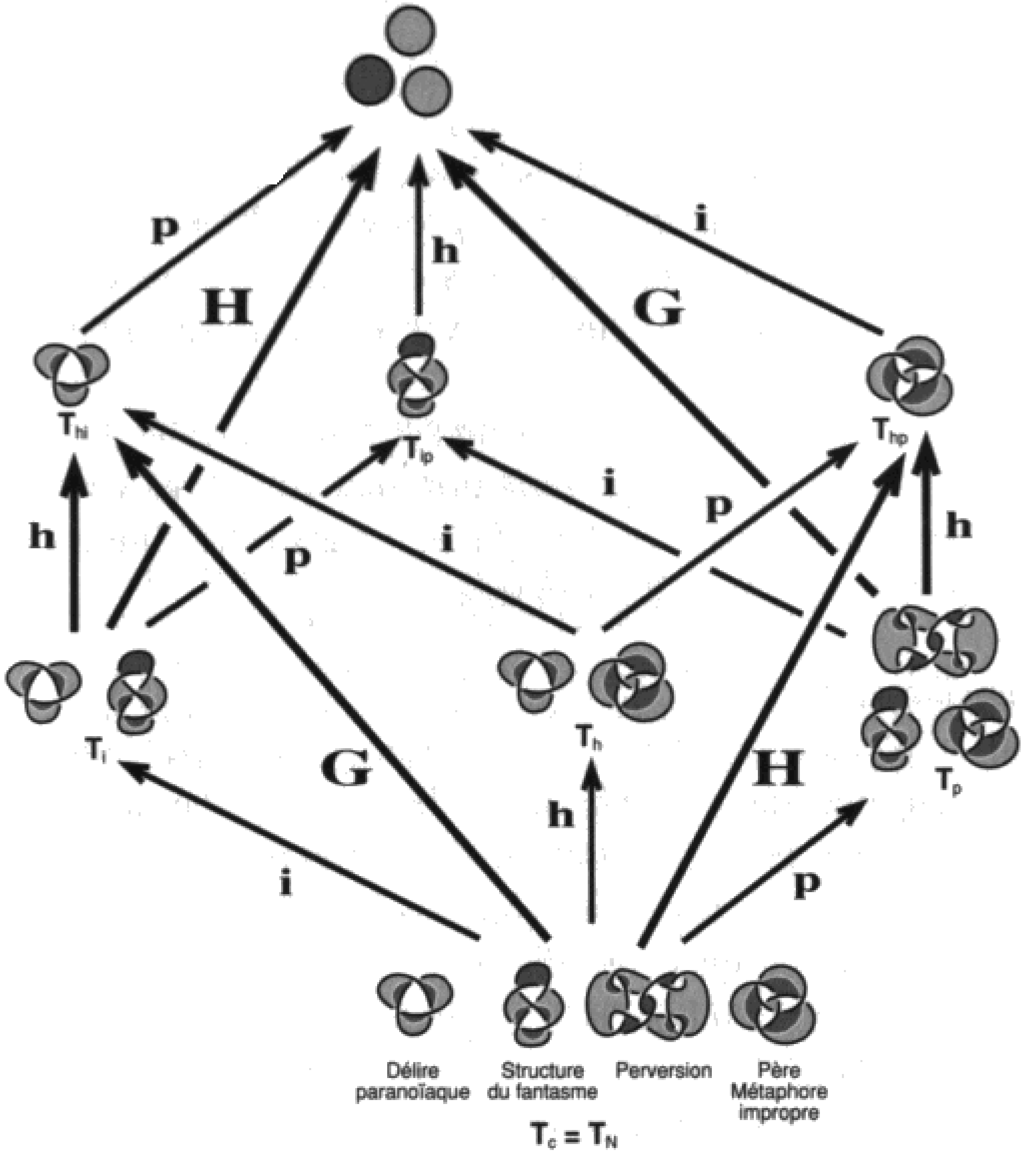
with the impossible death of the subject in the obsessive, who tries to prevent the subject from fainting in response to the jouissance that has overwhelmed him.

But to leave it at that would be to forget that psychoanalysis is an exercise in reading, not a search for a model. In order to enter the Freudian field, there is the opposite pole to that of neurosis, that of perversion, whose role is clarified by Lacan with the sinthome, as we have already mentioned above.

Clinical psychoanalysis

We note that entanglement, characteristic of non-knots, occupies all positions in the latticework of an asylum clinic.

The question now arises as to the existence of knot theories independent of non-knots. This would be a clinic without entanglements, a clinic of psychoanalysis.



Treillis des huit théories dans la psychanalyse

We will go on to show which theories are necessary when entanglements are set aside by the ethical envelope of analysis.

Second question raised by our presentation

To do this, we must first study, in the presence of the entanglement that causes unease in civilization, the relationships between these various theories.

Passage from the extrinsic to the intrinsic and reciprocal passage along the edges of the lattice of theories in the asylum with which we are therefore not finished. We must continue what has been undertaken here, isolating the non-knots among the knots.

This study, which should enable us to detach ourselves from the asylum, goes so far as to situate the function of the nodes of Lacan's node family and the clover node among the proper nodes. This is done by means of a relationship, called homology, which will be defined to conclude this chapter and this book, and consequently by taking into account the heterology of the knot.

This relationship will lead us to compare proper (homotopy) and improper Gordian movements with each other in the rest of this series.

This provides an opportunity to return to the notion of crossover in specific cases. This is where the main difficulty lies in presenting a clinical case in psychoanalysis, as opposed to commenting on a figure or style in literary criticism.

Let us now clarify the second point that we emphasized at the beginning of this presentation of Freudian structures. We will now explain what is at stake in this clinic and in this topology that makes them resonate together.

Clinical dimension

0 — We have already established the structure of the semblance, that of the imaginary function of the phallus. It presents a difficulty, known as alienation, for the subject. Indeed, if the subject adopts the male option, then there is no logical necessity (impossible) for it to be otherwise; only what he represses thus returns in his symptoms. Whereas, if they opt for the female version, by not repressing castration, they will undergo it, because having chosen it allows for its disappearance as logically probable (possible) and the necessary return to the male position.

It is true that it is contingent that there are women worthy of the name. It seems that there were some in Venice...

0.1 — The symptom realizes a figure of speech.

0.2 — It is not that the subject becomes aware of the repressed elements, nor of their meaning, that removes the repression. There may be an intellectual lifting (*Aufhebung*) of the repression through classical negation, but not an affective acceptance (*Annahme*) of the repressed.

Thus, we are led to insist on the characteristic feature of our analysis of denial, on what remains unnoticed in the students of Freud and Lacan.

There is an ambiguity regarding the function of the sign of negation as a marker in our reading of Freud's text.

Conversely, however, it is always a question of the subject holding together, in the same syntactic construction, the two terms of a more (foreclosure) or less (repression) exclusive contradiction. Otherwise...

"*A repressed content of representation or thought can therefore make its way (durchdringen) into consciousness, provided that it allows itself to be denied. Negation is a way of becoming aware of the repressed; strictly speaking, it is already a lifting (Aufhebung) of repression, but certainly not an acceptance (Annahme) of the repressed*."

There is the function of negation, which appears as a marker, and this difference between lifting and accepting the repressed. We will discuss the use of different negations, but this must be demonstrated logically. Freud clarifies the issue.

"*We can see how the intellectual function dissociates itself from the affective process. With the help of negation, only one of the consequences of the process of repression is reversed, namely that its representational content does not extend to consciousness. The result is a kind of intellectual acceptance (Annahme) of the repressed, while the essential remains subject to repression*."

This difference between lifting and acceptance becomes here the distinction between intellectual function and affective process.

0.2.1 — We return to these distinctions between lifting repression and accepting the repressed, intellectual function and affective process, because it is precisely the rhetorical aspect of the symptom, its stylistic and non-classical aspect, in the logical sense, that leads us to conclude that the affective process and the intellectual function are dissociated.

Far from words expressing ideas, sentences manifesting thoughts, and figures revealing feelings or passions or even affections, it is rather these categories of language that produce their effects even in narcissism and lead us to believe in the parallel existence of these entities of the mind.

However, we already have enough to do with deciphering the truthful fictions produced by the effective rhetorical use of language without having to invent additional ghosts. On the contrary, it is the impact of speech in its effectiveness that suggests these psychological additions hanging on the walls of an unlikely place, like family portraits in a gloomy mansion. The illusion of the self (onto) must find its logic, refusing to recognize itself in words (risk of delirious interpretation), sentences, and especially figures; it places itself on the other side, in what it believes itself to be, its ideas, thoughts, and passions.

1 — Thus, more than the subject's interest in his own style, the transfer reveals his passion for the principle (Oedipus) that a figure must satisfy in order to pass as rhetorical. This attractive and repulsive passion leads the subject to believe in the natural, even supernatural, dimension of his attachment to his privileged and misunderstood figures. Unrecognized, that is to say, more or less confusedly known and recognized.

We must therefore return to what a style, a figure, a literary device is.

The clinical issue lies in the importance of the logical structure of these figures, which are unsuspected by the subject himself.

But the question bounces back from the difference between meaning and style to the difference between rhetoric and logic.

Some consider these two disciplines and the constraints they study to be radically different, even incompatible.

We must explain why this is not the case, since they do sympathize, but yes, they do sympathize.

1.1 — This principle is a regression, specific to truth.

Principle

The difficulty in knowing, presented by truth, rhetoric, and speech, stems, for the scholarly commentary on scientific subjectivity, from the absence of markers that attest to the presence of truth, figures, or simply speech.

We can always attempt to introduce such markers in a scholarly manner, indicating, for example, the presence of a metaphor in the verse by Victor Hugo proposed by Lacan:

"His sheaf was neither stingy nor hateful."

If we mark (linguistically) this figure with a metaphorical predicate such as:

"His spray" is a metaphor

this predicate must be capable of satisfying the (rhetorical) condition of structure T:

"his spray" is a metaphor for his spray.

The logical equivalence connector "⇔" reads "if and only if."

For what makes a figure of speech powerful is its ability to surprise us in a special way, precisely because it lacks markers.

Thus, because of this condition, figures of speech are like opportunities to situate subjectivity in language [41] when using personal pronouns, deixis, and performatives.

This principle specific to truth (Tarski), although crucial in logic, is not widely studied in this discipline, where logicians prefer to examine the syntactic aspect of statements, which is apparently closer to algebra.

What can be said about linguists who, thanks to this privilege of order, have been able to introduce a little reason into their field but who, today, no longer dare to continue this advance in the right direction?

Dependent on philosophy professors, both are forced to participate in the desperate rescue of consciousness. This is a criminal pact of which our scholars are hostages. It results in a lack of criticism of psychoanalysis relevant to its rationale.

This principle, linked to speech, which makes every utterance imperative, plays on each figure to produce it as a figure, with the presence linked to the plasticity of the signifier and the absence of a marker, which is replaced by the act of saying that imposes it in the first place.

First means, before any criticism, the discourse of the master who always legislates even where it is outdated.

1.2 — We have already studied truth and the marker of truth. Negation traditionally appears as the marker of what is false.

1.2.1 — We thus find the function of negation, which is a marker function, already noted by Freud.

What is false would be the same as what is repressed. Freud's id is this antinomy. But what is repressed is untrue and irrefutable.

2 — The figure, like truth, conceals an inversion against a backdrop of dissolution.

This rupture of pretense causes a trickle of letters that erode the signified. Counting them produces rapture, a return to the pretense that binds.

2.1 — The construction of this involution requires the immersion of an object in a higher-dimensional space.

2.2 — Dimension is a topological invariant.

3 — All of these statements form a chain of reasons that situates the unavoidable aspect for the analysand of this topological structure, which is usually invested in narcissism.

We will call this structure of signifying involution, a copula that unites the identical with the different, the structure of semblance, denoted Φ, in the discourse of analysis, and which dominates the functioning of $-\varphi$ in the body image.

3.1 — From this, two modes of satisfying a function are revealed, two jouissances linked to the body.

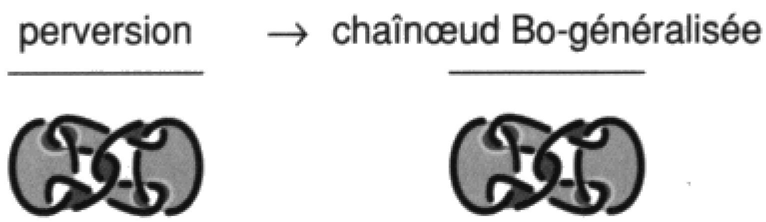
This is the reason for this distinction, on the side of structure rather than trickle-down, which Lacan takes up between the three-ring and four-ring Borromean chains.

Perversion, the sinthome

This brings us back to the link established by Lacan between perversion and the involution of the knot between the four-ring chain and the three-ring chain. We will make four-ring chains the subject of the next work[12] in the series.

For the purposes of this work, we will focus on the effect of this involution among objects constructed with one, two, or three circles. For this structure has a realization in the three. This is the question of the generalized Borromean knot. We will develop this in the last work[13] of the series.

The structure of perversion is the quaternary sinthome. We therefore propose to write this figure of the father-version (reality) as a knot:



given the effectiveness of the chain of three, which remains this unthinkable function of the father.

How to read this algorithm, given that the structure of perversion is fetishism. To rigorously articulate the sinthome as a tendency toward the father in the fetish requires establishing the function of semblant, as we did with Hans and Tarski.

Our reading of perversion in Lacan's Écrits is confirmed if we refer to his mathème as he proposes it [É a 29, p. 823]. It is indeed the fetish when a replaces Ⱥ in S(Ⱥ). Thus, the mathème of perversion is written $\Phi \cong S(a)$, not to be confused precisely with S(Ⱥ).

On the other hand, there is a consistency in Lacan's work, a consistency that is confirmed by the fixation of vocabulary in his later years. The fourth term takes its name from the third mode of primary identification, in the often-repeated equation:

symptom = love for the father

A key question arises when reading Lacan's writings, if we look closely, concerning this association of the paternal function with perversion, but the answer becomes clear.

If the structure of perversion is fetishism, fetishism is the establishment of semblance in the sense of metaphor. It is the master structure of language because it relates to truth. Lacan even makes it the discourse of the master, between the whole of the true that imposes itself imperatively in the assertion and the not-all of a truthful word.

We have shown through Hans and Tarski why there must be a penis where there is nothing.

The Freudian structures of clinical practice, stretched between neurosis and perversion, are implemented by Lacan from his seminar on the object relation [2 Sém IV].

This is indeed the opposition, absolute in structure, between phobia and fetishism. It is modulated on the opposition between metaphor and metonymy.

The first question lies between metaphor and metonymy. Indeed, we associate the fetish here with signifying condensation, whereas Lacan [É a 32, p. 877], when describing the consequences of discovering the mother's lack of a penis, uses the opposite expression to "reverting to a phobia," which is "restoring the fetish, albeit displaced."

Condensation or displacement, neurosis or perversion: like every pair of opposites in Lacan, these oppositions are susceptible to involution. We always respond with signifying involution instead of certainty, but this response is legitimate after we have established the doctrine in mathematical logic.

Then, and only then, can we assume that Lacan, in his Écrits and seminars, performs an involution between metaphor and metonymy, the line without a point of which is found in one of his Écrits. This is in "Radiophonie," when he answers the third question asked of him about linguistics. We thus open the way to a commentary on this answer according to the reasoning that led to its formulation as we read it.

In the structure, it is a matter of the pulsation between Borromean rings:

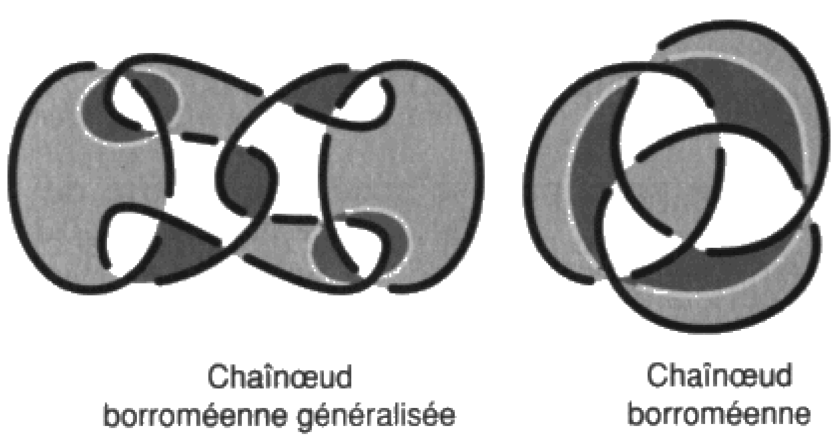


Fig. 31

In the above, there is metaphor, which plays an effective role in determining the other structures.

The improper metaphor

The metaphor of the father's name fulfills a particular function in psychoanalytic discourse. Let us note this result as follows:

This function is that of the effectiveness of the chain of three. We mean the effective function of the Borromean three insofar as it suffices, with any knot movement, to destroy or construct any object that is not a non-knot.

Insofar as it is a matter of immersing the theory of non-knots in the theory of the knot. From there, the analysis consists in diffracting this immersion into various theories, characterizing n spaces, thanks to the specification of the different knot movements.

In the Borromean ring with four rings, the knot that holds the three rings in the chain of three is explicit. The sinthome writes what is implicit in the three, and thus becomes that toward which the four tends without ever reaching it: the sinthome, perversion.

Thus, we will also write:

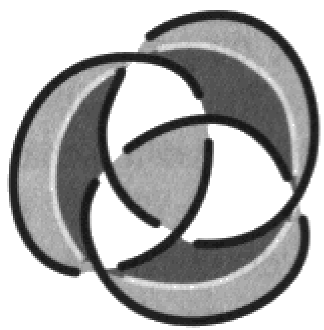
Lacan explains his attempt by pointing out that Freud sticks to the four in *Totem and Taboo* mainly, and he needs the myth of the Orangutan father where the structure shows another reason for the murder with the transition from four to three.

The mathème of metaphor undergoes variations in Lacan's Écrits, of which we retain the final formula:

$$\frac{S_1}{\$} \to \frac{S_2}{a}$$

If it turns out to be the master's discourse, it is indeed the imperative fact of the signifier articulated by this formula. Again, the function of saying.

We will therefore write it from this node, having established its function in topology.



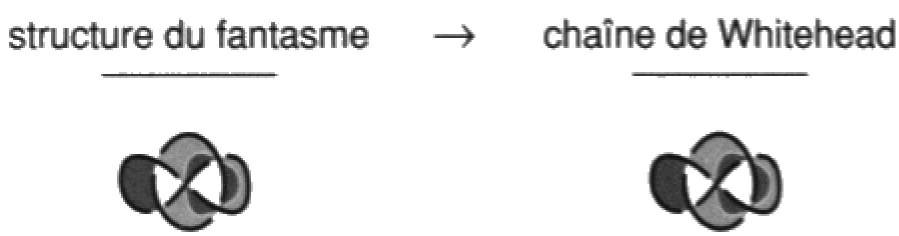
Chaînœud borroméenne

Fig. 32

We will now calculate, within this topology of the node, the relationship between fantasy and delirium in relation to metaphor. This calculation is based on cross-sections. The rigor of this calculation rests on the homology relationship, the definition of which we will establish in order to conclude.

The structure of fantasy

But first, the structure of fantasy. It is written as follows:

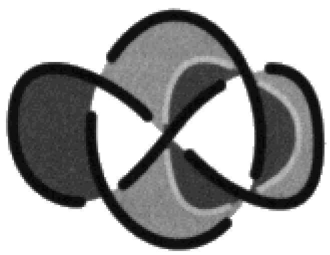


Whitehead's chain is obtained from the Borromean chain by a cross-section. We can read the function of the chains of this family in the lattice of theories.

The mathematical writing of the fantasy by the

formula: ($\$ \lozenge a$)

makes the connector [1] support the relationship between its two terms, the subject and the object. This relationship is found in the reversibility of the two rings of the chain in question [2 Sém XX, *Encore*].



Chaînœud de Whitehead

Fig. 33

The folded loop can be unfolded provided that the other loop is folded in the same way as the previous one. This is the reversibility of the loops; such a chain is said to be reversible.

This is not the case for all chains that we define as belonging to the Whitehead chain family, if we characterize them as 2-chains derived from 3-braids with an even part and an odd part (we are referring to the cut and non-cut parts of our graphical description completed in Chapter VI).

We will study their relationship to Borromean knots using the homology defined below. They are generalized knots, but considerably altered; however, there remains a trace of the opposition between the parts, which is necessary for knots.

Let us now move on to the defect of metaphor with the continuation of the alteration until the complete erasure of one of the parts of the chain knot in favor of the other, thus producing pure knots or pure non-knots.

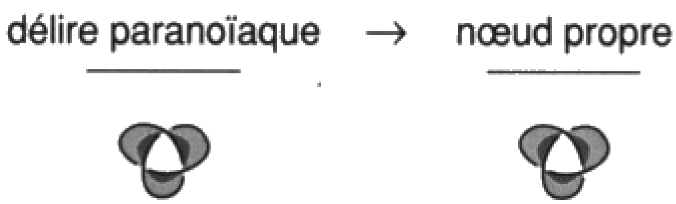
a2 - ***Psychic causality with alteration of the metaphor***

There are two aspects to be distinguished in the study of psychoses. These two aspects are additional, in other dimensions in the latticework of theories, to the pulsation of the structure that characterizes analysis in the terms in which we now formulate this clinic.

It is about delirium and the absence of metaphor.

The delirium of paranoid psychosis

The proper knot of paranoid delirium leads us to write:



We discussed this in terms of a hole in the period of surfaces with the schema **I** constructed by Lacan to illustrate his analysis of the Schreber case. Homology will allow us to clarify this function of proper knots, presented here using the clover knot.

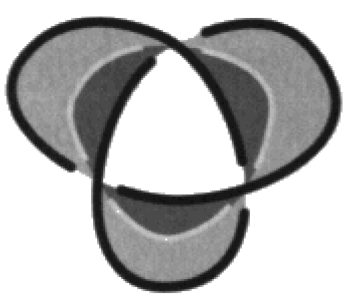
In psychosis, there is no signifier of a lack in the Other **S** (Ⱥ). The semblant is good for others, obsolete, obsolete, Lacan says, foreclosed. The Other lacks nothing, hence the identities taken as a whole:

S (A)□≡ F ≡ a

by default of the circulation of $-\varphi$.

We now clarify this by presenting paranoid psychosis as a freezing of the signifier with the continuity of the three consistencies of the Borromean ring, the real, the symbolic, and the imaginary. The term $-\varphi$ thus translates the distinction between the three rings in their homogeneity [2 Sém XXII, R.S.I.]. It is this homogeneity, which is neither identity nor continuity in the chain, that causes difficulty for Lacan's students of the generation that preceded us, as several have personally testified to me.

The clover knot is produced by connecting the loops of the chain knot using two cross sections. It is a neat knot similar to the chain knot, as we will see below:



Nœud trèfle Fig. 34

Scientific subjectivity tends to make this outcome commonplace, with a somewhat rigid, even completely blocked style.

Scientific subjectivity

We return to where we started at the beginning of this chapter with delusional subjectivity. It is characterized by three features, a delusion, an alibi, and a belief, which place it in the realm of social psychosis.

This absence of metaphor specific to delusion therefore extends beyond paranoid psychosis.

But it should be noted that mathematics, while it is a discourse without metaphor, is not for that reason a place of absence of condensation in the Freudian sense, that is to say, a rigidity of style. On the contrary, the mathematics of mathematicians, not that of apparatchiks or technocrats, is pure condensation, reduced to the letter, of what language deploys in metaphor. This means that there is a link between syntax and rhetoric.

But the social psychosis spread by scientific practice requires resolution in analysis through rigorous answers to the questions now posed in these terms. It is therefore necessary to respond with the mathème.

In the mathematical writing of non-rapport converges the only place where psychoanalysis borders on competing with science.

a3 - ***Psychic causality and condensation***

With these clarifications made, we can return to a crucial point at the time of the completion of psychoanalysis. After Lacan, one task remains to be accomplished.

Another question arises, which has been left unanswered, concerning perversion. When the reader wonders about the fetish in sadism, which is usually considered a perversion. Freud considers it as such.

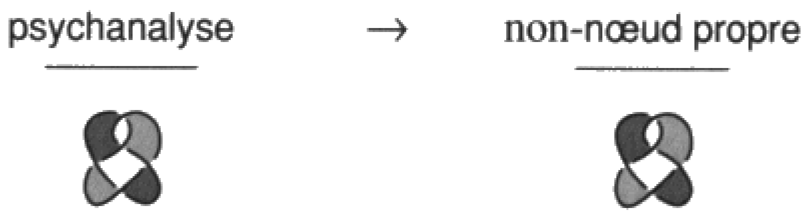
In sadism, it is the pervert himself who becomes the fetish. Lacan clarifies this in his essay entitled *Kant with Sade*.

But we must consider more than just the privileged instrument of perversion. This instrument has a function. It is a matter of making it satisfy the structure of the fantasy. Thus, the sadistic pervert attempts, with this object, to reach the subject of the torments he inflicts. This subject is the other for him, who is reduced to the function of agent. He tries to reach him in his subjectivity, that is, his split. If the subject faints, the scene breaks down.

But here there is a risk of great confusion, which Lacan's students who preceded us were unable to avoid.

The object of psychoanalysis

We will write the object a, from a final algorithmic formula:



This object is the argument of a detachment function characteristic of the position of the desiring subject.

Thus, we must not confuse, as most aspirants to the function of psychoanalyst do, the pervert with the desiring subject. The desiring subject does not shy away from the glimpse of lack in the Other, even on occasions when it is reduced to what is perceived of it, which Lacan formulates in writing:

$$\frac{\mathbf{a}}{-\varphi}$$

where the $-\varphi$ of castration runoff... slips under the object **a**.

The desiring subject, whose example Lacan points to in the character of Alcibiades at the end of Plato's *Symposium*, illustrates the springboard and outcome of transference.

The desiring subject is in a position radically opposed to that of the pervert, since he submits himself to the test of the structure instead of trying to subject others to it by making them experience this lack. He aims to enjoy witnessing this test in the other, instead of submitting himself to the test of the failure of lack in the Other. Our perverse sinthome is very close to madness here.

This writing also shows the opposition to the neurotic, for whom this term $-\varphi$ is inscribed, as we have already specified, under the matheme of the subject:

$$\frac{\$}{-\varphi}$$

But at the same time, this writing promotes the risk of confusion that we denounce here among Lacan's readers between the perverse and the desiring. Since neurosis and perversion are opposed as opposites, as we have said, as metaphor and metonymy also in a rotating structure. This opposition between neurosis and desire can be confusing, and it has not

. See the reminders from certain little Lacanians to their colleagues, always the others, not to slip into perversion by becoming heroes of desire. In fact, it is always a question of control and conformity, by perverts themselves.

It is therefore a common mistake to confuse perverse with desiring, which very few have refrained from doing when it comes to themselves in their attempt to regulate the Freudian field after Lacan. These are the risks of imitation in this field; there are acts that cannot be mimicked.

We return to analysis to conclude, to its end. It is characterized by the recognition of a radical lack that the subject can confront:

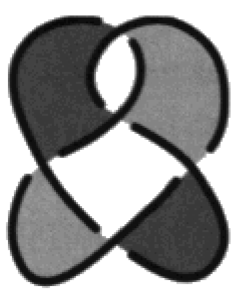
S (Ⱥ)

whereȺ is not replaced by **a** as in the mathème of perversion and the recognition of the structure of fantasy:

(*$* ◊ a)

where **a** is not replaced by **D** as in the formula of the drive, producing a substitution characteristic of neurosis.

This restoration of letters to their proper place applies to the recognition of the object. Finally constructed as an agent, we write this object a, after involution, thanks to a non-knot that owes nothing to entanglement. This is Lacan's knot [2 Sém XXII].

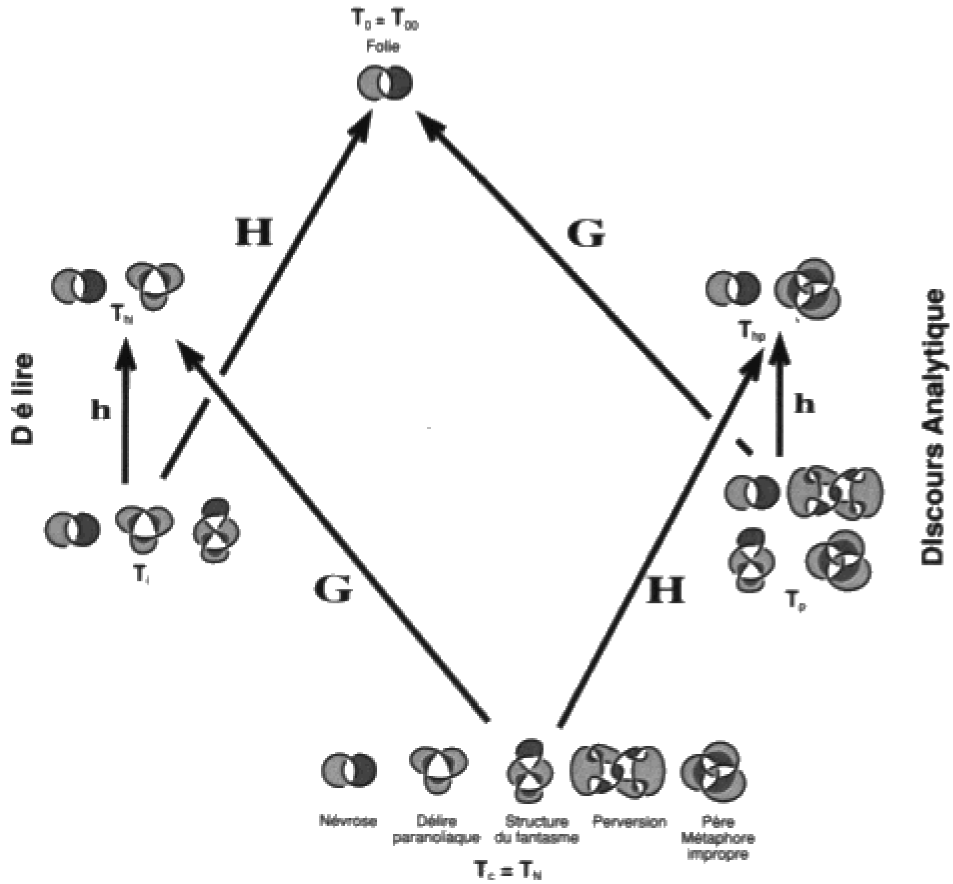


Nœud de Lacan Fig. 35

But let us not believe that we are already done with the torus of neurotic madness, that is, with entanglement. It still acts in this context in a subtle way and puts the subject at risk of returning to it at any moment.

Our task now will be to locate it in the right place.

To this end, we want to return to the difference between Lacan's and Soury's approaches as a first study. That is, to the comparison of proper (homotopy) and improper Gordian movements.



Portion du Treillis montrant le chiasme qui fait difficulté

P. Soury, in dealing with homotopy chains T_{ph}, placed himself in a strictly Freudian perspective. It is necessary to undertake the study of T_p theory at the same time, because an involution takes place between these two theories due to the generalized Borromean movement, i.e., the hybrid knot movement.

Based on our points of contact between knot theories and analytical discourse, we can see that this is the privileged locus for the study of neuroses and perversions from which Freud set out to invent psychoanalysis.

Lacan practiced T_{ih} theory, albeit improperly, in solitude, since it was not enough for him to say so for anyone at the time to realize it. For the same reasons, we must turn, on this side, to T_i theory, which is in involution with the former through an interposed hybrid movement.

This is the structural locus of predilection for delirium and madness, between paranoia and delusional subjectivity, shall we say.

Thus, as participants in discourse—Lacan in analytical discourse, Soury in scientific subjectivity—each deals more specifically with discourse that is supplementary to that to which he or she belongs, that to which the other belongs.

Third question raised by our presentation

If we follow the latticework of theories we have arrived at, we can reveal the chiasm that was difficult at the time and that must be taken into account between proper Gordian (homotopy) and improper Gordian.

This chiasm has a reason in the form of generalized Borromean movement, once again hybrid movement. We can thus establish, some fifteen years later, what our contribution to this debate was. It will be obvious to anyone that we did not understand at the time what was going on there, but that with robust categories such as those produced by Lacan, we can wander through this field in a relevant way. Without the risks of false prudence that the so-called serious people of realpolitik impose on others.

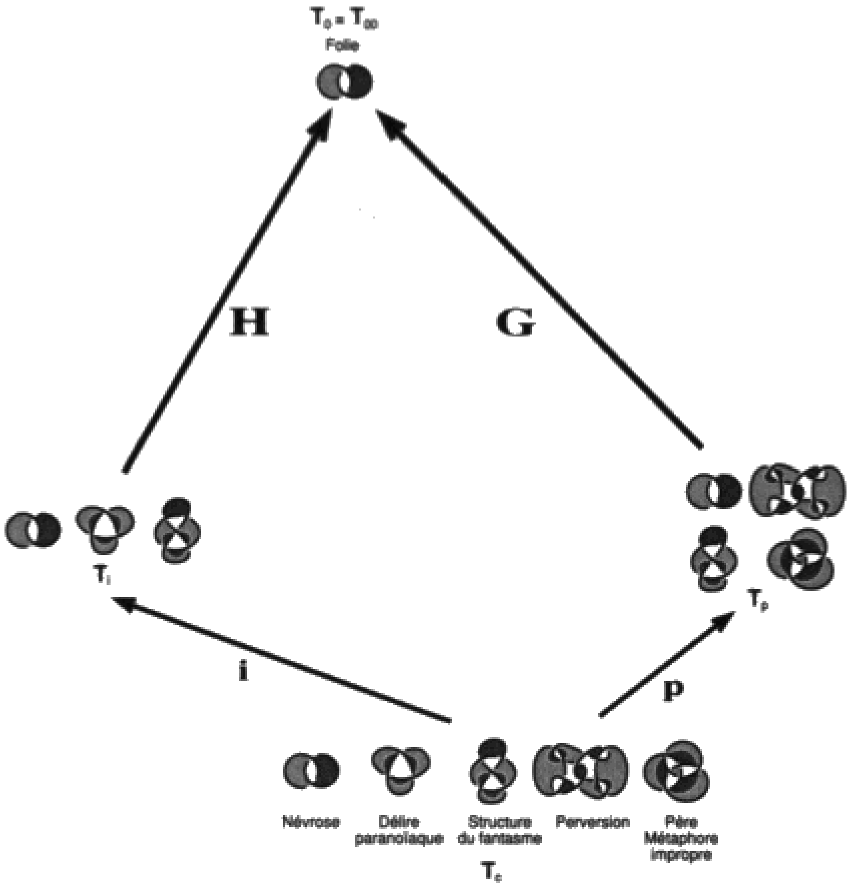
Structural analysis requires time to be illuminated, as does political analysis or historical analysis.

4. **Heterology of the knot**

This chiasm concentrates what is heterogeneous about the knot in relation to the homology we now want to establish between the theories.

We return to the aesthetics of this letter, after passing through the difficult stage where we had to reverse something in the study of these theories.

We therefore obtain the following diagram for the study of homology.



Portion du Treillis pour l'étude de l'homologie

We are now ready to study the homology announced in its place, between these two theories T_i and T_p.

a1 - ***Homology (general reminder)***

In order to define a final equivalence relation between the objects of our knot theories, a relation that will produce an equivalence indifferent to the number of linked or knotted circles, let us recall a notion already presented[14] in our previous work. This is the homology of paths in topological surfaces.

Consider two oriented paths on the surface of a simple torus:

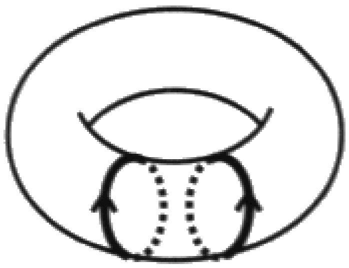


Fig. 36

Beyond the intuitive reason we may have for thinking that these two paths, in our example chosen for this purpose, resemble each other in some way, given the surface of the torus in which they are immersed, we want to define this resemblance. This amounts to specifying how they resemble each other to the point of being identifiable in an equivalence class. This requires that the relationship between them be one of equivalence. We will see this later.

Let us call *a* and *b* our two directed paths given as examples and now consider the case of the two paths *a* and – *b*:

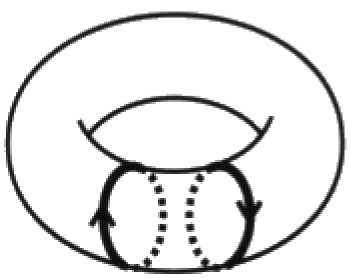


Fig. 37

where the minus sign indicates a change in direction of the path, shown by an arrow in the drawing.

These two objects, *a* and – *b*, must be combined:

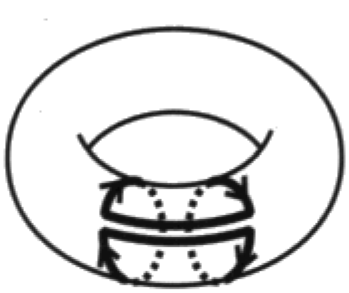


Fig. 38

according to a method of composition reminiscent of our cross sections, that we will define the relationship between them.

We cut them both to join them together with a strip whose edge goes back and forth. This is a portion of an adjustable surface, so as to create a new path named (*a* – *b*). This composition is carried out, in cases where the respective orientations of the two paths allow it, and taking into account the surface on which we are constructing this compound.

If, then, these conditions are already met, the composite path is the edge of a disk, a portion of a sphere that can be taken from the surface where we are located, which is the case here:

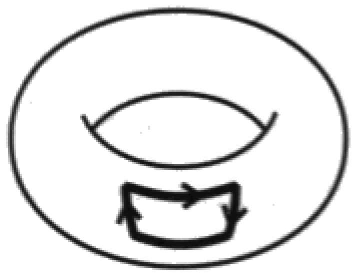


Fig. 39

We will say that (*a* – *b*) = *d* and that the two paths *a* and *b* are homologous.

Definition

Two oriented paths are homologous in a topological surface if the composite of one with the inverse of the other forms an edge (of a disk).

Note that if we consider disk edges as a class playing the role of neutral element for composition, the relation constructed geometrically between oriented paths is written between classes of paths ($a^*– b^*$) = 0 and produces a group structure between these classes. Then the homology relation between two representatives of two homology classes is written:

$(a – b) = 0$, or $(a – b) + b = b$, or $a = b$

where the reader can grasp the need to introduce the inverse of one of the paths in the definition.

Let us give another example of homology on the surface of a torus presented as a sphere with a handle. The aim is to show that the composite of two paths is indeed homologous to a single path.

Let us consider two oriented paths and their composition according to the process we have just described:

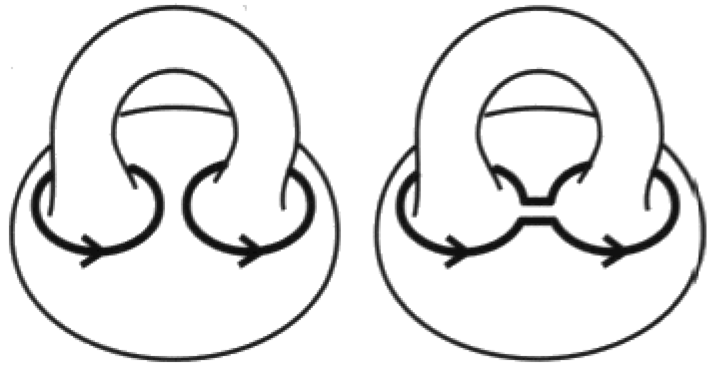


Fig. 40

This composite is equivalent, through intuitive deformations, to the following path:

Fig. 41

To ensure that this intuitive equivalence is indeed contained in the notion of homology, consider the composition of our two starting paths with the inverse of its supposed similar:

Fig. 42

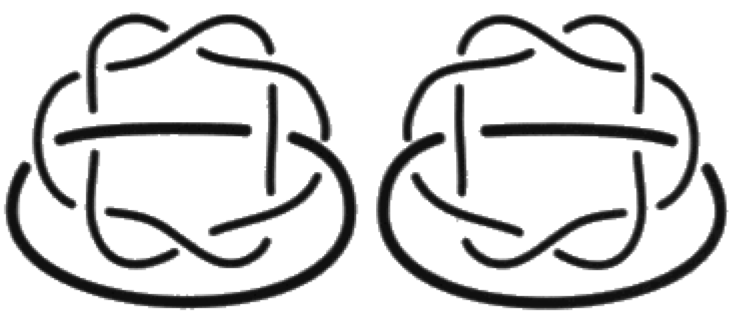
Indeed, the combination of the three is indeed the edge of a disk:

Fig. 43

Let us return to this intuitive notion in our nodal considerations, in order to clarify the definition of a new equivalence relation which, like path homology, will disregard the number of chain components and allow us to compare proper knots and chain knots.

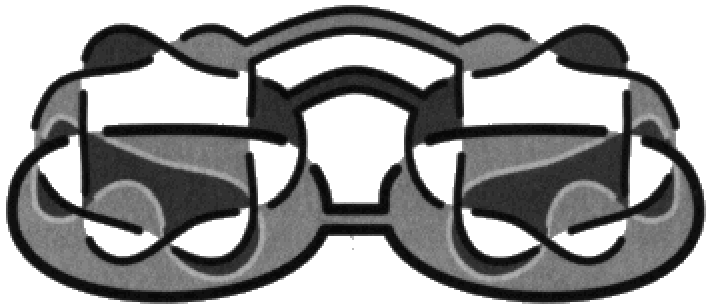
a2 - ***Trivialization by symmetry***

It happens, but this quality is not necessary, that a chain s_r of r rounds is such that, when composed with the chain that is symmetrical to it s_r^{-1} = sym (s_r) (in an axis symmetry):



S^r S^{r-1} Fig. 44

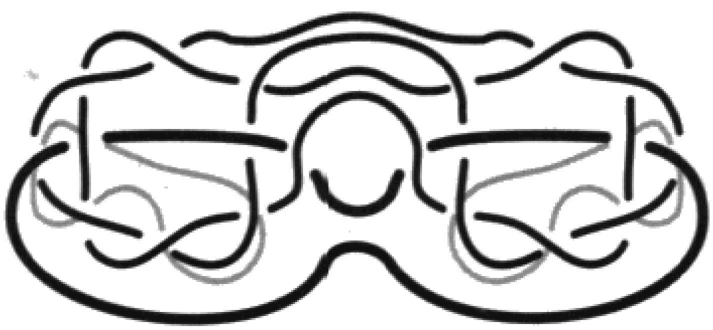
Using ribbons that match the colors of the empan surfaces in the two presentations given, we note $\#_r$ the composition:



$S^r \#_{\mathbf{r}} S^{r\,-1}$

Fig. 45

The whole is trivialized into disjointed, unlinked circles, which we denote **er**. We draw your attention to the fact that we have indicated the cut in this configuration to remind you that the assembly is consistent in terms of coloring, but it is not an additional edge circle that insists. It is an edge that consists[15].



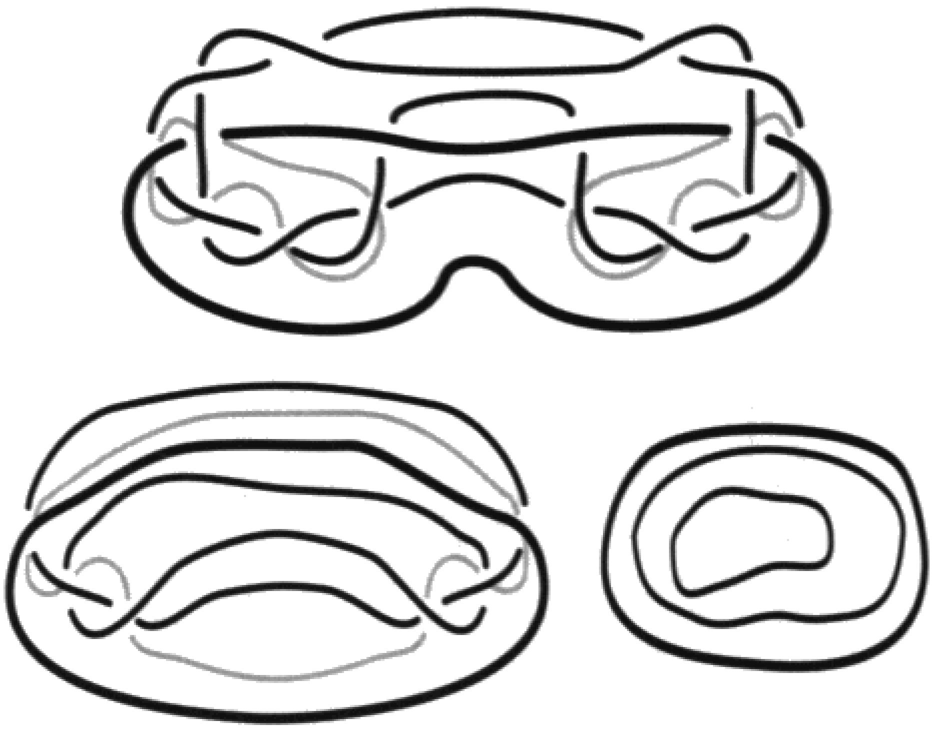


Fig. 46

This means that we can write about this construction: $[(S_r) \#_r (S_r^{-1})$

$= e_r]$

The number of ribbons and the number of circles

When this trivialization does not occur, we can add **r'** ribbons such that we obtain:

$[(S_r) \#_{r+r'} (S_r^{-1}) = e_{r+r'}]$

This gives us a first indication of the regular assembly from which a given object originates.

The additional ribbons will be an indication to look for an r+r'-chain that maintains the type of homology relationship we want to study here with the given chain.

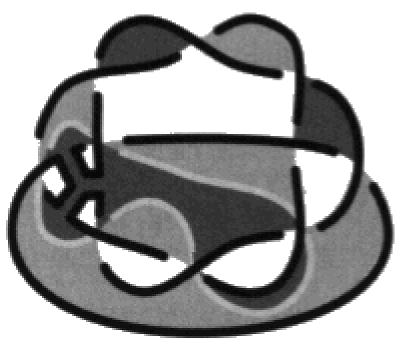
But this indication is insufficient, because there may be a higher-numbered solution, even when r' = 0. There may be other solutions with r' > 0, with changes in the presentation of the chains involved.

a3 - ***Cross sections***

Defining, according to a process analogous to the homology of paths in a surface, from this trivialization, a first relation between the nodes and the chains, and between the chains themselves, requires some clarification.

Definition of a system of cross sections

We denote by c_q a system of **q** cross sections that respect a coloring of the span surface of a chain or knot presentation. These sections are made between the circles that constitute edges that lie on the span surface.



un système $\mathbf{c_q}$ dans $\mathbf{S_r}$

Fig. 47

We propose that these sections follow the boundaries of the span surface, not the edges that consist of it, i.e., without torsion, even if not apparent[16].

Definition of a regular system of cross sections

A system of **q** cross sections is regular for a given r-chain with $0 \leq q < r$, when for any order on q it passes from the r-chain to an (r – 1)-chain, to an (r – 2)-chain... up to an (r – q)-chain.

To meet this requirement, each cross section is performed between distinct pairs of rounds in the chain.

a4 - ***Sections that do not hinder trivialization***

Let us then define a relation between chains and nodes that uses ribbon montages, trivialization, and these section systems.

Definition of the order relation of degree m: denoted $\leq_m$

We will say that two nodes or chains s_n and s_m, with **n ≤ m**, defined up to Reidemeister moves, are linked by a relation of type $\leq_m$, or that $(s_n \leq_m s_m)$, if and only if:

$$s_n = s_m + c_{(m-n)}$$

and, if $[(s_m) \#_m (S_m^{-1}) = e_m]$, then $[(s_n) \#_m (S_n^{-1}) = e_n]$.

The section system $c_{(m-n)}$ does not interfere with the trivialization defined by the assembly $[(s_m) \#_m (S_m^{-1}) = e_m]$.

a5 - ***Number of rounds in the maximum chain***

Even in cases where it suffices to take the number of ribbons equal to the number of circles for a chain to trivialize with its opposite, which we write as follows with the relation $\leq_r$:

$$(s_r) \leq_r (s_r) \Leftrightarrow [(s_r) \#_r (s_r^{-1}) = e_r]$$

We may wonder about the maximum number of circles, if it exists, and the number of ribbons required that we will retain in the theory after discussing the case, i.e., such that :

$$(s_r) \leq_{max} (s_{max})$$

This gives us a second indication of the regular assembly from which a given object originates.

For a knot or chain of **r** loops, the number **p** of parts (knotted and unknotted) can serve as a second indication, given the description of chains and knots that we have already proposed in Chapter VI of this book.

Under these conditions, if s_{max} is a string s_m with **m** characters, then this number is such that:

$$\mathbf{p + 1 \leq m}$$

a6 - ***Partial strings of a given string***

In order to formulate the heterogeneous presence of the node in non-nodes from three circles, we can consider, in a chain, its partial chains or partial chains.

A partial chain is characterized by a quotient of the set of colorings of a given chain.

When we consider the most discrete partition of this coloring set, each coloring corresponds to what we will call partial nodes of the given chain.

This means that in the case of knotting the chain as a whole for each fixed orientation, i.e., the chain itself, but whose orientations of the rings are made integral by this very fixation, we will refer to the partial knots of the given chain.

It is these partial objects that account for the presence of knots in non-knots. Non-knots also have partial knots that are homologous to trivial knots.

It is clear that the choice of presentation and orientation of the object plays a role in establishing this homology relationship.

a7 - ***Definition of the homology relation and homologous knots***

Let us define the type of homology relationship we are using here in order to isolate the knot.

Definition of the H_{Sn} relationship

We will say that two knots or chains **S** and **S'**, defined by movements in theory T, are homologous in T or linked in T by an $H_{Sn\text{-type}}$ relation, or that (S H_{Sn} S'), if and only if there exists an n-chain s_n such that:

$\mathbf{S} \leq_n s_n$ and $\mathbf{S'} \leq_n s_n$

that is, there exist c_p and c_q such that:

$\mathbf{S} = s_n + c_p$ and $\mathbf{S'} = s_n + c_q$

and if $[(s_n) \leq_n (\mathrm{sym}\ (s_n))^{-1} = e_n]$

then:

$[(\mathbf{S})\square\leq_m (\mathrm{sym}\ (s_n))^{-1} = \mathbf{e}_{(n-p)}]$ and $[(\mathbf{S'})\square\leq_n (\mathrm{sym}\ (s_n))^{-1} = \mathbf{e}_{(n-p)}]$

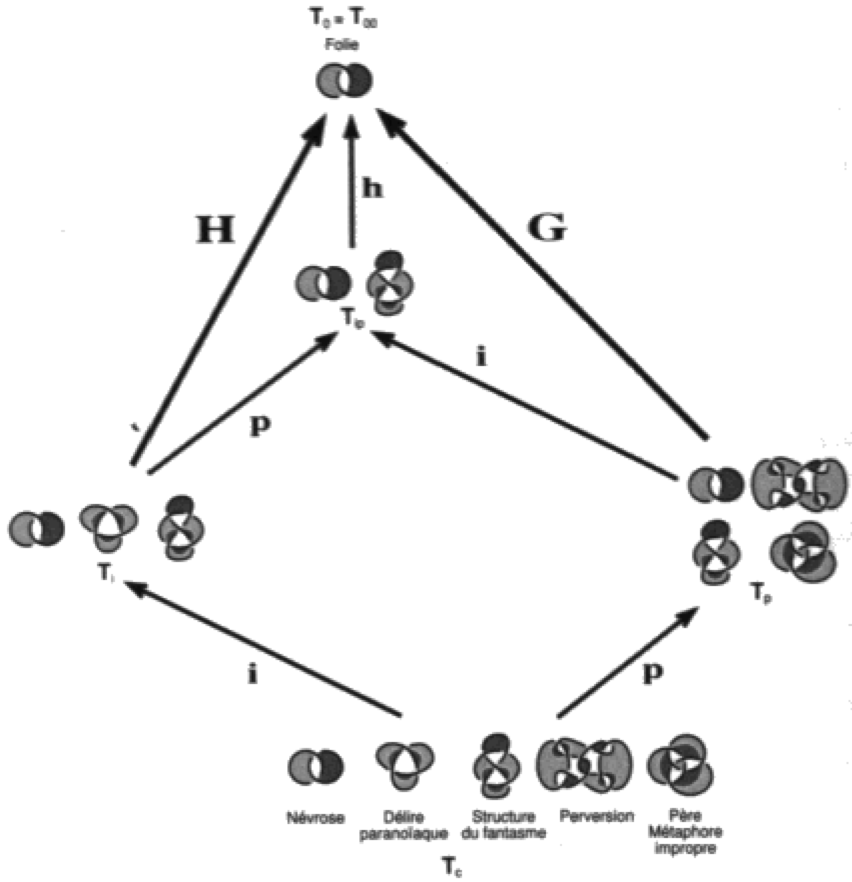
a8 - ***Properties we want to study next***

Invariance of the number of nodes

It is very clear and simple to understand that if these cuts preserve the number of nodes, we are satisfied.

Homologous movements

It also happens that between theories T_i and T_p, the homotopies that trivialize an object of theory T_i into its contained non-knot are homologous to the Gordian knots that trivialize a chain of theory T_p, homologous to this object, also into its contained non-knot.



Portion du Treillis pour la question du rôle des Whiteheads

We can then consider that, in a certain way, to be specified below, homotopies are halves of Gordian knots.

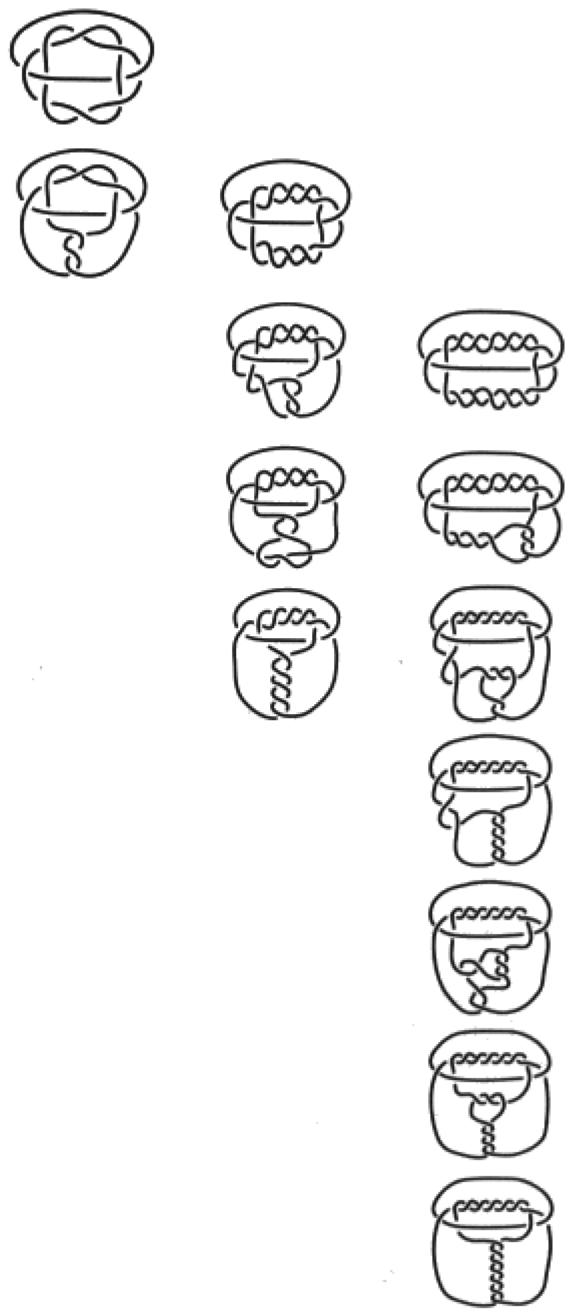
We can conclude this work on these perspectives with a first outline of results that reveal the role of the knots of the Lacan knot family in this theory of homology, in contrast to the generalized Borromean knot, which plays a role closer to heterology.

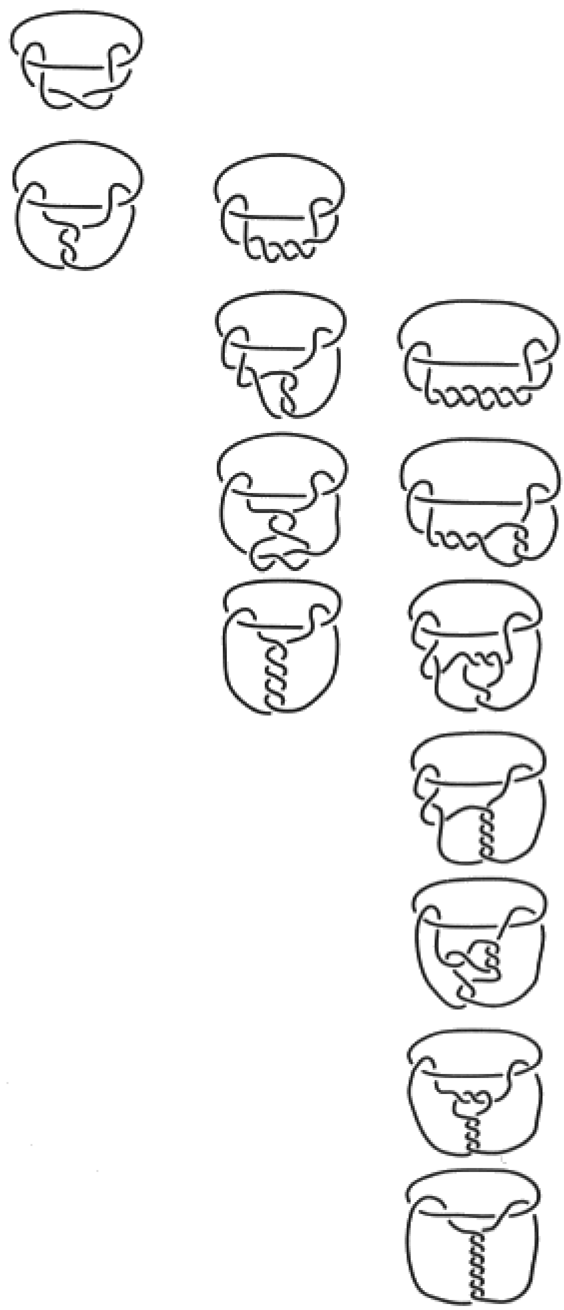
a9 - ***Series of homologous chains and knots***

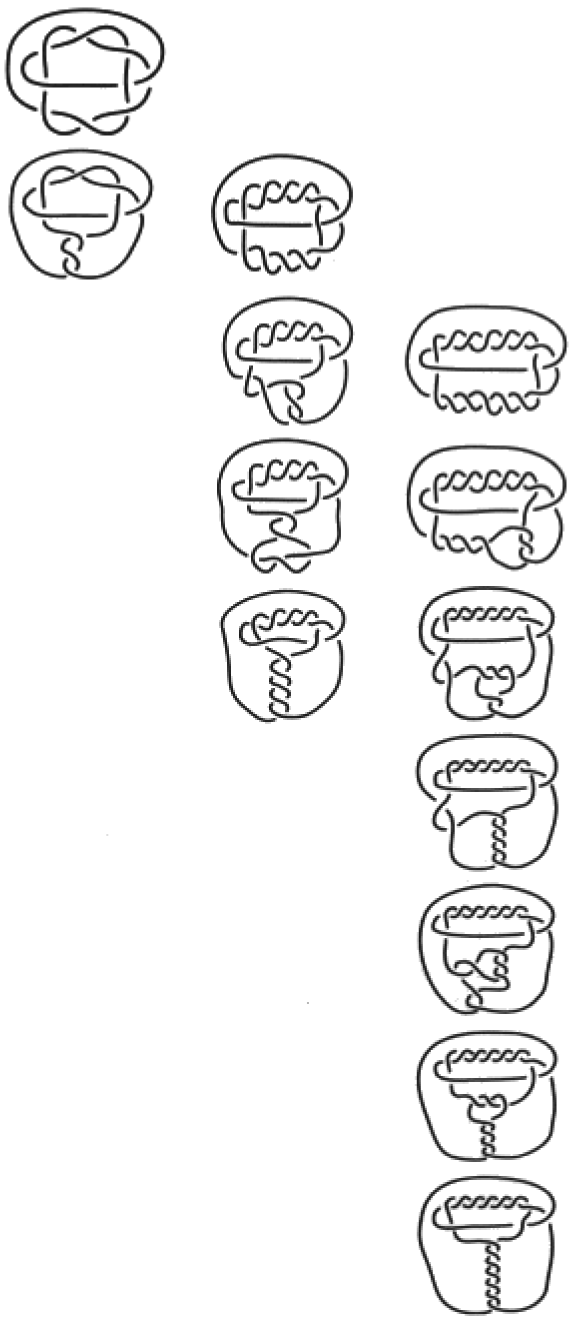
Finally, we give the main series of pure trefoil knots and Lacan *knots* (*proper knots*) which are homologous to Whitehead knots in T_i theory (*2-chain* knots) and Borromean knots (*3-chain* knots), homologous to the same Whitehead knots in T_p theory.

Homologous objects correspond term by term according to their position, from one drawing board to another.

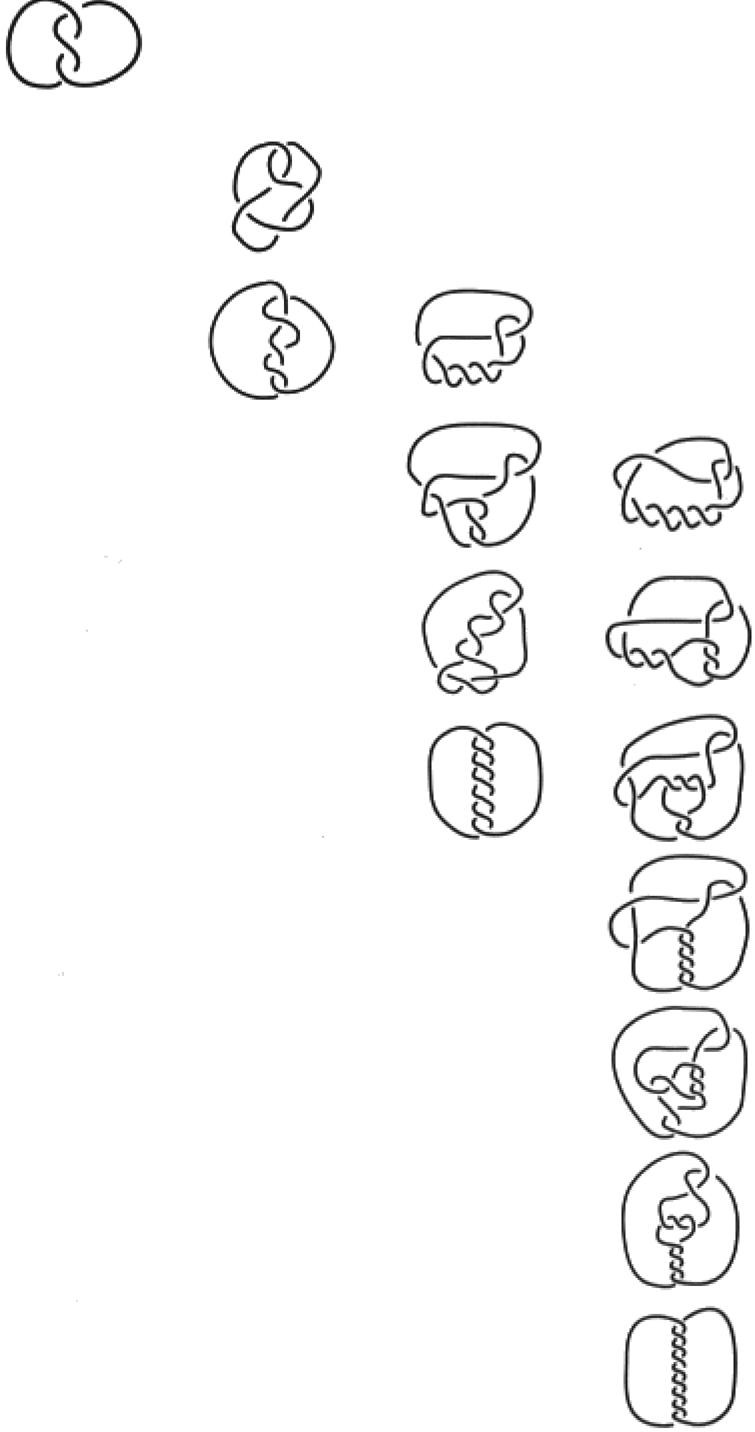
3-CHAÎNŒUDS







Nœuds propres



Appendix

Elements for a theory of representation and the object

Chapter I

Coloring and Orientations

The act of spreading color to paint areas is not a trivial gesture, as it might seem to a hasty mind. If we compare this coloring to what a semblance of calculation requires, such as orienting cycles, we are surprised by the exact translation that insists between the results of these seemingly very different activities.

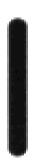
The difference between them lies in their size: two for the colored flat area, one for the oriented paths.

We are tempted to believe that the extended surface lends itself more to representation and resemblance, which are characteristic of analogy, while the line drawn in writing and meaning are more suited to metaphor. However, here we will show that they intersect in a strict translation.

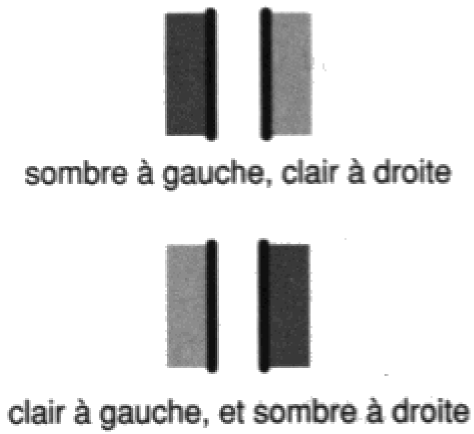
1. **Coloring**

We will not dwell on this further here, since coloring is the main focus of our initial investigation of the knot. Let us simply emphasize that the effect of the color spread across the areas is that, after establishing the distinction between full and empty spaces, it marks the string elements on one side with a color. There are therefore two primary differential pairs in this process: dark/light and right/left.

For a single string element in this case:



This gives four colored cases:



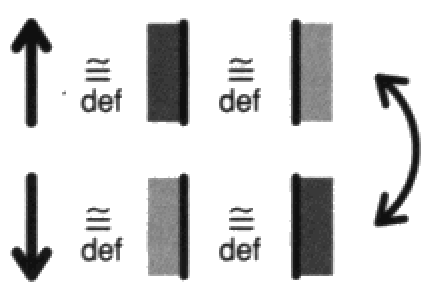
We want to compare this quartet to the pair of orientations of the string element.

2. **Orientation**

As we pointed out at the beginning, a string element taken from a cycle is capable of two orientations, in this case up or down.

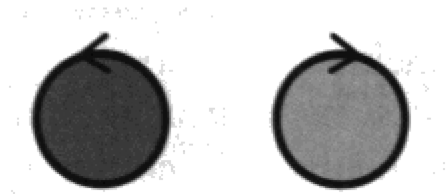
3. **The translation**

It is based on a proposed equivalence between the previous indicators, which are composed in accordance with this relationship:



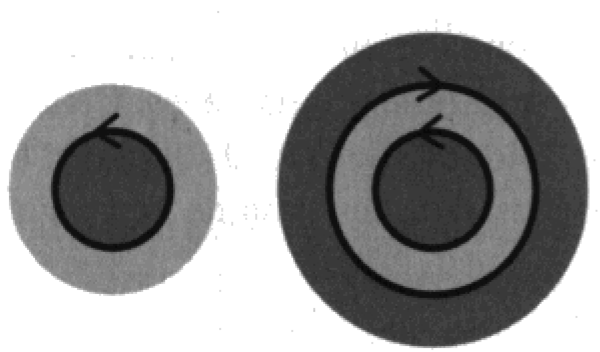
For coloring, the dark/light change or the right/left change is equivalent, according to this translation principle, to a change in orientation from top to bottom. A double change returns to the initial situation.

We can establish this correspondence using two basic cycles, which are condensed into two small drawings:

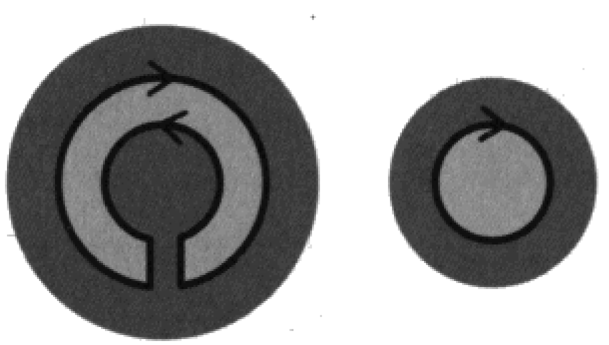


4. **Accumulation**

We can see that this method is consistent with a series of concentric circles alternately oriented in one or the other of the two directions:



And that this approach is also consistent with the cross sections that we are likely to produce between these oriented circles:



These cross sections respect the composition between them of the orientations of the cycles.

We can even generalize this practice of coloring and orientation to concentric circles with different orientations. But then we must add a third element to the coloring in the form of a cut that forms a border between the colors. We call this type of cut cycle a border.

The cross sections cannot cross this edge, because they would then no longer respect the cycle orientations. But this edge itself is susceptible to orientation.

These colorings and this cut remain consistent with the cross sections if we add twisted sections between the circles. This means that we can consider the initial cycles as Seifert circles.

We can also take into account only one coloring of every other zone by introducing the notion of solid and empty zones.

We remain astonished by the consistency of this set of facts, knowing that with well-defined coloring principles, we can dispense with cycle orientations, which remain uniquely determined based on the choice of given base cycles.

We can conclude that well-defined colorings or orientations are at play. The processes are identical. This identity plays a role in our node coloring algorithm, which obviously comes from the theory of surfaces conceived as piecewise-oriented tilings[1].

Chapter II

The object petit a in Chinese

Formalization

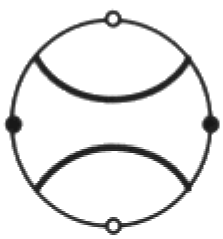
We propose to use the following expression where $x \in Z2$ and $y \in Z2$: $f(x, y)$

$= (x + y + 1)\, 1 + x\, (y + 1)\, \lambda + (x + 1)\, y\rho$

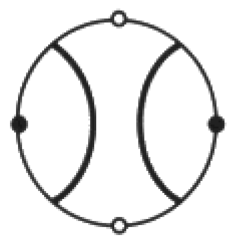
1. **The elements**

Any configuration can be broken down by a graph (called a Terrasson graph) into two types of pieces, and only two types, which we will call generating punches.

We designate the two generating punches by the values:



a = f (0, 0) = 1



a' = f (1, 0) = λ

and formulate the two modes of composition in the case of more general punches.

a1 - ***General form of the punch***

A punch has the following general form:

$P(x, m, n) = B(x) \, . \, V(m, n)$

where $V(m, n) = \lambda^m \rho^n$ denotes the number of circles formed inside the punch with $m \in N$ and $n \in N$, and:

$B(x) = f(x, 0) = [(x + 1) + x\lambda]$ forms its edge. a2

- ***Generating punches***

The generating punches are therefore:

$P(0, 0, 0) = f(0, 0) . V(0, 0) = f(0, 0) = 1$

and

$P(1, 0, 0) = f(1, 0) . V(0, 0) = f(1, 0) = \lambda$

2. **The compositions**

The laws of composition of the punches can therefore be

written as follows. a1 - ***Composition by solids***

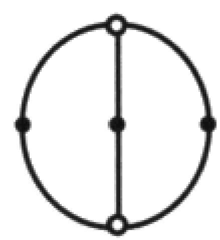
For the composition by the full :

$$P(x, m, n) \wedge P(x', m', n') = B(x'') . V(m'', n'')$$

where $B(x'') = F(x \vee x', 0)$

and $V(m'', n'') = F(x \wedge x', 0) . V(n, m) . V(n', m')$

That is:



composition selon un point de plein

a2 - ***Composition by empty sets***

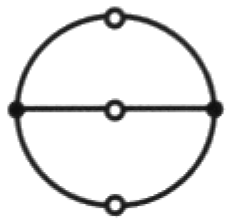
For the composition by the : :

$$P(x, m, n) \vee P(x', m', n') = B(x'') . V(m'', n'')$$

where $B(x'') = F(x \wedge x', 0)$

and $V(m'', n'') = F(x \vee x', 1) . V(m, n) . V(m', n')$

That is:

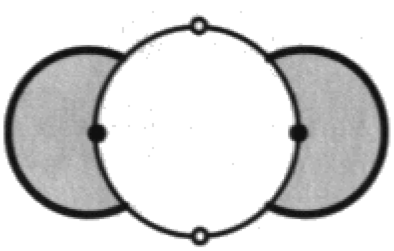


composition selon un point de vide

a3 - ***The closer***

Finally, the closer introduces a multiplier factor:

$F [P] = \lambda\square . P$

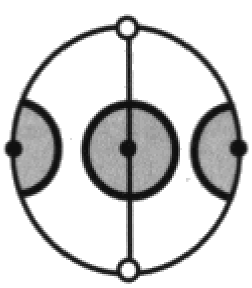


Le fermeur F

3. **Examples**

a1 - ***Two punches (a')***

The composition of two punches (a') adds a solid circle:



We compose two punches P (1. 0, 0) knowing that :

$$P (1, 0, 0) = f (1, 0) \,.\, V (0, 0) = f (1, 0) = \lambda.$$

$$P1\,(x'', m'', n'') = P\,(1, 0, 0) \wedge \Box P\,(1, 0, 0) = B\,(x'') \,.\, V\,(m'', n'')$$

where $B\,(x'') = f\,(1 \vee 1, 0) = f\,(1, 0) = \lambda$

and $V\,(m'', n'') = f\,(1 \wedge 1, 0) \,.\, V\,(0, 0) \,.\, V\,(0, 0) = f\,(1, 0) = \lambda$

thus: $P1 = P\,(1, 1, 0) = f\,(1, 0) \,.\, f\,(1, 0) = \lambda^2$

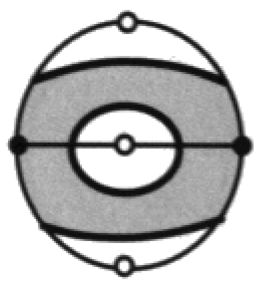
and becomes, if we add a closer:

$F\,[P1] = \lambda \Box .\, \lambda^2 = \lambda^3$

We leave it to the reader to verify that P1 with a closure does indeed have three circles surrounding solid areas.

a2 - ***Two punches (a)***

On the other hand, the composition of the empty spaces of two punches (**a**) adds a circle of empty space:



We compose two punches P (0. 0, 0) knowing that :

$$P(0, 0, 0) = f(0, 0) .\, V(0, 0) = f(0, 0) = 1$$

$$P2\,(x'', m'', n'') = P\,(0, 0, 0) \wedge \Box P\,(0, 0, 0) = B\,(x'') \,.\, V\,(m'', n'')$$

where $B\,(x'') = f\,(0 \vee 0, 0) = f\,(0, 0) = 1$

and $V\,(m'', n'') = f\,(0 \wedge 0, 1) \,.\, V\,(0, 0) \,.\, V\,(0, 0) = f\,(0, 1) = \rho$

thus: $P2 = P\,(0, 0, 1) = f\,(0, 0) \,.\, f\,(0, 1) = \rho$

This becomes if we add a closer:

$F\,[P2] = \lambda \Box .\, \rho = \lambda\rho$

Here again, we leave it to the reader to verify that P2 with a closure does indeed have two circles, one surrounding a solid area and the other surrounding an empty area.

Reading principle

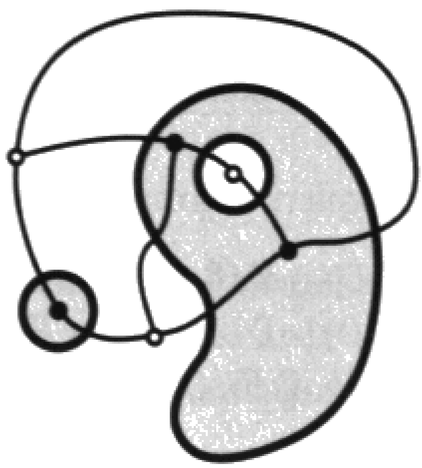
The principle to adopt when reading the result in the figure is as follows: We read λ and ρ for each circle depending on whether it encloses a solid or a void.

The action of the closer can be described by the formula:

$^{F}[P\ (x, m-1, n)] = \lambda\square.\ P\ (x, m-1, n) = \lambda^{m}\ \rho^{n}$

which states that the closer always completes a circle enclosing a solid shape in the punch.

a3 - ***The little object a in Chinese culture***



$F\ [P\ (1, 0, 0) \wedge (P\ (0, 0, 0) \vee P\ (0, 0, 0))\ ]$

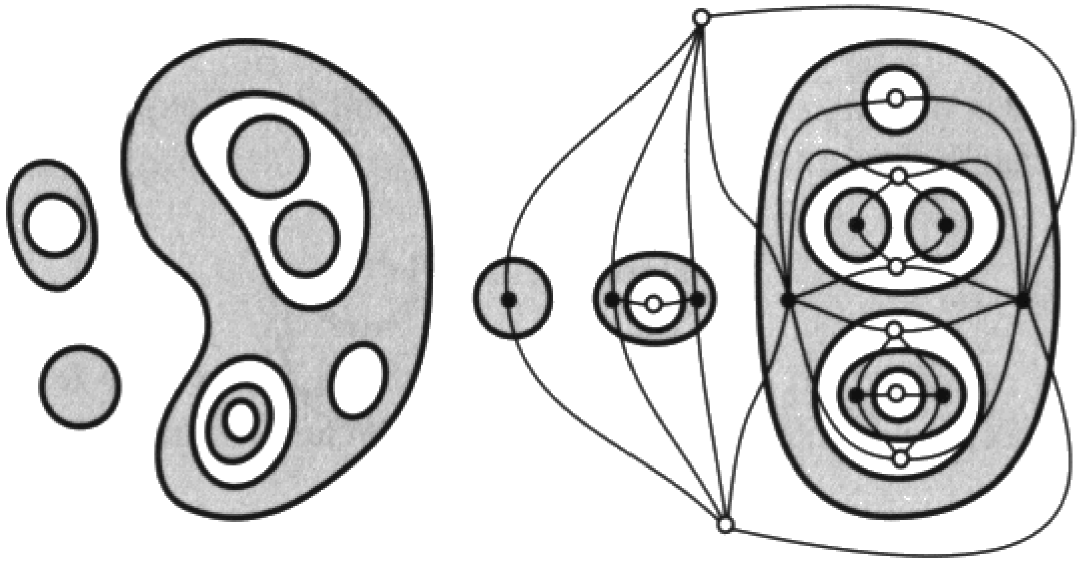
$\lambda\ [P\ (1, 0, 0) \wedge P\ (0, 0, 1)]$

$\lambda\ [P\ (1, 0, 1)]$

$\lambda\ [\lambda\ .\ \rho] = \lambda^{2}\ \rho$

a4 - ***The proposed exercise***

We can formalize the solution to the proposed exercise:



We can describe this solution linearly, even though it is drawn in two dimensions on the plane.

F **[**P (1, 0, 0) ∧ **(**P (0, 0, 0) ∨ P (0, 0, 0)**)** ∧ P (1, 0, 0)

∧**(**P (0, 0, 0) ∨ P (0, 0, 0) ∨ **(**P (1, 0, 0) ∧ P (1, 0, 0) ∧ P (1, 0, 0)**)**

∨ P (0, 0, 0)

∨□**(**P (1, 0, 0) ∧ (P (0, 0, 0) ∨ P (0, 0, 0)) ∧ P (1, 0, 0)**)** ∨ P (0, 0, 0))**]**

We mark the change of direction in the drawing with bold parentheses. We have chosen to present the graph in a way that highlights this change of direction in the composition. It is equivalent to swapping the composition signs ∧ and ∨.

We perform the first groupings that are legible in the figure:

λ [P (1, 0, 0) ∧ **P (0, 0, 1)** ∧ P (1, 0, 0)

∧ **(**P (0, 0, 0) ∨ P (0, 0, 0) ∨ **P (1, 2, 0)**

∨ P (0, 0, 0) ∨ **(**P (1, 0, 0) ∧ **P (0, 0, 1)** ∧ P (1, 0, 0)**)** ∨ P (0, 0, 0)**)**]

This helps us find the breaks that correspond to the parentheses and changes in direction in the drawing:

λ [P (1, 0, 0) $\wedge$ **P (0, 0, 1)** $\wedge$ P (1, 0, 0)

$\wedge$ **(**P (0, 0, 0) $\vee$ P (0, 0, 0) $\vee$ **P (1, 2, 0)**

$\vee$ P (0, 0, 0) $\vee$ **P (1, 1, 1)** $\vee$ P (0, 0, 0)**)**]

Let's perform the most significant grouping on the right side of the figure and begin to express the punches obtained in terms of edges and numbers of circles:

λ [P (1, 0, 0) $\wedge$ **P (0, 0, 1)** $\wedge$ P (1, 0, 0) $\wedge$ **P (0, 3, 4)]**

λ [B (1) **.** 1 $\wedge$ **B (0) .** ρ $\wedge$ B (1) **.** 1 $\wedge$ **B (0) .** λ^3 ρ^4 **]**

We actually obtain the following punch, the expression of which can be seen in the figure:

λ [P (1, 5, 5)]

λ [B (1) **.** λ^5 ρ^5]

It contains five circles of each kind, which, together with the closer, gives six circles surrounding a full circle and five circles surrounding an empty circle:

$^{\lambda}[\lambda^5 \ \rho^5] = \lambda^6 \ \rho^5$

We used the general form of the punch:

B (1) λ^m ρ^n

^{or}B (0)□λ^m ρ^n

which, at the end with the closer, is no longer distinguishable:

$^{\lambda}[B (1) \lambda^m \ \rho^n] = \lambda \ [B (0) \lambda^m \ \rho^n] = \lambda^{m+1} \ \rho^n$

Chapter III

Polynomials

The simplest way to introduce knot and link polynomials into calculations is undoubtedly through L. Kauffman's polynomial. We follow this mathematician in [18. d and c].

1. **From Kauffman's polynomial to Jones' polynomial**

This calculation requires a smoothing decomposition of the object, as in every case since Conway, when it comes to *skein calculus*, in order to obtain one of these polynomials.

a1 - ***Smoothing***

This involves decomposing each crossing of the object into two smoothings:

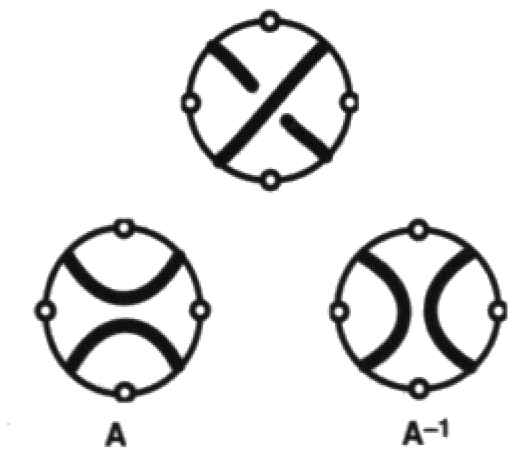


Fig. 1

Each smoothing is provided with an index.

Let's look at an example of such a decomposition, using the trefoil knot:


Fig. 2

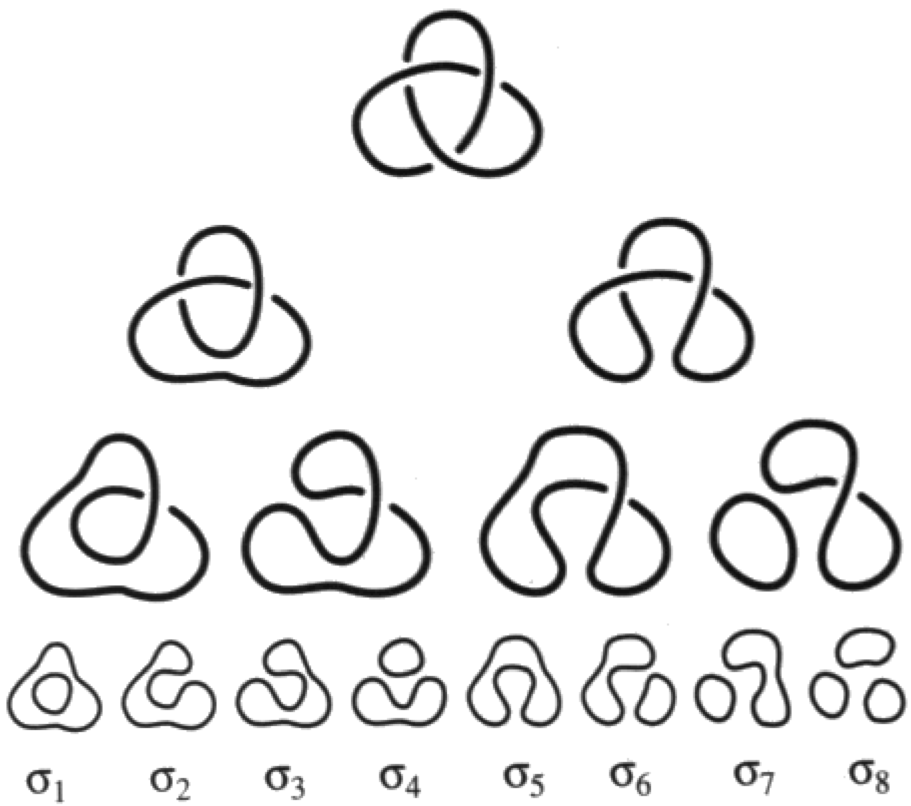


Fig. 3

Each state σ resulting from the decomposition of K has a product of the indices of successive smoothings and a certain number of circles.

a2 - ***The bracket polynomial***

Let us give the expression of the polynomial bracket constructed by Kauffman < K > of a given object K:

$$< K > = \sum_{\sigma} < K \mid \sigma > d^{\|\sigma\|}$$

where < K | σ□> denotes the product of the indices attached to σ, and ||σ|| denotes the number of circles of σ,□minus one, and let us set:

$$d = (- A^2 - A^{-2})$$

This polynomial is therefore a linear combination of the letter *d* raised to the power of the number of circles in each state, minus one—that is, this number is reduced by one—a combination whose coefficients are the products of the indices of each state.

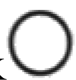
It should be noted that Kauffman's bracket polynomial is established in the case of an object that is not oriented with respect to the circles. It is an invariant of chains and unoriented knots up to regular isotopies. This means that it is sensitive to loops (B1).

From this polynomial, it is easy to obtain others that sometimes correspond to different *skein* calculations, but this is not always the case.

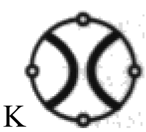
a3 - ***His interlacing calculation***

It is itself the product of an interlacing calculation that allows for the definition we have given.

Based on the polynomial of the trivial knot:

< K  > = 1

Smoothing can be rendered by the formula:

< K  > = A < K  > + A^{-1} < K  >;

This is what we call a *skein calculus*.

Let's move on to the polynomial corresponding to the ambient isotopy deduced from Kauffman's bracket polynomial.

a4 - ***The regularized polynomial of the Kauffman polynomial***

This polynomial is an invariant of oriented objects with respect to ambient isotopy. It does not correspond to a skein calculation:

$L_K(A) = (-A^3)^{-v(K)}$ < K > (A)

where v(K) is the twist—the sum of the characteristic of all crossings—of object K.

This is a necessary step to obtain the Jones polynomial. a5 - ***The***

Jones polynomial

The Jones polynomial was obtained independently of the bracket polynomial, but it can be obtained from the latter by a change of variable.

If we set: $A = t^{-1/4}$

we obtain the Jones polynomial: $_{VK}(t) = {}_{LK}(t^{-1/4})$

This polynomial is the subject of an interlacing calculation, which we will give later.

Its merit is that it distinguishes between the two trefoil knots, for example. It provided the opportunity to construct the Homfly polynomial, which is simply the Jones polynomial with two variables, and which in turn demonstrated its value to readers of the Bulletin of the American Mathematical Society. They themselves had not realized this until four different teams of mathematicians produced the Homfly polynomial at the same time. We mention this anecdote in the history of scientific publications today because of its indicative value for the state of scientific communication. There is a real difficulty in reading, and boasting is useless in hiding the inability to assume this domination of the letter, whose subjects are puppets in industrial production.

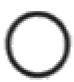
2. **The semi-oriented 2-variable polynomial (Kauffman polynomial)**

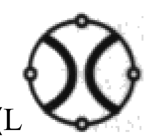
We can relate the Jones polynomial to another polynomial related to oriented objects with respect to circles. Not being directly obtained by a link calculation, it comes from a polynomial susceptible to such a calculation.

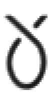
a1 - ***The*** $_{LK}$ ***polynomial and its linking calculation***

We graph it as: $_{LK}(\nu, \zeta)$

It is invariant to regular isotopy. Here are the elements needed to calculate it:

L  =1

L  + L  = ζ (L  + L )

L  = ν□L L  = ν^{-1} L

a2 - ***The semi-oriented 2-variable polynomial*** $_{FK}$ **(Kauffman polynomial)**

This is the regular polynomial of the

polynomial $_{LK}$: $_{FK}(\nu, \zeta) = \nu^{-\nu(K)}\ {}_{LK}(\nu, \zeta)$

where v(K) is still the twist of object K.

a3 - ***Correspondences***

The Kauffman bracket polynomial is a special case of L, and the Jones polynomial is a special case of F of the Kauffman polynomial.

< K >;(A) = $_{LK}$ ($-A^3$, A + A–1)

where: $\nu = -A^3$

ζ = A + A–1

and: $_{VK}$ (t) = $_{FK}$ ($- t^{-3/4}$, $t^{-1/4} + t^{1/4}$)

with: $\nu = -t^{-3/4}$

$\zeta = t^{-1/4} + t^{1/4}$

These changes of variables give the relationship encountered above in the change of variable:

$A = t^{-1/4}$

3. **The Jones polynomial and its derivatives**

We already have a way to calculate the Jones polynomial from the calculation of the Kauffman bracket polynomial:

$_{VK}$ (t) = $_{LK}$ ($t^{-1/4}$)

But it is susceptible to interlacing calculation.

a1 - ***Interlacing calculation for the Jones polynomial***

Here are the formulas needed for this calculation:

V  = 1

t V  $- t^{-1}$ V  = $(t^{1/2} - t^{-1/2})$ V 

Using these two equations, we can calculate this polynomial from the figure by decomposing it through successive inversions and smoothings.

a2 - ***The 2-variable oriented polynomial (Homfly) and its interlacing calculation***

This is the polynomial: P_K (a, z)

obtained by calculating its decomposition using the formula:

P  = 1

a P  $- a^{-1}$ P  = z P 

It corresponds to the following change of variable: a

= t, and $z = t^{1/2} - t^{-1/2}$

to the Jones polynomial:

$V_K(t) = P_K(t, t^{1/2} - t^{-1/2})$

and by the simpler variable change: a = 1 to the

Alexander-Conway polynomial:

$\nabla_K(z) = P_K(1, z)$

a3 - ***The Alexander-Conway polynomial and its link calculation***

There is an interlacing calculation for the Alexander-Conway polynomial:

$\nabla_K(z)$

which is easy to establish based on what we have just said:

∇  □= 1

∇  □– **∇**  □= z **∇**□ 

Today, it only has a historical role to play, since it was this polynomial that started things moving in this direction of research, the polynomial for Alexander and the calculation of links for Conway.

4. **Derivatives of the Homfly polynomial**

The Homfly polynomial is invariant for ambient isotopies. a1 - ***Its***

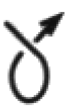
regularized polynomial HK (a, z)

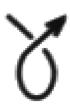
PK (a, z) = $a^{-v(K)}$. HK (a, z)

This expression defines it, but it also has an interlacing calculation.

H  = 1

H  –H  =z H  H

 =a H 

H  = a^{-1} H 

Where we see that it is invariant only for regular isotopies. It is sensitive to the presence or absence of loops and corresponds to objects placed on the sphere that are subject to deformations integral to this sphere M2 and T3, which differentiates them from ambient isotopies that are not integral to any support.

a2 - ***The oriented rotational polynomial***

This is the last polynomial we will consider here among the known polynomials. It is obtained through a change of variable:

$a = (\alpha\beta)^{1/2}$ $a' = (\alpha\beta^{-1})^{1}$

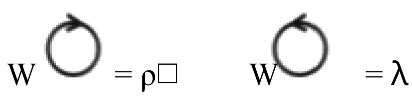
Using a polynomial **H**~K (a, z) obtained from the previous one by a slight modification
:

WK (a, a', z) = $(a')^{rot(K)/2}$.□**H**~K (a, z) where: $\mathbf{H}\tilde{}_{K}(a, z) = [(a - a^{-1}) / z]\ _{HK}(a, z)$

and where rot(K) is the sum of the oriented cycles produced by Seifert's algorithm, defined in the first chapter.

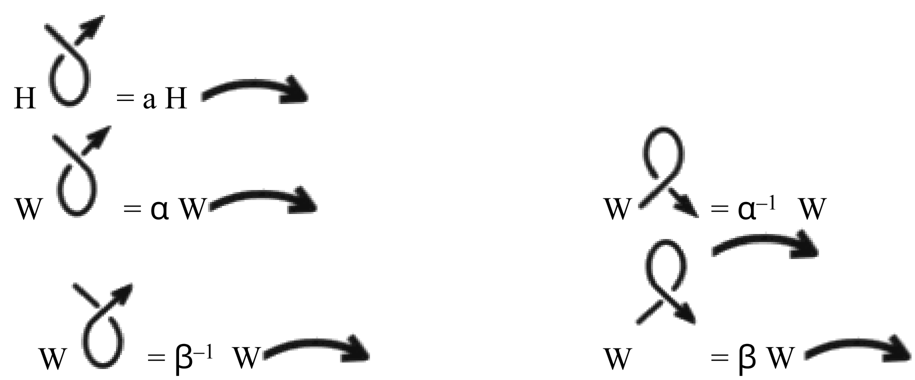
It corresponds to the oriented objects placed on the plane, i.e., the hollow sphere, and subjected to the deformations associated with this plane M2 and T3.

This polynomial can also be calculated from the figure of the object by a declination in inversions and smoothings of its crossings, i.e., the calculation of interlacing defined as follows:



with $\rho = (\alpha - \beta^{-1}) /□z$ and $\lambda = (\beta - \alpha^{-1}) /□z$

and the following distinctions:



The interest of this last polynomial lies in its very close proximity to the figure it connotes and its sensitivity to what we have themed under the title of duality. The duality factor corresponds to a displacement of the hole in the sphere. It suffices to refer to the two families of areas determined by the first step of our algorithm. The displacement of the hole in the sphere occurs from an area belonging to one of these families to an area of the other. This is equivalent to the change from full to empty in duality, i.e., the change from empty around the figure of one family of areas to empty around the figure of another.

We have agreed to consider areas of the same family as the one surrounding the object in question to be empty.

5. **The polynomials of our colorings**

We multiply each polynomial by the factor **μC**(K) or **a'C**(K), which requires the presence of the span surface, i.e., the distinction between solids and voids, because the number of crossings **C**(K) is oriented by the torsion.

${}_{L*K}(\mu, \nu, \zeta) = \mu$□$^{C(K)}\ .\ {}_{LK}(\nu, \zeta)$

thus: ${}_{F*K}(\mu, \nu, \zeta) = \nu^{-\mathbf{v}(K)}\ {}_{L*K}(\mu, \nu, \zeta)$

and: < K >$^*(A) = {}_{L*K}(A,$□$-A^3, A + A^{-1})$

${}_{V*K}(t) = {}_{F*K}(t^{-1/4},$□$-t^{-3/4}, t^{-1/4} + t^{1/4})$

Whereas: ${}_{P*K}(a', a, z)$□$= a'^{\mathbf{c}(K)}\ {}_{PK}(a, z)$

thus: ${}_{P*K}(a', a, z) = a^{-\mathbf{v}(K)}\ .\ {}_{H*K}(a', a, z)$

and: ${}_{V*K}(t) = {}_{P*K}(t^{-1/4}, t, t^{1/2} - t^{-1/2})$

Now we will construct the modified oriented rotational polynomial, which is slightly different from those in the previous series:

${}_{W*K}(\mu, \nu, a, a', z) =$

$\mu^{\mathbf{C}(K)}\ .\ \nu^{-\mathbf{v}(K)}\ .\ (a')^{\mathbf{rot}(K)}/2\ .\ [(a - a^{-1})/z]\ .\ {}_{HK}(a', a, z)$

6. **Variable changes**

We list the correspondences we have reported:

We can deduce that:

$A = a^{-1/4} = a'$

${}^{\nu}= -a^{-3/4} = -(\alpha\beta)^{-3/8}$

Two remarks

1. First, regarding the conversion from ζ to z via t. In the actual calculations, this remark applies:

$z = t^{1/2} - t^{-1/2} = (t^{1/4})^2 - (t^{-1/4})^2$

$z=(t^{1/4}+t^{-1/4}).((t)\ 1/4(-)\ (t)^{-1/4()}(=)(\zeta)\ (.)((t)\ 1/4(-)\ (t)^{-1/4()}$

2. Secondly, a simple calculation to keep in mind:

$a = (\alpha\beta)^{1/2}$

$a' = (\alpha\beta^{-1}$

$)^{1/2}$

$a^2 = (\alpha\beta) \quad \alpha = (aa')$

$a'^2 = (\alpha\beta^{-1}) \ \beta = (aa'^{-1}$

$) \ \alpha = (aa')$

$\beta = (aa'^{-1})$

We have introduced a new variable μ that we couple with ν. In the polynomials of the colored cases, we find other correspondences through successive variable changes:

$\mu = A$

$\mu = t^{-1/4}$

Therefore:□$\mu = a^{-1/4} = a' = (\alpha\beta^{-1})^{1/2}$

In order to find, between μ and ν with two other variables x and y, the same type of calculation as between a, a', α, and β, we set:

$\mu = (xy^{-1})^{1/2} \ \ \mu^2 = (xy^{-1}) \ x = (\mu\nu)$

$\nu = (xy)^{1/2} \ \ \nu^2 = (xy) \ y = (\mu^{-1}\nu)$

Now, considering the exponents assigned to these variables in the polynomial expressions:

$\mu^{\mathbf{c}(K)}$

ν^{-}

$\mathbf{v}(K)$

where **c**(K) is the number of crossings oriented by torsion and **v**(K) is the twist, i.e., the number of crossings oriented by the characteristic, we can follow the effect of this calculation in the expression **π** defined by the product:

$\boldsymbol{\pi} = \mu^{\mathbf{c}(K)} . \nu^{-\mathbf{v}(K)}$

$\pi=(xy^{-1})^{1/2\,\mathbf{c}(K)}.(xy)^{1/2\,(-\mathbf{v}(K))}$

$\pi=x^{1/2\,[\mathbf{c}(K)-\mathbf{v}(K)]}.y^{-1/2\,[\mathbf{c}(K)-\mathbf{v}(K)]}$

However, according to our definitions corresponding to the figures:

$[\mathbf{c}(K) - \mathbf{v}(K)] = 2\ _{\mathbf{kiS}}$

$[\mathbf{c}(K) + \mathbf{v}(K)] = -2\ _{\mathbf{k*iS}}$

therefore: $\pi = x^{\mathbf{kiS}}\ .\ y^{\mathbf{k*iS}}$

where we find the cut and non-cut parts of our flat diagrams. Or again: $\pi =$

$(\mu\nu)^{\mathbf{kiS}}\ .\ (\mu^{-1}\nu)^{\mathbf{k*iS}}$

Let's return to the polynomials where we will make use of these calculations.

7. **Interlacing calculations for our polynomials**

Among the polynomials that we introduce with our coloring pages, some of them have an interlacing calculation. This means that they can be calculated on the figure or at least based on it.

These are the following polynomials.

First $_{L*K}$ (μ, ν, ζ), then < K >*(A) and $_{V*K}$ (t) which are derived from it. Next, P*K $_($a', a, z) and its regularized form H*K $_($a', a, z), from which V*K $_($t) can also be deduced. On the other hand, the modified oriented rotational W*K $_($μ, ν, a, a', z) requires its own calculation.

The other newly defined polynomials do not require any calculation; they can be deduced exclusively from these.

a1 - ***The polynomial*** $_{L*K}$ ***and its interlacing calculation***

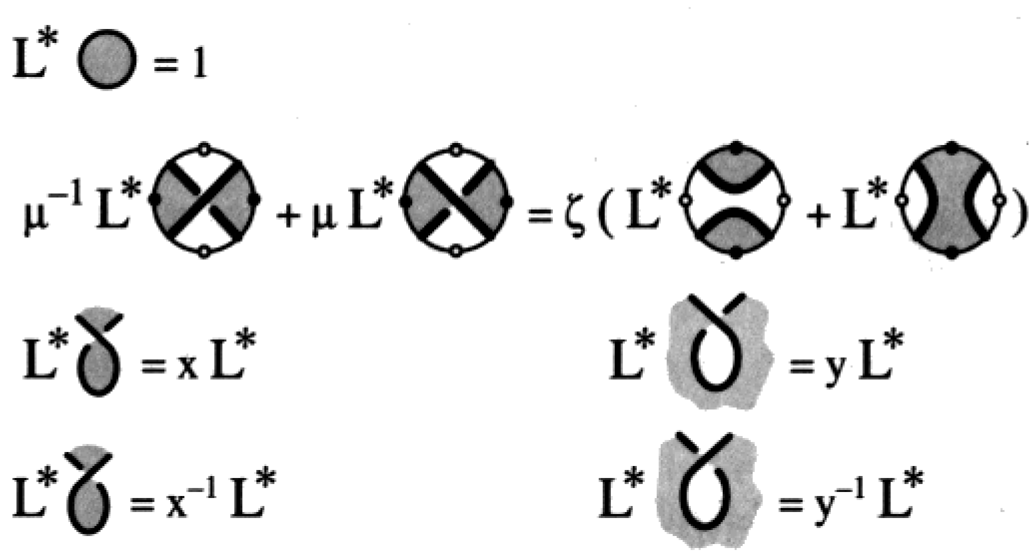
We will begin by giving: $_{L*K}$ (μ, ν,

ζ)

Recall that we defined it as:

$_{L*K}(\mu, \nu, \zeta) = \mu\square^{C(K)}\ .\ _{LK}(\nu, \zeta)$

It is calculated using the following formulas:



Thus, we can calculate $_{F*K}$ (μ, ν, ζ) using the following expression: $_{F*K}$

(μ, ν, ζ) = $\nu^{-v(K)}$ $_{L*K}$ (μ, ν, ζ)

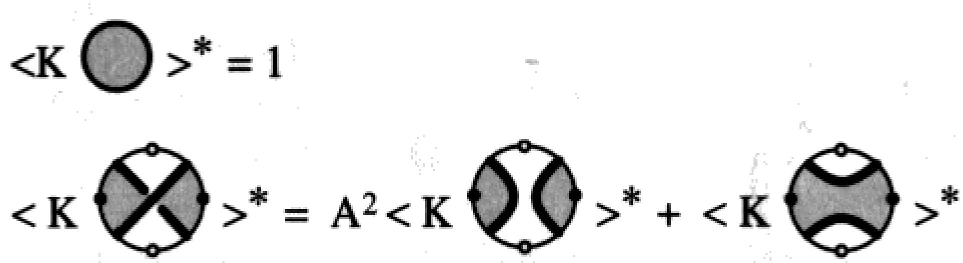
and: < K >*(A) = $_{L*K}$ (A, $-A^3$, A + A^{-1})

$_{V*K}$ (t) =□$_{F*K}$ ($t^{1/4}$, $-t^{-3/4}$, $t^{-1/4}$ + $t^{1/4}$)

a2 - ***The modified polynomial***

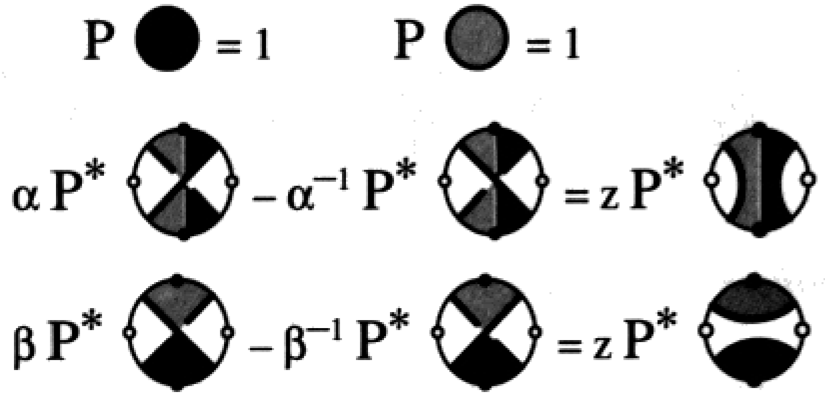
bracket

< K >*(A) can be calculated using the following formulas:



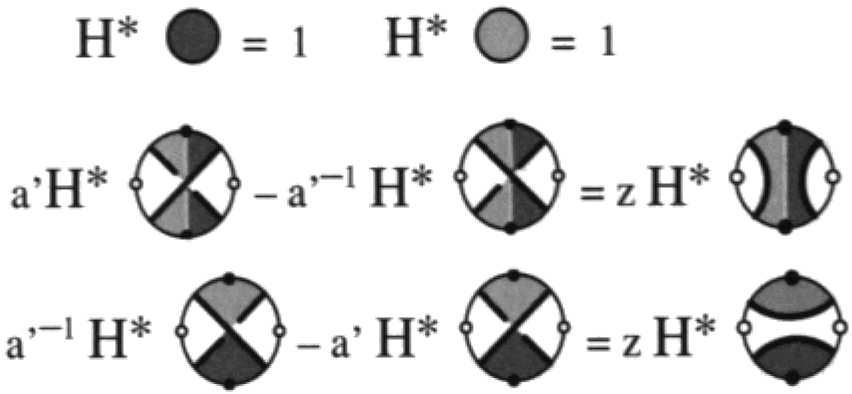
a3 - ***The modified 2-variable oriented polynomial (Homfly) and its interlacing calculation***

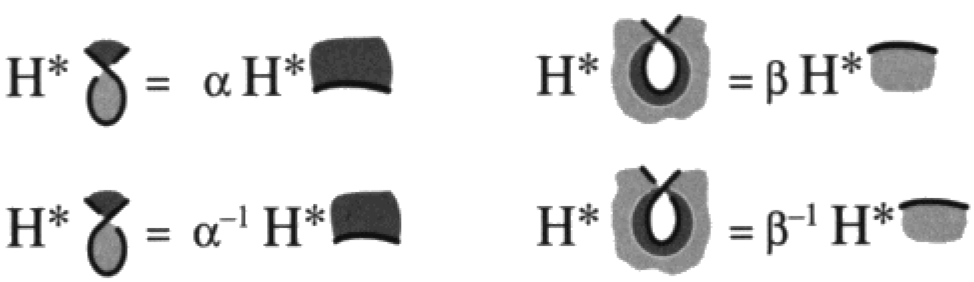
On the other hand, we can determine $_{P*K}$ (a', a, z) using the following calculation:



a4 - ***Its regularized and modified polynomial*** $_{H^*K}$ (a, z)

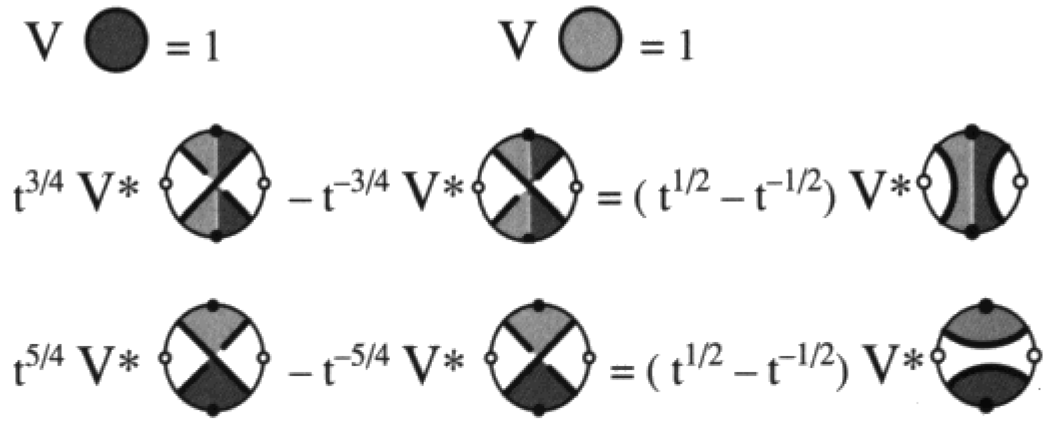
We can also calculate $_{H^*K}$ (a', a, z) using a similar process:





a5 - ***Interlacing calculation for the modified Jones polynomial***

And $_{V^*K}$(t) is obtained by calculation according to the following data:



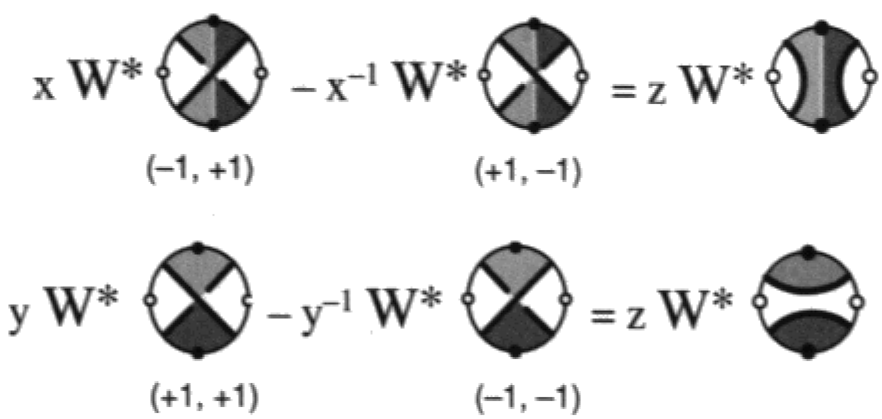
a6 - ***The modified oriented rotational polynomial***

And finally, the modified oriented rotational

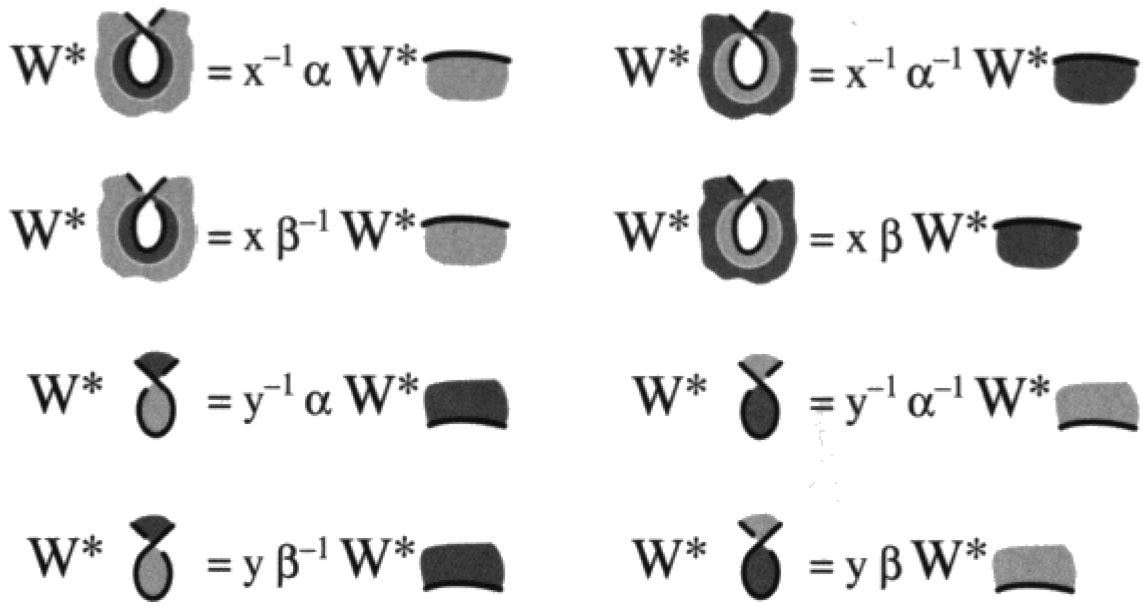
polynomial: W*K (μ, ν, a, a', z) = $\mu^{c(K)}$. $\nu^{-v(K)}$. WK

(a, a', z)

where: ρ = (α − β^{-1}) /□z, and λ = (β − α^{-1}) /□z



and the following distinctions:


W* = x⁻¹ α W*
W* = x⁻¹ α⁻¹ W*
W* = x β⁻¹ W*
W* = x β W*
W* = y⁻¹ α W*
W* = y⁻¹ α⁻¹ W*
W* = y β⁻¹ W*
W* = y β W*

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