

La topologie. une mathématique de la qualité utilisée par Lacan

Marc DARMON

trans. Quinn FOERCH

Throughout his theoretical endeavor, Lacan maintained a dialogue with the sciences—linguistics, of course, but also the "hardest" sciences: logic and mathematics.

If Lacan ultimately came to formulate that psychoanalysis is not a science, it is because he positions psychoanalytic discourse as being out of phase with the discourse of science—within a formalization we shall discuss, wherein these two discourses are distinguished from the discourse of the Master and the University discourse. Nevertheless, science remains necessary to psychoanalytic discourse because it shares with it the same subject: a purified subject, a mere cut that Lacan designates as S. Indeed, Freud required the prior, abrasive work of science to enable him to take his inaugural step—as evidenced by *Project for a Scientific Psychology*. A study of Lacan's texts and seminars reveals a constant quest for the literal formula, the formal schema, or the topological figure—a quest undertaken as a kind of imitation of science. It would be possible to demonstrate how Lacan's discourse is organized around a handful of formulas, graphs, and figures—a set actually quite limited in number and remarkably consistent across his various seminars. In his later years, Lacan suggested that his

*Psychoanalyst, Psychiatrist—1 rue Bourbon-le-Château, 75006 Paris

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topological figures were intended to ensure the transmission of psychoanalytic discourse—a transmission that would thereby proceed independently of his own person. One might object that this concern for transmission is contradicted by the difficulty of the texts, and by the fact that the various formulas and figures are not introduced as explicitly as they would be in a scientific text; thus, a portion of the construction often remains undisclosed—as is the case, for instance, with the construction of the graph depicted in the *Écrits* text "The Subversion of the Subject and the Dialectic of Desire," which is derived from the sequence $\alpha, \beta, \gamma, \delta$ introduced in "The Seminar on 'The Purloined Letter'." This construction is neither reproduced in the written texts nor explicitly detailed in the seminars; it must be, quite literally, reconstructed. Let us propose that truly explicit demonstrations are to be found only in academic theses—works written in the *après-coup* of a genuinely scientific endeavor, which itself entails certain shortcuts during the initial process of blazing a trail. Furthermore, the constructions employed by Lacan often appeared to his audience as mere artifices, thereby acting as obstacles to the incisiveness of the very discourse they served to organize. It was deemed more important to put these formulas or diagrams to the test within a discourse that utilized them in act—specifically through its engagement with the experience of psychoanalysis—than to provide the key to their construction like a common recipe, thereby running the risk of fostering the belief that if everything could be reduced to that alone, one could simply dispense with the rest; or that if it amounted to nothing more than that—a mere assemblage of letters or a drawing, childlike in its simplicity and starkness—one could, ultimately, do without that as well. Lacan's formulas and diagrams—much like Einstein's formulas within scientific discourse, for instance—require the

entire surrounding linguistic context, as well as the ongoing pursuit of the analytic experience as a form of social bond, in order to sustain themselves. It is evident that while, in his later seminars, Lacan could pursue his research by silently sketching knots before an attentive audience, he could not have begun in this manner during the early years of his teaching. This presupposed a certain amount of preparatory work.

We will take two examples of Lacanian topology: the Moebius strip and the formula of the four discourses.

The Moebius Strip

It is to illustrate certain paradoxes inherent in the logic of the unconscious—paradoxes from the standpoint of mathematical logic—that Lacan, in his seminar on identification, employs topological models and introduces the torus, the Moebius strip, and the cross-cap.

Henri Atlan proposes a criterion of demarcation to distinguish between rational and irrational discourse, independently of the notion of truth; a rational discourse adheres to certain logical principles: the principle of identity and the principle of non-contradiction—namely, that A is A and cannot simultaneously be both A and non-A.

Must we, then, reject the *Traumdeutung*? Indeed, Freud acknowledges that the dream does not adhere to these principles, and he sets forth a logic that rejects the principles of non-contradiction and identity. For the unconscious, in fact, one must write: $A \neq A$ and $A = \bar{A}$.

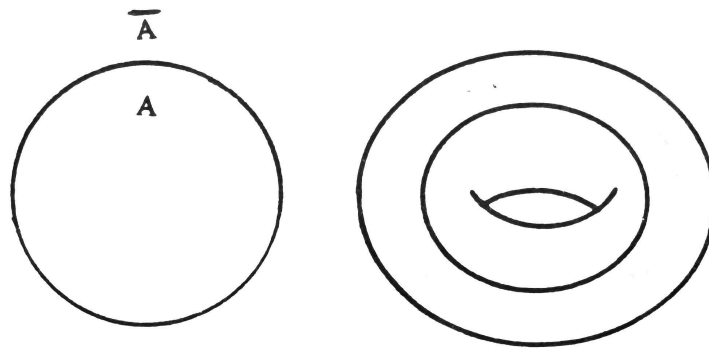
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This is, in fact, the rule in language; one need not—as Freud did—refer to Abel to discover, within the most commonplace speech, expressions that are antonymous to themselves: "that's a fine mess," "that's a real piece of work," "that's brilliant"—or even the term "queen," which in Middle English signified both a monarch and a prostitute.

Unlike the formalized signifier of mathematical logic, the natural signifier is, in principle, non-identical to itself; here, equivocation is the rule.

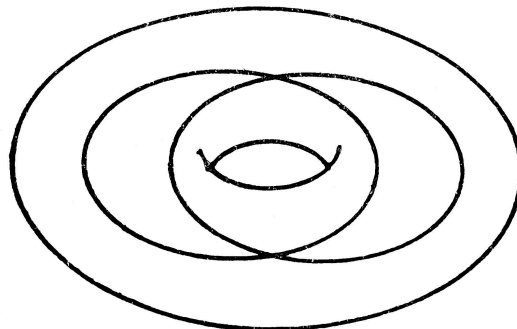
What, then, becomes of our logical relations—and their customary illustration via Euler circles—in such a case? A circle nested within another circle: such is the model for the logic of classes. To illustrate this model is not the same thing as asserting that something is a "non-man" situated within the class of animals. Yet what applies to zoology cannot adequately encompass the domain of language.

Indeed, the Eulerian illustration already presupposes a specific topology: the topological fact that a circle partitions a plane or a sphere into an interior region and an exterior region remains implicit. However, were this circle to be drawn upon a torus in a particular manner, it would fail to divide the torus into two distinct parts; the circle's interior would, in that instance, communicate continuously with its exterior. One could then write: $A = \bar{A}$.



It is possible to inscribe—in the form of offset circles upon a torus—a signifier that is "different from itself," thereby bringing to light, on the one hand, the emptiness of the space where the object is ostensibly circumscribed by the circles' cut, and on the other, the field of what Lacan terms the signifier's difference from itself.

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Let us illustrate this with an example—the simplicity of which, I trust, you will forgive: an analysand arrives at her session complaining of "a headache" [*un mal de tête*] and immediately follows up with an embarrassing fantasy of fellation that had surfaced in the subway during her commute just prior to the session.

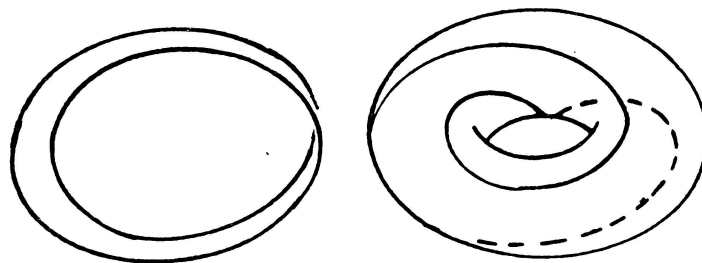
We can see here how interpretation may play upon the signifying equivocation in its sexual sense. We can position upon our two Euler circles, on the one hand, the signifier "head" [*tête*]; and on the other, that same signifier—but as it appears within the phrase "a male one sucks" [*un mâle qu'on tète*—at the very moment it produces the hysterical symptom as a metaphor. As we stated at the outset, the commonplace properties of the "natural" signifier constitute, for the logician, so many paradoxes. It is in this light that Lacan interprets the famous paradoxes identified by Russell.

Let us return to the well-known example of the set of all sets that do not contain themselves: does this set belong to itself, or does it not? It goes without saying that if we identify this set with itself—that is to say, insofar as it subsumes other sets on the one hand, and insofar as it is itself

such a set on the other—we are confronted with an impossibility; however, if we consider that a signifier is distinct from itself, it becomes apparent that this set is not the same entity in both instances. To take the analogous example of the catalogue of all catalogues that do not list themselves: this catalogue is not the same thing when it is listing other catalogues as it is when it is itself being listed. Lacan speaks here of "internal exclusion": the set of all sets that do not contain themselves is in a state of internal exclusion *vis-à-vis* itself.

The double loop serves to illustrate this internal exclusion, inasmuch as the central region—circumscribed by this double loop—remains in continuity with the exterior of the set. Drawn on the plane by means of a representational device—the over-under crossing—this figure requires not three dimensions, but at least the two dimensions of the torus.

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On the torus, the double loop thus represents the internal exclusion of the object—an object thereby circumscribed, yet missed, through the incision of the signifier, which is distinct from the object itself. This model allows one to grasp, at a single glance, how demand—represented by the incision of the signifier—is situated in a dimension distinct from that of desire; for Lacan assigns to demand the circuits circling the peripheral hole of the torus, and to desire, the circuits circling the central hole.

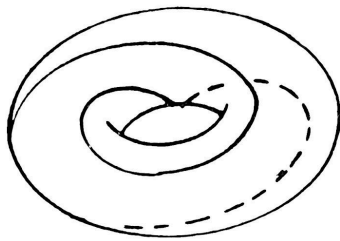
This demand, as it completes its single circuit, has in fact executed two circuits around the central hole—the very aperture through which the object of desire is missed.

Above all, it is crucial to emphasize how this dialectic between demand and desire is generated as an effect of language. It is precisely because the signifier is pure difference that the metonymic movement of desire comes into being—a movement in which each signifier refers back to another signifier—and that the object *a* of desire is distinguished from the need in which demand originates. It is by naming the object of need that the signifier of demand—in completing its circuit—effectively executes two additional circuits around the object of desire. By the mere fact that this demand passes through the defile of language, it undergoes a bifurcation, splitting itself between need and desire. We may formulate this as follows: need + signifier = desire.

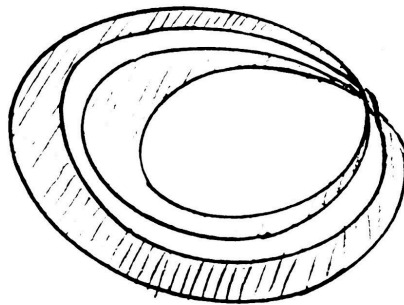
Since the signifier is distinct from itself, it implies a space of difference that can never be filled; and even the simplest of signifiers, in looping back upon itself, can only bifurcate—splitting itself between its own identity and that other which it simultaneously is to itself.

The torus invariably implies a complementary torus to which it is coupled. The peripheral hole of the one serves as the central hole of the other. Paraphrasing the title of a book mentioned earlier, we might say that the reasoning of the couple is the torus. For Lacan, the torus represented neurosis; it is striking to note that for the subject governed by this torus, what circulates within the peripheral hole—circumscribed by his demand—is the object *a* of the Other.

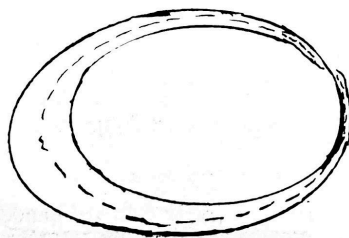
The double-loop cut—should it occur—transforms this torus into a two-sided, twice-twisted band, which is the double of a Moebius strip; that is to say, there exists a topological identity between the torus cut in a double loop and a Moebius strip that has undergone a single-turn median cut.



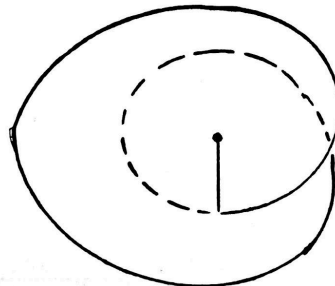
Tore découpé par une double boucle



Bande de Mœbius bipartie



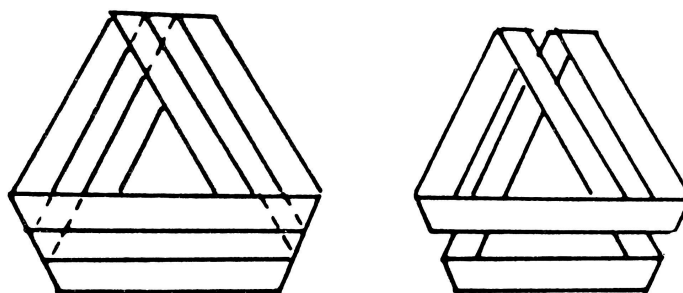
Bande de Mœbius



Objet *a*

For Lacan, this transformation and this identity represent the schema of an analysis: the neurotic torus, cut into a double loop, becomes a bipartite Moebius strip, and subsequently a simple Moebius strip whose single edge encloses object *a*—an object that is, this time, truly circumscribable.

This transformation is the specific effect of the double-loop cut alone; but what becomes of this cut when applied to the Moebius strip itself? If one cuts a Moebius strip along its edge, one traces out just such a double loop and detaches a two-sided strip from a central Moebius strip; if one repeats this operation while moving away from the edge, then—at the limit—our double-loop cut becomes a single-turn cut, yielding nothing but the two-sided strip. There is no longer a central Moebius strip; hence the conclusion: this Moebius strip is the cut itself.



This equivalence between the cut and the Moebius strip provides a non-substantialized support for what Lacan means by the subject *S* as a pure cut.

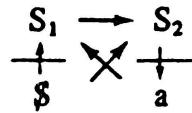
When a formation of the unconscious occurs, it takes place within the flow of ordinary discourse; at this level, there is no distinction between inside and outside. The saying itself constitutes a cut; and if—thanks to interpretation—this cut closes back upon itself, the Moebius strip transforms into a two-sided strip endowed with both an inside and an outside. It is in this sense that interpretation produces the unconscious as the reverse side of discourse.

Here, we grasp how Lacan's topology requires no underlying substance, but rather grounds itself upon the cut effected by the saying—the Moebius strip (whose relationship to the torus we have already highlighted) being itself constituted solely by "*ligne sans points*," or cuts. The Moebius strip is, therefore, uniquely suited to symbolize the subject *S*. It combines the highly peculiar properties of being simultaneously a surface and a pure cut—of uniting, at every point of its surface, the inside and the outside—yet of being capable of separating them by means of a single cut, by assuming a toroidal form. The Moebius strip thus makes it possible to conceptualize a subject *S*—quite distinct from the *Myself*—and to explain how, without invoking anything other than the cut of the said, a topological transformation is possible, representing the analytic process itself.

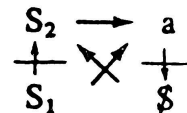
The construction of the four discourses reveals to us an exemplary aspect of Lacan's approach regarding his relationship to formal structure.

In place of the usual, simplified notation for these four discourses, I present to you this version—rarer, yet more precise—which I discovered in various seminar notes and, most notably, in the transcription of a lecture delivered in Milan in 1972 on the subject of psychoanalytic discourse.

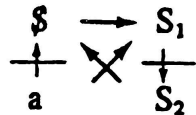
Discours du Maître



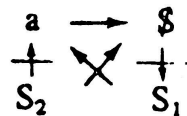
Discours de l'Université



Discours de l'Hystérique



Discours de l'Analyste



Through this algebraic notation, the aim is to demonstrate the organization—in its most reduced, yet irreducible, form—of the discourses that govern us.

The Discourse of the Master is modeled upon the formula: "a signifier represents the subject for another signifier."

It is structured around the Master Signifier, which commands a hapless slave—S2—to produce *a*: *plus-de-jouir*. However, for this Master, access to *jouissance* is barred; there is no downward-pointing arrow. Indeed, the Master has renounced *jouissance* by accepting the risk of death—a point Lacan underscores in his reading of Hegel. It is the slave who enjoys [*jouit*].

It is not my intention here to comment at length upon these formulas; rather, I refer you to Charles Melman's book on hysteria to fully appreciate their astonishing clinical relevance. We shall content ourselves with tracing the logical steps Lacan followed in establishing them.

The Discourse of the Master establishes a set of fixed places which, in the other discourses, will be occupied by different terms.

These are the places:

l'agent

l'autre

la vérité

la production

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The terms are:

S1: the signifier

S2: knowledge

\$: the subject

a: object *a*—the *plus-de-jouir*

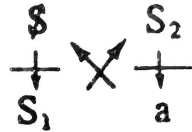
Let us note that it is the hysterical discourse that produces knowledge; hence Lacan's idea—at first glance astonishing and provocative—of situating scientific discourse precisely within the hysterical discourse. But do we not recognize here the position of the scientific subject \$—who counts for nothing in his own discovery—and who defies the Master Signifier S1: authority, established theory? Knowledge is produced in S2; yet what remains unknown is object *a*—the cause of desire—situated in the position of truth. Situating scientific discourse within hysterical discourse in no way diminishes the merits of science, nor is there any reason to place it elsewhere—in the discourse of the university, for instance. In the latter, what is produced is not knowledge; rather, the university purports to produce a subject—the student—whose course credits confer upon him a certain *plus-value*: a subject divided between knowledge and truth. What remains hidden within this discourse is the Master Signifier, which the subject is powerless to attain.

One cannot help but be struck by the fact that such a simple construction can account—in a non-trivial manner—for matters of such complexity. Yet, if these matters are constituted by language—and if language rests, in the final analysis, upon the principle that a signifier S1 always refers to another signifier S2; that the subject is born solely from the gap between these signifiers; and that object *a* is both the residue of this operation and the very element that reignites it as desire—then it is not unreasonable to conclude that the specific arrangement of these elements determines the organization of discourses which, when actually articulated in speech, prove to be remarkably rich and complex.

Lacan frequently emphasized that the number of such discourses is strictly limited to four. The terms circulate within the positions like a circular permutation, and no commutation is possible. Yet Lacan spoke of a fifth discourse: the discourse of the capitalist, which he touches upon very

briefly in his seminar, describing it without formally inscribing it. The only written formulation he provides for it—to my knowledge—appears in Milan, in the lecture previously cited.

Discours du Capitaliste



A discourse in which—unlike the others—no term is isolated; rather, each term fuels the next in a chain reaction whose inherent tendency is to spiral out of control.

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The discovery of a fifth discourse may shed light on Lacan’s approach to his use of mathematics. Indeed, in introducing this fifth discourse, is he taking liberties with a formalism he himself introduced solely to simplify the notation of the discourses—treating it merely as a form of shorthand—or are there inherent constraints at play? We shall demonstrate that this formalism was introduced for specific reasons, and that the discourse of the capitalist—which appears to elude this framework—is, in fact, a logical consequence of it.

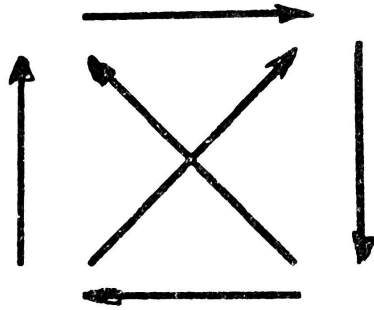
To do so, we must reconstruct a formal construction of the discourses—one that remains implicit in the surviving notes.

With these four discourses, we enter the realm of elementary topology—specifically, graph theory—which traces its historical origins to Euler’s solution to the *Königsberg* bridge problem.

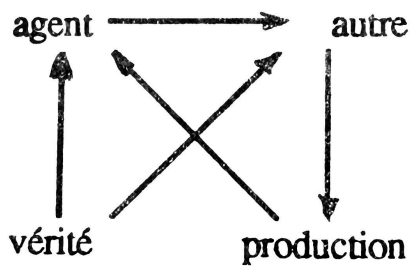
The graph in question here is tetrahedral. Lacan had taken an interest in graph theory from the very inception of his seminars; this particular graph serves, among other functions, as the structural support for the operations of the Klein group. Lacan refers to the tetrahedral graph in a lecture delivered at Sainte-Anne on February 3, 1972, wherein he designates it as the Graph of the Four Discourses.

Let us note, first of all, that if the edges of this tetrahedron are oriented, one can verify—as an exercise—that there exists only a single possible way to orient this tetrahedral graph such that a continuous circulation is established; that is to say, such that every vertex receives an input, and every vertex is capable of feeding one or more other vertices.

The resulting tetrahedral graph appears as follows:

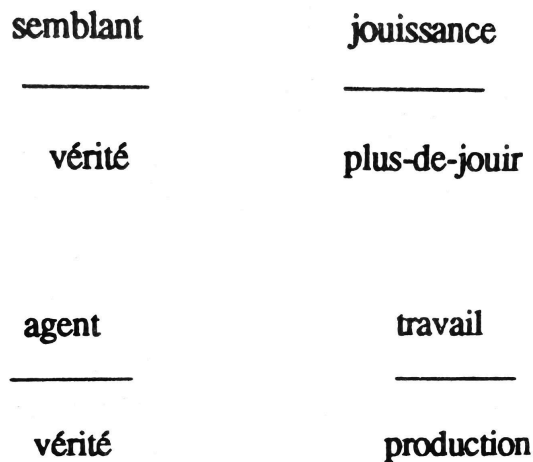


Lacan then removes one of the arrows from the tetrahedron, thereby deriving the specific arrangement of the positions within the Four Discourses.



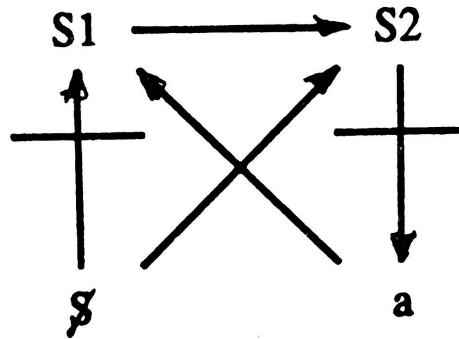
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One also finds:



The significance of this arrangement lies, on the one hand, in defining a position from which there is no return—that of truth—and, on the other, in defining three positions that form a circuit. At each vertex of the tetrahedron, the four letters—linked by a circular relation—come to be inscribed: S1, S2, a, \$.

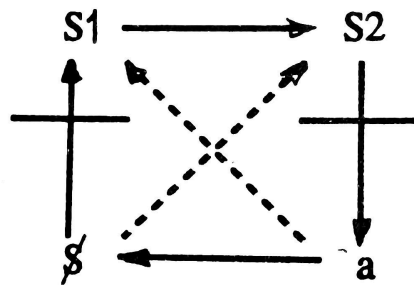
Thus, the Discourse of the Master is written as follows:



How, then, is the fifth discourse—that is, the Discourse of the Capitalist—derived? It is possible to deduce it through reconstruction. We have previously noted the Master's impotence in appropriating the *plus-de-jour*; this impotence is symbolized by the absence of an arrow between a and \$.

In the capitalist discourse, this *plus-de-jour* is quantified and assigned a value. At a certain moment in history, production is no longer termed *plus-de-jour* but rather *plus-value*. This surplus-value is accreted to capital; the impotence alluded to above is thereby overcome. It is therefore necessary to reinstate the lower arrow.

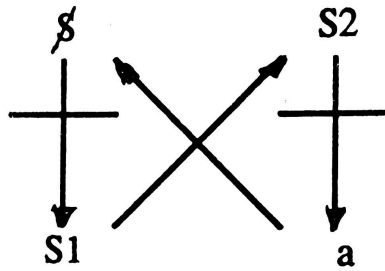
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The S1 of the Master's discourse—from which it originates—becomes the truth of the Capitalist's discourse, and the subject (\$) is thrust to the forefront of the stage, just as in the Hysteric's discourse.

This arrangement is made possible by the suppression of the oblique arrows—that is to say, of two edges of the tetrahedron.

And the configuration of the Capitalist's discourse is finally obtained through torsion:



We find the exact notation for this discourse in the Milan lecture, complete with its highly distinctive arrangement of vectors. Let us note that in this fifth discourse, the position of truth is no longer protected; the four vertices feed into one another. This implies that, within this discourse, the denunciation of truth serves only to reinforce the process—a point for which Lacan cites the role of Marx himself as an example. Here, then, is an unexpected yet interesting and verifiable consequence of those little formulas I was just commenting upon as an illustration of Lacanian topology.

In the course of examining these examples, we have been able to gauge the justified liberty Lacan took with mathematics—but also the rigor with which he strove to follow his chosen formalism through to its most extreme consequences, identifying it, for a time, with structure itself.

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