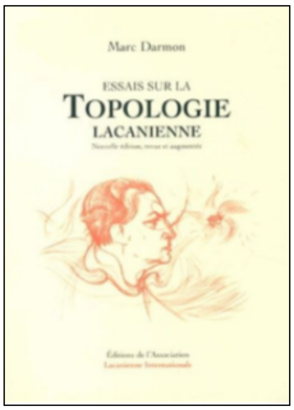
LES SCHEMAS R ET I

from Essais sur la topologie lacanienne

Marc Darmon



In the R et I diagrams of the *Question préliminaire à tout traitement possible de la psychose*, Lacan gives a spatial representation of the subject's structure that is no longer limited by the Euclidean plane. Remarkably, Lacan looked to non-Euclidean geometries for instruments more suited to his object. But these diagrams only take on their true dimension in the dialectical relationship with the discourse that accompanies them, a discourse that takes over from Freud's, or even Schreber's, and whose structure they merely underline, with that imperfection indissolubly linked to the necessary flattening of these representations. This is why Lacan warned against using them (1).

In a 1966 note, Lacan points out that the topology of the projective or cross-cap plane is already indicated, albeit in enigmatic form, in the R-Schema. This topology was developed in the seminar on Identification (1962). The text of *Écrits* takes up part of the seminar on *Les Psychoses* (55-56), but also includes contributions from the seminar on *La Relation d'objet* (56-57) and the contemporary seminar on *Les Formations de l'inconscient* (57-58).

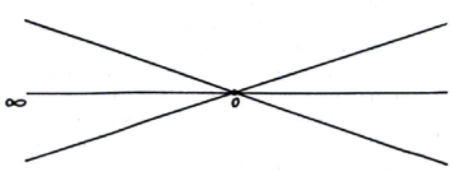


Figure 1

The projective plane discussed here has proved to be of fundamental importance in geometry. Starting from a conic of this projective plane, the only three possible geometries are defined: hyperbolic geometry, elliptic geometry and Euclidean geometry. It was this remarkable result that led Klein to transform the very concept of geometry, in his Erlangen program, into the analysis of group structures.

The projective plane is made up of all the straight lines in space space passing through the origin O; the set of points on each line each line, except O, being subject to an equivalence relation (2).

This projective plane was developed from the realization that points at infinity or a line at infinity could be added to the usual plane.

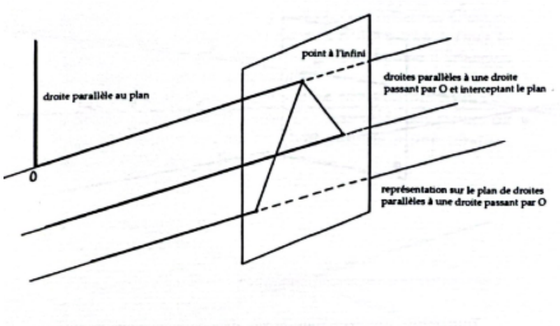


Figure 3

straight line at infinity. This is the vanishing point of classical perspective, where parallel lines meet in the picture.

Any point located on one of these straight lines is projected onto a same point on the picture, at the intersection of the line and the plane plane. And all straight lines parallel to one of these lines converge on the same point x at infinity located on this plane. But even if we consider an array equivalent to an infinite plane plane, we can see that the straight lines parallel to the board cannot intercept it. The projective plane is therefore the generalization to all lines passing through the center of this equivalence equivalence given to all points on each line.

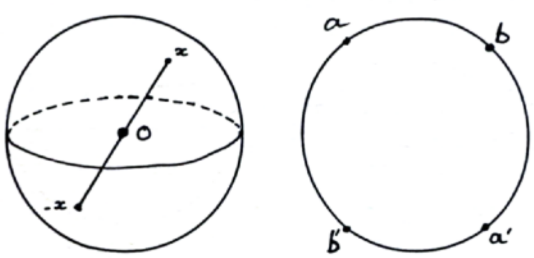


Figure 4

To construct a representation of this projective plane, we can first drag the points of each line, including the point at infinity but excluding 0, onto the points of abcissa $ 1 by the function — TxT (x on absolute value of x), then identify all the antipodal points of the unit sphere centered on O thus obtained. (Figure 4)

This last operation can only be imagined in our space space only by admitting an immersion of this projective plane with an an interpenetration line. This imaginary line allows the surface to cross itself without any real intersection. This means that a path on this surface crosses this line without leaving the part of the surface to which it locally belongs; it cannot abruptly branch off onto the other part.

From the sphere, we can initially identify the antipodal points of the two hemispheres without taking the equatorial line into account. We thus obtain a hemisphere or a disk (these surfaces being flexible) bordered by the equatorial line, whose opposite points now need to be identified. It's possible to start with two opposite points, bringing the two lips of this O-shaped edge together to give it the shape of oo. This is the central point of the cross-cap, then simply join the other points of this edge by making them cross by the interpenetration line. (Figure 5)

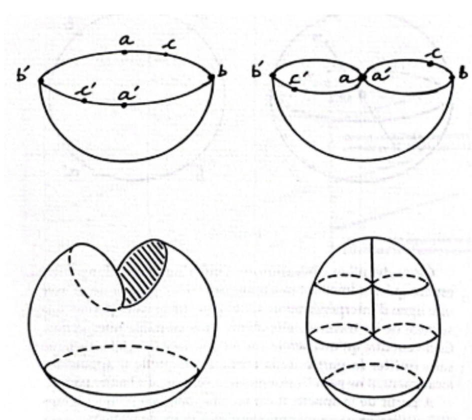


Figure 5

Another method consists in detaching an equatorial strip from

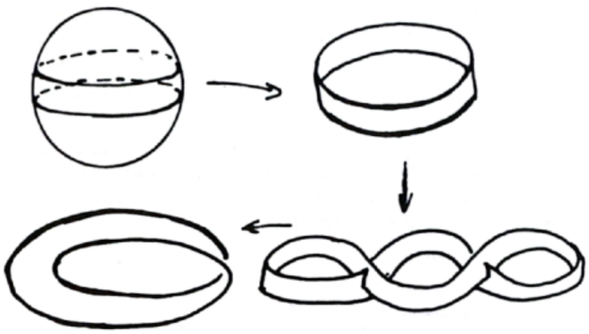


Figure 6

the two polar caps, and transforming it into a strip with two edges and two complete twists. This strip can overlap itself to form a two-sheet Möbius strip. All you have to do is merge the opposite points to obtain a simple Möbius strip. (Figure 6)

This Moebius strip is glued back to the disc resulting from the identification of the two polar caps by the common edge. The projective plane is thus composed of a Möbius strip and a disc.

By comparing these two methods, it is easy to deduce the equivalence of the Möbius strip and the equatorial cut. This paradoxical equivalence of the cut and the Möbius strip is underlined in the 1966 note and later repeated in the 1972 text *L'Etourdit*. In the latter text, Lacan uses "the bipartite Möbius strip" to demonstrate this equivalence, which is none other than the two-twisted strip encountered above. This strip can also form a Möbius strip by sewing one of its edges to itself, either directly or via an intermediary.

itself, either directly or via another Möbius strip (3).

The projective plane possesses some very remarkable properties: — if it seems possible to have a local right side and a local left side, these right side and left side can in fact be joined everywhere; — if we define a direction of orientation by an oriented circle, this circle, by a simple continuous displacement, sees its orientation reversed.

The projective plane is therefore, like the Möbius strip, a non-orientable surface.
non-orientable surface. It's this property of a non-orientable, edge-less surface that Lacan uses to explain what he means by "non-specularity". (Figure 7)

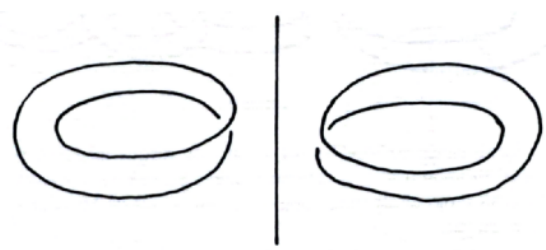


Figure 7. La bande de Mœbius a une image spéculaire

Although this notion was explained in quite different ways in the seminars on Identification and Anguish, it's worth trying to account for it. It is possible to orientate and colour a surface, even a symmetrical object, to differentiate it from its mirror image; this is also the case with the Möbius strip. In fact, the Möbius strip, although not orientable, is immediately very orientated, as there is a right and a left side. This surface, with no need to color it or arrow its edge, is irreducible to its mirror image and remains the same when turned upside down. At first glance, this would also seem to be the case for the washer that supplements it at the center of the cross-cap. If we trace the double-loop cut on the cross-cap, in the shape of a Möbius strip edge, there are two possibilities, one levorotary and one dextrorotary. So both the projective plane and the Möbius

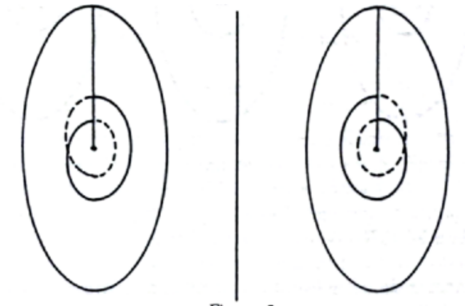


Figure 8

strip appear to be distinguishable from their mirror images. (Figure 8)

In fact, this is not the case. In fact, the levorotatory cut can be continuously transformed into a dextrorotatory cut. The cross-cap has the very special property of not having a mirror image, whatever the artifice used to give it one. This is what Lacan calls non-specularity.

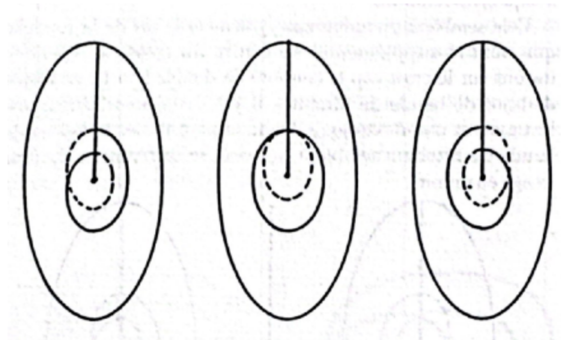


Figure 9

Since this property applies to the complete cross-cap and not to the Möbius strip, which nevertheless seems to possess it in its potential state, Lacan attributes it to the washer, which he identifies with object a. (Figure 9.)

The R schema

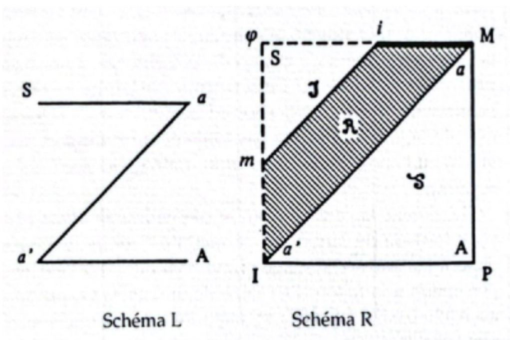


Figure 10

Armed with this topology, let's turn to the description of schema R. Schema R contains the Saa'A path already encountered in Schema L of the schema L of the seminar on The Stolen Letter, where the symbolic relation of Subject S and Other A is doubled by the imaginary relation of the ego a' and its objects a,. In this place, we must see not the object a as such, whose concept and topology will only emerge later, but rather the reflections of such an object a. (Figure 10)

Thanks to the seminar La Relation d'objet, contemporary with the writing of this text, we can trace the construction lines of the field of reality in this R(4) diagram.

The first axis of this reality is the symbolic Mother-Child relationship. But this symbolic relationship, from the outset, is not reduced to dependence on the satisfaction or non-satisfaction of needs; the child is dependent on the Love of this mother, i.e. on the desire of her desire. So it's the mother's desire that constitutes the third element, the symbolic axis, in relation to which the subject has to find his bearings. Mélanie Klein, Lacan tells us, sensed, but did not actually identify, this double pole of opposition between the good mother and the bad mother. But in the Kleinian perspective, reality is essentially hallucinatory and phantasmatic. In this conception, then, there is a "fundamental homogeneity of psychosis with the normal relationship to the world". Lacan, on the other hand, stresses the importance of language in the constitution of the field of reality, right from the most primitive stage. In fact, the memory inscription that hallucinatorily responds to the need is a sign, already a signifier that not only relates to the need or the object, but above all to “the absence of this object”.

Let's go back to the construction of this quadrangle of reality. It is the mirror stage that introduces a certain dialectic into this primitive system, offering the child a perception that is both real and unreal, a captivating and alienating image (i). The prematurity of this image opens up a gap in the imaginary, which responds to another gap in the symbolic, on the side of the relationship to the Other who is there, witnessing the scene. M designates this real Other, this primordial maternal object, the support of “the Thing”. The image i thus constitutes a point of support, a limit of reality. This point of reference offers the subject the possibility of entering, in the opposite direction, for the identifications of the ego (m), into another field constituted by the miM triangle, homologous and inverse to the miM triangle. These successive identifications are made in the direction of the Symbolic, where the ego takes on the function of a series of signifiers, limited by the ego ideal I, at the paternal level. The miMi field of reality is thus constituted in the direction of the Symbolic, and is strewn with signifiers. Identification with the Ego Ideal on the paternal side enables, says Lacan, “a greater detachment from the imaginary relationship than from the relationship with the mother”.

The subject's identification with the imaginary phallus, at the apex of the imaginary triangle i @m, as the object of the mother's desire, must be “destroyed” correlatively to the unveiling in A, the locus of the Other, of the Nom-du-Père P, at the apex of the symbolic triangle IPM destined to cover the imaginary triangle.

The 1966 note allows us to identify the R diagram with a spread-out projective plane; indeed, it's possible to join the antipodal points on the edge of this square. This is already suggested by the dotted lines and the arrangement of the letters mM, il. (We can imagine that locally m is placed on the reverse side of M, i on the reverse side of l, but this reverse side being in fact on the same face as the right side). We need to operate in the same way as for a disc, as we saw above. In this operation, the quadrangle miMl is transformed into a Möbius strip, and the triangles S and I become a single disk or disc, resting on the Möbius strip thanks to the common border. This common boundary is formed by the single cut mi, MI, which is effectively the only real cut on the surface, the edge of the square being in fact only artificially figured as it is intended to be glued back to itself, each solid line corresponding to the antipodal dotted line.

This cut isolates a Möbius strip that covers the field of reality. We've already stressed the paradoxical identity of this cut and the Möbius strip from a topological point of view. This is why, on this strip, “nothing is measurable that can be retained from its structure”, i.e. al
width of the strip has no structural value. Through this cut, the Real constitutes the boundary between the Imaginary and the Symbolic, which are nonetheless on the same edge.

To sum up: the imaginary field, on the other side of the symbolic field, is both separated and united on the same side by the cut that is the Möbius strip. Only the topology of the projective plane can support the paradoxes contained in this sentence.

The scaled vectors that measure the intrusion of the imaginary into reality in the direction of the symbolic express the fact that these identifications consist of a symbolization of the imaginary. Lacan warns us against seeing this as a return to the aforementioned theoretical conception of a real world based on narcissism. Considering the overall topological structure of the surface, this series of identifications, vectors and signifiers doesn't distance one edge of the strip from the other, which doesn't exist; it merely displaces the cut, which remains irreducible. If the screen of fantasy closes off the field of reality, it does not erase the cut of the Real, which remains marginal. Indeed, it's this cut that provides the framework, the structure of fantasy; the cut of the projective plane is symbolized both in the subject bar $ and in the rhombus 0, which articulates $ to the object, $ 0 a, in the fantasy formula. Here, object a corresponds to fields 1 and S, to the washer, and $ corresponds to the strip, i.e. the cut.

Diagram I

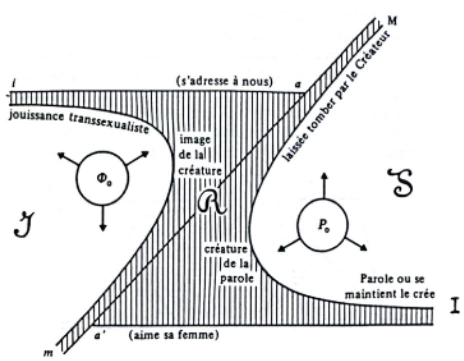


Figure 11. Schéma I.

In the seminar on The Formations of the Unconscious, Lacan gives us some clues to explain the from schema R to schema I in Schreber's psychosis.

In psychosis, the field of reality is reshaped. First, there is a topical, structural regression. (Figure 11, Diagram I)

Starting with the iMm and mMI triangles, we need to think in the opposite the movement of intrusion at the level of the i boundary of the image of the own body in the R field, and, at the level of the ego, an unleashing of signifiers. These two movements distort the field of reality, still limited by the mi and MI lines. The foreclosure of the paternal signifier forms a chasm on the symbolic side, to which another chasm responds on the imaginary side. These two holes bend the mi and Ml lines, sending the subject's four fundamental markers m, i, Met I to infinity, the latter, Created I, taking the place of P as if attracted by the void, in an accelerated movement on an infinite trajectory. It's worth noting that, in this universe of general relativity, we can conceive of the curvature of the lines as primary to the existence of the supposed holes. It's easy to recover the general form of diagram I by this transformation of the R field, conceived as formed by two homologous and inverse triangles. (Figure 12)

This transformation implies a radical change in the topological relationship between the places of m and M. M and m come to stand on opposite sides of the symbolic and imaginary sides of the main line, the axis of this diagram, which constitutes their common asymptote in their race to infinity in space and time. Remember that Lacan refers here to Freud and his term asymptotisch

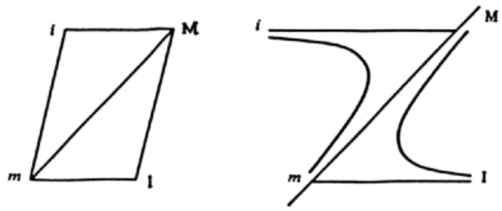


Figure 12

to qualify the desired conjunction of the delusional self and its God (5).

But how can we characterize as fully as possible the geometric and topological structure of schema 1, for which Lacan gives us no other key?

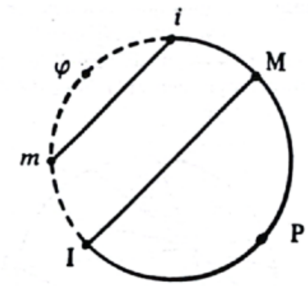


Figure 13

As we've seen, it's possible to relate the reference points of Schema Rs to the edge of a disk, and to constitute the projective plane by identifying the antipodal points of this edge (i.e. the pairs mM, il and P. Only the order of the points on the edge of the disk matters. (Figure 13)

In the same way, the four reference points i, m, I, Msur can be placed on the edge of a disc, this edge being set at infinity. The lines, the strings im and MI then become the straight lines, or rather the geodesics, of a hyperbolic plane with which this surface can immediately be identified. Geometrically, the hyperbolic plane is defined from a conic of the projective plane. The projective plane and the hyperbolic plane can be modeled by a disk. In the case of the projective plane, the circular edge is the straight line at infinity; this straight line at infinity is included in the disk and each of its points must identify with the diametrically opposite point. In the case of the hyperbolic plane, the edge of the disk placed at infinity is excluded from the surface, there is no identification of the opposite points and we find the remarkable properties of the hyperbolic plane, properties that oppose it to Euclidean geometry. Thus, the chords of this disk represent straight lines; it is then possible to pass through a point an infinite number of straight lines that do not intersect a given straight line, i.e. an infinite number of non-intersecting straight lines. And there are two strings, i.e. two boundary lines that separate intersecting straight lines from non-intersecting straight lines: these are the hyperbolic “parallels”. These particular straight lines join the given straight line on the edge of the disc. (Figure 14)

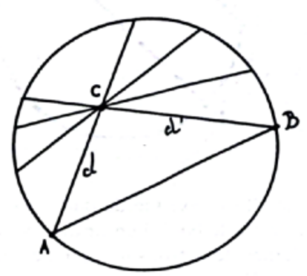


Figure 14. Modèle du plan hyperbolique. Par C passe une infinité de droites non sécantes à la droite A B; d et d' sont les deux *parallèles hyperboliques*

Let's consider the strings, i.e. the im and MI lines. The straight lines mM and il join im and Mi at the edge of the disc, which is infinite and in fact excluded from the hyperbolic plane. The straight lines mM and il are “parallel” to the hyperbolic sense at both im and MI. This means they can be modeled as branches of hyperbolas. (Figure 15)

The point of intersection of these asymptotes is the only finite point determined in this construction. We can identify it with the specular couple and deduplicate it by sliding the curves

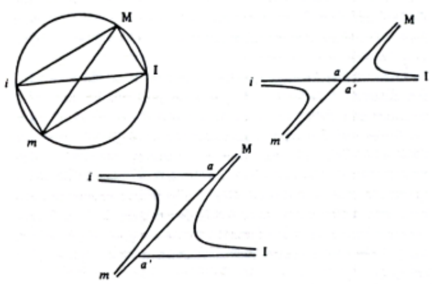


Figure 15

along mM. This deduplication is justified by the persistence of this mirror relationship with the little other, despite the psychotic reworking. This concerns the role of Schreber's wife or the function of the reader of the Memoirs.

Schreber gives us a true description of this space, this "hyperspace" as Lacan calls it, which is in fact that of the signifier (6). In his companion piece to The Hallucinations, Schreber describes the very particular path of these "thread-rays" in the direction of his head. These "voice-bearing" threads describe a curve, a "loop" or a "parabola", as if they were going around a "milestone" or a "post". These rays from God would melt directly onto his body, flooded with "soulful voluptuousness", if they were not "somewhat restrained" by a mechanical force that Schreber links to "earthboundness" and which induces this curve.

It's this hyperspace that Lacan seems to reproduce in his Diagram I, in the light of which we can read all the relationships he draws from Schreber's text. In these lines, we find a striking description of this signifying journey from Creator to Created, culminating in the Creatures of the Word. Lacan taught us to read the constancy of the real in the "docking with the earth" that deflects the rays here. The real is to be found precisely in the schema delimited by the curvature of the hyperbola. And finally, at the symmetrical point in the imaginary of the impact of these rays on the Created, the "transsexual jouissance" that invades Schreber is inscribed.

The topology of Schema R and Schema 1, as described above in relation to the projective and hyperbolic planes, implies certain consequences that can be verified in Schreber's case. The subject's fundamental reference points, placed at infinity, and the foreclosure of the Nom-du-Père, give the Real, Symbolic and Imaginary fields the geometry of a hyperbolic plane. The MI, mi cut of the R schema, identical as we've seen to the Möbius strip, is transformed into hyperbolic lines whose asymptotic limits can only be traced. Instead of a single, well-defined cut with a single edge, we have an indefinite overflowing of the fields of the Symbolic and Imaginary, making the status of the field of the Real highly precarious and infinitely variable?

For Schreber, this situation corresponds to the period preceding November 1885. By then, his psychotic world had truly collapsed; all that remained for Schreber, at the end of his struggle, was to resolve to occupy the place of "carrion" that his voices kept calling him. And it is by accepting this true death that he can embark on the process that will make him God's wife. This work of transformation is well symbolized by Schreber's habit of going out at night, in the rain, with his feet out through the bars of the window, thus giving birth to The Woman.

This operation involves only the imaginary axis and the relationship to the mirror, as the diagram shows. The doubling of the specular couple, of points a and a,' enables this shift on the imaginary axis of asymptotes, giving the field of the Real a certain thickness and anchorage. It is at this price that the Real can, for the subject, become habitable, despite the movement caused by the intrusion of the Symbolic and Imaginary fields. This is the role played by those states of transsexual jouissance that Schreber achieves by spending much of his time in front of the mirror, his upper body bare, with a few feminine accessories (7).

One of the consequences of the topology of Schema I, if it is indeed a hyperbolic plane, is that, unlike the projective plane, Schema l is orientable. As we have seen, it is the non-orientability of the projective plane that accounts for the lack of object a in the mirror. In the case of scheme I of psychosis, object a can actually appear in the mirror. This is indeed what Schreber's states of jouissance in front of the mirror clinically verify. Indeed, Schreber affirms the existence, made visible in the mirror, of the intermittent swelling of his female breasts, depending on whether he is approaching or distancing himself from God. In this Other jouissance demanded by God, it is at the very level of Schreber's body in the mirror that the sublimated object a is revealed. Previously, Schreber had occupied this position as a waste product.

The party wall phenomenon

Charles Melman has given this topology an illuminating clinical application: the “party wall” phenomenon.

As we know, it's not always easy to distinguish genuine paranoid persecution from a persecutory episode in hysteria. Charles Melman has noted the remarkable clinical fact that, in paranoia, the persecutor is not located in an indifferent part of the neighborhood, but preferably behind the party wall, either right next to it, or even above it, on the other side of the ceiling. In other words, the persecutor is always on the other side of the wall, and there's no way of finding him there. The plane of the wall thus determines an absolute right and wrong. This is not a Moebian topology, but a flat, hyperbolic topology with two sides. The persecutor stands at a doubly infinite distance; he is both infinitely close, behind a plane without thickness, and infinitely distant, since to reach him we would have to cross the inaccessible edge of the plane.

These particularities of this infinite Other behind the wall, capable of reading and divining thoughts, lead us to suppose that it is indeed the subject himself.

This is not the case with the neurotic, who maintains a relationship with his unconscious that is either toric, i.e. his interior, or mœbian, the other side being reachable at any point.

TORUS, MÕBIUS STRIP, CROSS-CAP

—Topology from the seminar on *Identification* (1962) to *L'Etourdit* (1972).

Lacan uses topological models in his seminar on Identification, introducing the torus, the Möbius strip and the cross-cap, ostensibly to illustrate certain paradoxes of the logic of the unconscious — paradoxes for mathematical logic — but in fact to lay the foundations for it.

Classical logic respects certain principles, the principle of identity and the principle of non-contradiction: Ais Aand cannot be both A and not A. But in the *Traumdeutung*, Freud admits that the dream does not respect these principles, and sets out a logic that rejects the principle of non-contradiction and identity. For the unconscious, A # A and A = A must be written. This is in fact the rule in language, and we don't need to refer, as Freud did, to Abel to find in everyday language expressions that are antonyms to themselves: “c'est du beau”, “c'est du propre”, “c'est intelligent” or even the term “queen”, which in Middle English meant queen as well as prostitute.
Unlike the formalized signifier of mathematical logic, the natural signifier is in principle non-identical to itself, and equivocation is the rule. So what happens to our logical relationships and the usual illustration of Euler circles?

A circle inside another circle — that's the model of class logic. So it's not the same thing to assert that something is “non-human” or “non-human within the class of animals”. But what applies to zoology doesn't necessarily apply to language.

The Eulerian illustration in fact already presupposes a topology; the topological fact that a circle separates the plane or sphere into an inner and an outer part is implicit. If this circle is drawn on a torus in a certain way, it cannot cut this torus into two parts, and the inside of the circle is found to communicate continuously with its outside; A= not A, we can then write.

A signifier different from itself can be inscribed on the torus in the form of shifted circles to reveal, on the one hand, the emptiness of the space in which the object is supposedly encircled, and on the other, the field of what Lacan calls the self-difference of the signifier to itself.

Let's take an example. In the margin of his handwritten notes on The Rat Man, Freud writes the first name Dick vertically opposite the passage in which he discusses his patient's compulsion to lose weight. From this symptom, in which the subject shows his division, Freud gives the “password”: it was in order not to be “Dick” that the Rat Man went to such lengths, “Dick” meaning “fat” also being the first name of his cousin, the hated rival of his beloved. The symptom represents the subject for this signifier, knowing what escapes it. The symptom is thus the result of a purely signifying articulation, hence the attempt to resolve it through an interpretation that plays on equivocation. We can place on our two Euler circles, on the one hand, the signifier “dick”, and, on the other, the same signifier insofar as it refers to the rival's first name, thus different from himself. (Figure 16)

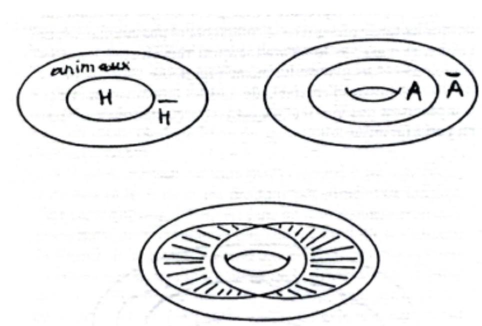


Figure 16

The topology of the subject

The symptom represents the subject in this way. The symptom represents the subject for this signifier, knowing what escapes him. The symptom is thus the result of a purely signifying articulation, hence the attempt to resolve it through an interpretation that plays on equivocation. We can place on our two Euler circles, on the one hand, the signifier "dick", and, on the other, the same signifier insofar as it refers to the rival's first name, which is therefore different from himself. itself.

As we said at the outset, the banal properties of the natural signifier are paradoxes for the logician. This is how Lacan interprets Russel's famous paradoxes.

Let's take the well-known example of the set of sets that don't understand themselves: does this set belong to itself or not? It goes without saying that if we identify this set with itself, i.e. insofar as it subsumes other sets on the one hand, and insofar as it is such a set on the other, we are faced with an impossibility. If we consider that a signifier is different from itself, it appears that this set is not the same in either case.

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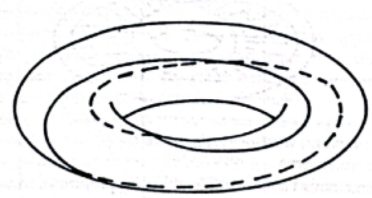


Figure 17

the two cases. To take the analogous example of the catalog of all catalogs that do not understand themselves, this catalog is not the same when it indexes other catalogs and when it is itself indexed. Lacan speaks here of internal exclusion: the set of all sets that don't understand themselves is in "internal exclusion" with respect to itself. (Figure 17)

The double loop illustrates this internal exclusion, since the central part, enclosed by the double loop, is continuous with the outside of the set. Drawn on the plan using a representational device known as the over/under passage, this figure requires not three dimensions, but at least the two dimensions of the torus.

On the torus, the double loop represents the internal exclusion of the object circled and thus missed by the cutting of the signifier different from itself. This model allows us to see, at a glance, how the demand represented by the cut of the signifier is situated in a different dimension from desire, since Lacan attributes to demand the turns around the peripheral hole of the torus, and to desire the turns around the central hole. When this demand completes its single turn, it has in fact completed two turns around the central hole, through which the object of desire is missed.

Above all, it's important to emphasize how this dialectic between demand and desire is produced by an effect of language. It is because the signifier is pure difference that the metonymic movement is produced, and the object a of desire is distinguished from the need from which the demand originates. It's by naming the object of need that the signifier of the demand, in completing its turn, actually completes two further turns around the object of desire. By the very fact that this demand passes through language, it splits into need and desire. We can write: need + signifier = desire.

The signifier, being different from itself, implies a space of difference that cannot be filled; and the simplest of signifiers, by closing in on itself, can only be split between itself and the other it is to itself.

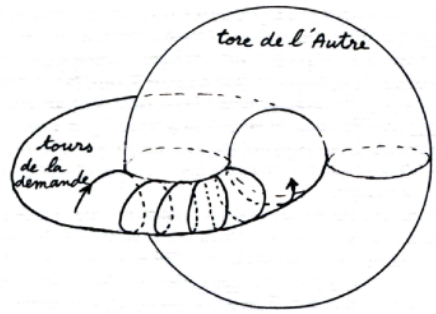


Figure 18

The torus always implies a complementary torus to which it is coupled. The peripheral hole of one is the central hole of the other; we can say that the reason for the couple is the torus.

For Lacan, the torus represents neurosis, and it's striking to note that, for the subject governed by this torus, what circulates in the peripheral hole circumscribed by his demand is the object a of the Other. (Figure 18)

The double-loop cut, if it occurs, transforms this torus into a double-sided, twice-twisted strip that is the double of a Möbius strip; in other words, there is a topological identity between the double-loop-cut torus and the Möbius strip that has undergone a single-turn median cut.

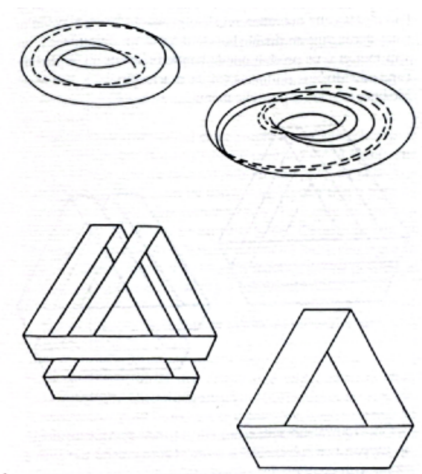


Figure 19

For Lacan, this transformation and identity represent the schema of an analysis: the neurotic torus cut into a double loop becomes a bipartite Möbius strip, then a single Möbius strip whose single edge encloses the object a, which this time can really be identified. This is the operation described in *L'Étourdit*. (7). (Figure 19)

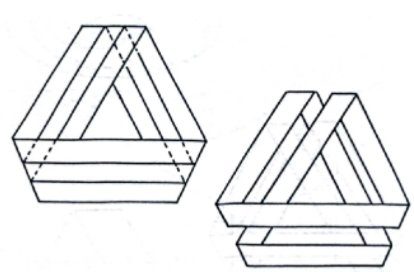


Figure 20

This transformation is the only virtue of the double-loop cut; what about the cut on the Möbius strip itself? If we cut a Möbius strip along its edge, we describe such a double loop and detach a two-sided strip from a central Möbius strip. If we repeat this operation away from the edge, our double-loop cut ultimately becomes a single-loop cut, producing only the two-sided strip. There is no central Möbius strip. Hence the conclusion that this Möbius strip is the cut itself. (Figure 20)

This equivalence between the cut and the Möbius strip provides non-substantive support for Lacan's understanding of the subject $ as a pure cut. When a formation of the unconscious occurs, it occurs in everyday discourse. There's no distinction between back and front at this level: this discourse makes a cut, and if this cut closes in on itself thanks to interpretation, the Möbius strip becomes a two-sided strip with a back and a front, and it's in this sense that interpretation produces the unconscious as the back of discourse.

We can see here how Lacan's topology requires no substance, but is founded on the cut of the spoken word, the Möbius strip whose relation to the torus we've emphasized being made up of nothing more than “pointless lines” or cuts.

The Möbius strip is thus apt to symbolize the $ subject; it combines the very strange properties of being both a surface and a pure cut, of uniting right and left at every point on its surface, yet being able to separate them with a single cut by taking on a toric form. The Möbius strip thus makes it possible to conceive of a subject $ quite distinct from the Ego, and to explain how, without involving anything other than the cut of the said, a topological transformation is possible and represents the analytic process itself.

On some fundamental points from the seminar on Identification

In Lesson 20 of the seminar on Identification (May 16/1962), Lacan sets out some fundamental points of reference for introducing topology as a structure of psychoanalytic discourse. It's remarkable to find in this lesson several allusions to the knots to be inscribed on the torus or cross-cap, and this ten years before the introduction of the topology of the Borromean knot.

Contrary to common intuition, which starts from the surface to consider the cut, Lacan shows here that it's the cut that's the starting point. It is the cut itself that organizes the surface. We've just seen how the Möbius strip was itself a cut, i.e., it's the cut that entirely defines the structure of the Möbius strip. In this lesson, Lacan uses the different organizations of the cut to define all kinds of surfaces.

A frequent objection to Lacan's use of topology is that topology is concerned with continuity, whereas language is made up of discrete, discontinuous elements. We saw in our reading of Saussure how crude this opposition is, and how it short-circuits the issue. In Lesson 20 of the Seminar, Lacan takes the signifier, which he sees as a cut, as his starting point for introducing topology. “Can a signifier,” he says, ”in its most radical essence, be envisaged only as a cut > <in a surface...” Topology makes it possible to account for the essence of the signifier, which is both discontinuity and difference. Discontinuity in its vocal incarnation, but also scansion, which relates to what Lacan called the function of haste in logic (8). Difference is the synchronic dimension of the signifier, as distinct from simultaneity. The fact that the same signifier, by being repeated, is inscribed as different from itself implies that the phrase “a is a” only expresses identity by first positing difference, as can be seen in commonplace examples such as “life is life” and “war is war”.

It is the necessity of inscription that introduces the topological dimension. The inscription of the inner eight, the double loop, highlights a cut that intersects itself after a second turn. The two loops express the difference of the signifier, which repeats itself as different from itself. The cut itself introduces the dimension of the Real as always returning to the same place. Lacan insists that the cut is to be considered intrinsically as the structure of the signifier, before any reference to a surface.

To comment on this difficult passage, we need to refer back to Saussure, to his reference to the linear structure of the signifier, and to the fact that in language there are only differences without a positive term. This means that, topologically, the utterance is a cut and that, in its most intimate structure, there is no isolable element but differences, i.e. pure cuts.

The cut. In topology, a space is said to be connected when it has no separation, i.e. it is impossible to separate this space X in such a way that there exists a pair ABof non-empty subsets of X such that Au B= X and A n B = D. The line and the plane are related, i.e. these sets cannot be simply divided. Given a segment (a, b), if cis a number such that a<c<b, c cannot divide (a, b] without "sticking" to one of the resulting segments (a, c) and (c, b] or without missing both segments (a, c) and [c, b]. In the latter case, the two sets are dehorned, and something is lost in the middle, point c, which constitutes the cut.

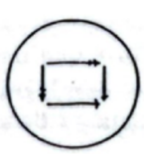
Here we find the same structure as in the exclusive vel studied in the chapter on logic. It's also a reading of the rhombic punch: 0 of the fantasy formula ($ 0 a) or of the drive ($ 0 D); this punch is a cut.

We seem to be dealing with this type of cut, i.e. a cut that slices through and not a cut that is already installed, as in a separate space, such as the space formed by the union of two non-intersecting circles in the plane. In the latter case, it would be pointless to make a cut, as there would already be two distinct pieces.

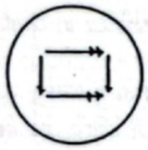
We must therefore take into account the highly paradoxical topological structure of the signifier. Each signifier is intimately linked to all the others, and is itself a pure cut. In other words, we need to conceive of a connection without substance, a connection made of pure differences.

The hole. So it's from the cut that Lacan deduces the surface, and not the other way round. To do so, he relies on the notion of the fundamental polygon. By giving the hole a positive value, we can deduce the surface as "the organization of the hole".

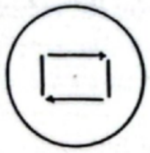
Indeed, starting from the cut, it is possible to show how the different surfaces are deduced from the way in which this cut does or does not glue back to itself. The starting point is a cut on the sphere. This cut is oriented by small vectors located at the edge of the hole; we then simply glue each vector to the corresponding vector, respecting their direction (the vectors to be glued have been indicated identically by a single or double arrow).



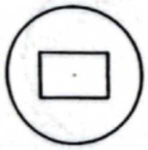
1) the edge closes in on itself itself like a wolf trap and we obtain a sphere.



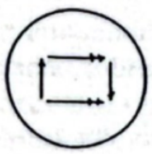
2) this is the torus, where each vector vector is joined to the vector vector facing it.



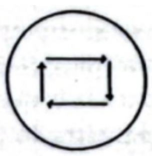
3) the vectors are not the vectors are not shown on the entire edge a free edge. This is the band.



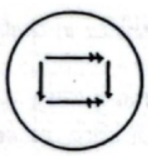
4) is the hole that can be reduced to zero, but also can encompass the entire sphere. In fact, you have to imagine that this sphere is disc or membrane.



5) Klein's torus.



6) the cross-cap; the the edge cannot be in our space without immersion, which is also is also the case with Klein's torus, and we must admit that the surfaces surfaces can intersect.



7) The torus. Lacan uses the torus to symbolize the dialectic of demand and desire. Unlike a sphere, certain circles on the torus cannot be reduced to zero. These are the circles that go around the peripheral void or the central void, or both.

• On this torus, the full circles, as they wrap around each other, will represent the demand D, and the empty circles the object, or the outline of the metonymic object of desire d.

• After the repetition of the unary trait, of the turns of the demand, the loop can close in on itself, and the subject who has traversed this loop has necessarily made a mistake in his account by one turn, i.e. the turn of the torus itself, which escapes subjectivity, or which subjectivity can only grasp through the detour of the Other. We see here the (-1) at the foundation of subjectivity. Lacan reminds us that, in order not to lose the salt of this topology, we need to consider the surface itself, and think of the infinitely flat subject that moves around on it, and is therefore mistaken by one turn in its counting. The (-1)

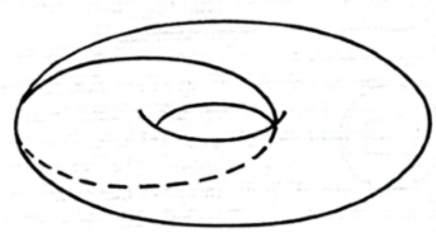


Figure 21

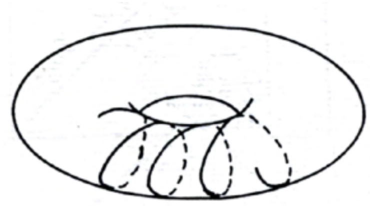


Figure 22

highlighted by the topology of the torus is linked to that of privation. It's the (-1) underlying every universal affirmation. If a class is constituted by the presence of a feature, it's because that feature may be missing (-1), hence the fact that the exception not only confirms but is the very principle of the rule. (Figure 22)

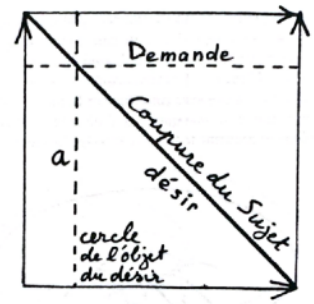


Figure 23

Symmetry and asymmetry on the torus. On the torus, we can represent the fondomental polygon, the loop of the demand returning on itself after having gone around the central hole, i.e. around desire a. (Figure 23).

Note that the torus we've drawn in (a) is not superimposable with its image in (b). superimposable with its mirror image drawn in (b); the torus therefore torus therefore has ed disymmetry properties (Figure 24.)

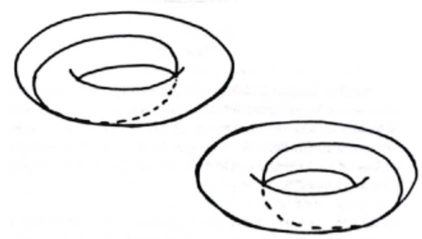


Figure 24

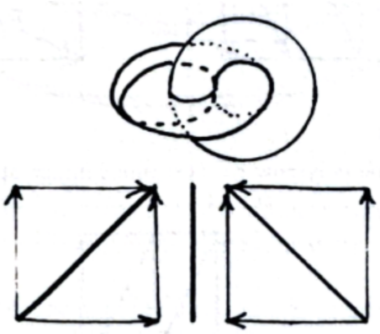


Figure 25

With the two chained toroids, Lacan represents the subject and the Other in the neurotic dialectic. The significance of circles d and D is reversed: the demand of the one is the desire of the Other, the desire of the one is the demand of the Other, in other words, the pattern of frustration. This is worth clarifying. The simple turn (D +d) on one of the toroids can be reproduced on the torus of the Other, demonstrating that the two toroids are then superimposable. (Figure 25)

This is a simple 90° flip. The images of the polygons are specularly symmetrical. If, however, we now perform on the torus, not the single loop but the double loop whose function we've learned is the real demand, we obtain this on the torus of the Other. (Figure 26).

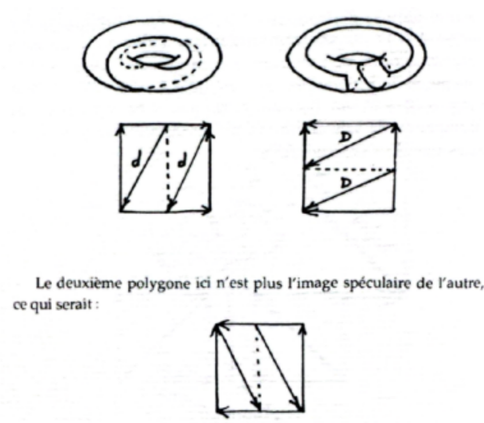


Figure 26

The toroids are no longer superimposable. Demand and object are inverted at the level of the Other. The subject's demand corresponds to the object a of the Other, and the subject's object a becomes the demand of the Other. For the obsessional, the emphasis is on the demand of the Other, for the hysterical on the object of the Other.

(Figure 26. The second polygon here is no longer the specular image of the Other, which would be: (figure)

It's not just the illusion, but the specular error that makes the neurotic look for object a through the specular image i(a). It's not that he confuses the two, but that he seeks one through the destruction or fixation of the other, of i(a), often resulting in the destruction of the desire of the Other. This is the sadistic fantasy of the obsessional who, as Lacan says, aspires to perversion without being perverse.

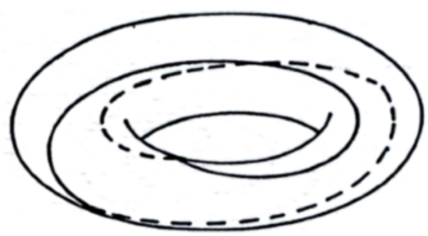


Figure 27

Quelques notes sur la topologie dans le séminaire *La Logique du fantasme (1966)*. Le support du fantasme

This particular structure of desire is revealed by the realization of the double turn which, according to Lacan, corresponds to the function of the object and to transference through the transfer onto the other torus. Dissymmetry only appears when there is a genuine demand, i.e., two turns.(Figure 27)

A few notes on topology from the seminar *The Logic of Phantasm* (1966). The support of fantasy

To introduce a logic of fantasy, Lacan relies on Euler circles at the start of the seminar, but immediately demonstrates their inadequacy as a representation. Euler circles accompanied the first developments in mathematical logic, easily illustrating Morgan's relations. In the seminar on Identification, Lacan had found a starting point that it's worth briefly revisiting here. The circle offers immediate support for intuition when it comes to representing the seizure of an object by a signifier. But the fact that the signifier's cut, forming a circle around the object, distinguishes two parts, an inner and an outer, is already a topological property that is the subject of a theorem, Jordan's theorem. This theorem specifies that the circle-cutting operation only applies to the sphere plane. In fact, it's enough to place the circle on a torus in a certain way to show that A reduces to non-A, and that the object escapes encirclement on the torus; what remains possible to grasp on the torus is not the object itself, but the difference between signifiers, and in the case of a signifier, its difference with itself, i.e. its self-difference. (Figure 28)

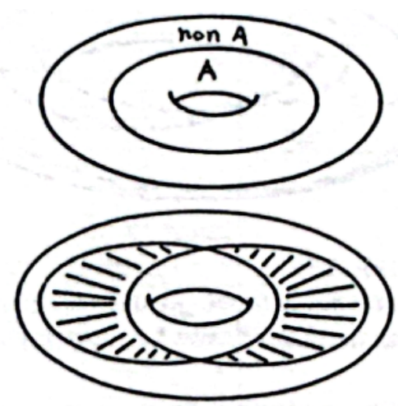


Figure 28

In the intersection, the object escapes, since there is continuity between this intersection and the outside of the circles. The double loop on the torus shows how, in the repetition of the demand that turns back on itself, the circumscribed object is missed, the inside revealing itself to be homogeneous with the outside (a? = -a).

On the other hand, this cut on the cross-cap detaches two pieces, the object a washer and a Möbius strip. The cross-cap is the very space of the signifier, whose two faces (the symbolic and the imaginary) are able to join everywhere. It's the surface Lacan uses to support fantasy. It's worth following this passage, commenting on it and illustrating it point by point (seminar Nov. 16, 1966).

What carries fantasy has two names, says Lacan: desire and reality. "Primordially, desire and reality have a textural relationship without a break, so they don't need stitching, they don't need to be re-stitched. The fabric of reality is thus entirely woven by the threads of desire, the same fabric where reality and desire would be right side up and upside down. "And yet this cloth is woven in such a way that, since it is seamless, we pass from one side to the other without noticing". This is the structure of the projective plane represented by the cross-cap. So there is no primitive separation of reality and desire. But this is the place of the Other, before any subject. The subject only begins with the cut. Lacan introduces a cut that crosses the imaginary line of interpenetration between the anterior and posterior walls of the cross-cap. "Any cut that crosses this imaginary line institutes a total change in the surface, namely that the entire surface becomes what we've learned to carve out of this surface under the name of object a. In other words, the entire surface becomes a flattenable disk, with an obverse and a reverse, of which it must be said that you can't pass from one to the other except by crossing an edge". (Figure 29)

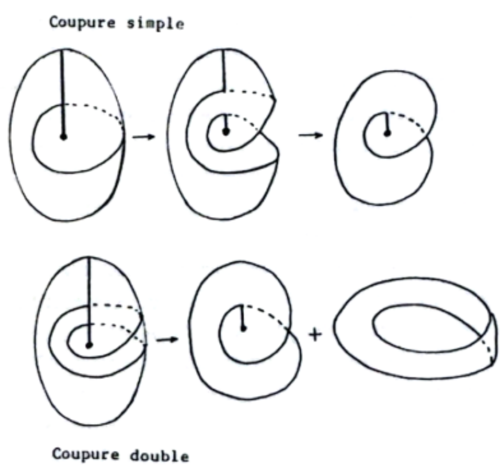


Figure 29

In these drawings, as we continue the operation, we have highlighted the relationship between a single cut and a double cut. A single cut through the imaginary line is enough to transform the entire cross-cap into object a, i.e. a disc with a right side and a left side. Reality and desire are then separated by an impassable edge. This operation shows how object a retains a fundamental relationship with the Other, since the subject has not yet appeared with the single cut that the signifier establishes in the real. This means that the subject is by no means first; at the outset there is no "being-there" (Dasein), other than the object a cut by the first signifier. The subject requires a double cut to finally appear. By splitting into two, this cut has the property of coming together. Lacan remarks: "It's the same thing to make one cut or two. We can consider the gap between the two turns, which are only one, as the equivalent of the first cut". When we pull the lips apart after the first simple cut, we see that they continue into each other, forming the single edge of a Möbius strip. Here we find the extraordinary equivalence of the simple cut and the Möbius strip. This equivalence is most easily seen on the Möbius strip itself, when a double cut is made close to its edge; a new, thinner Möbius strip is then detached, which remains linked to a double-sided strip. If the cut is made further from the edge, the result is an even narrower Möbius strip. But if we make the cut in the middle of the band, all we get is a two-sided band, and the Möbius band disappears; we can then say that it's just the cut itself. In the same way, on the cross-cap, if I make a double cut, as Lacan says, “I release, I restore what was lost in the first cut, namely a surface whose right side continues with the reverse. I restore the primitive non-separation of reality and desire”. The double cut restores what was lost with the single cut, i.e. continuity between right side and left side. What restores this continuity?

It's not the object a, the double-sided washer, but the other component, the Möbius strip, also cut in this operation. Thus, for the speaker, reality, i.e. psychic reality, is only the reverse side of desire, and this place and this reverse side are in continuity; the phantasm screens all perception of the real itself, as distinct from reality, of course.

In this lesson from the seminar on The Logic of Phantasm, Lacan explores step by step the effect of the cut on the cross-cap. First, there's a simple cut that slices through the object, but in this case there's no more than the object, and desire and reality are no longer in continuity, but on two sides separated by an edge. In a second stage, a double cut detaches the object from a Möbius strip, restoring the unilaterality of reality and desire. But Lacan is quick to point out that this Möbius strip was in fact already present in the simple cut. However, this identity between the simple cut, which was already there, and the Möbius strip can only appear after the double cut.

Following in Charles Melman's footsteps, we can ask whether this topological device does not condition the clinic of hysteria, where we find the separation of desire and reality, with desire always on the other side, behind the door or under the bed, or even inside the body, unknown, and not in the head. The hysterical woman suffers from being entirely an object, and strives to make her subjectivity exist, while evading herself as an object in order to maintain the desire of the Other, from which she remains suspended. The obsessional, on the other hand, seems to correspond to the case where the double cut fails to close in on itself. He thus remains in contact with an object from which he is unable to separate, in a jouissance that is both continuous and atrocious.

The physical topology of L'Etourdit

In L'Etourdit (9), written in 1972, Lacan defines topology as “the very fabric of psychoanalytic discourse”. He gives a summary of his topology at the point at which he is at in his discourse, in his contribution to psychoanalytic discourse. This text is remarkable for the absence of topological figures [Transformations and surfaces are described without any recourse to drawing. Lacan regrets being obliged to use images all the same, and not resort to pure mathematical formulas.

puns, in the grammatical structure itself, in ...

[END of TEXT, p. 2]