

Presentation of the Borromean knot

1. The monochrome Borromean knot, the gyrie

The Borromean knot is one of the solutions (Lacan repeatedly emphasizes that it is not the only one) to the Borromean problem, which is: how to tie three rings together in such a way that when any one of them is cut, all three rings are untied.

The simplest way to draw a Borromean knot is shown in Figure 1:

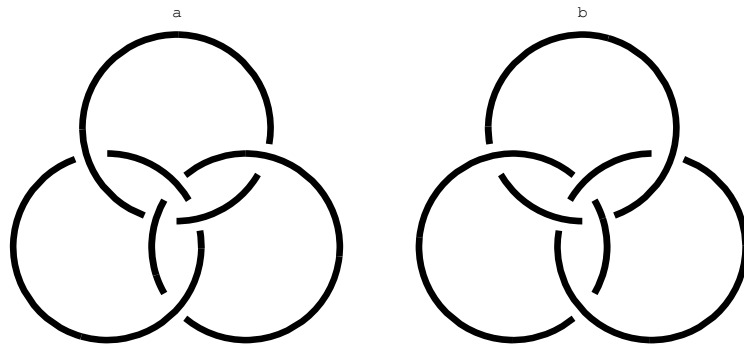


Figure 1: Levogyre and dextrogyre Borromean knots. (nfigO.eps)

In this figure, the rings are not distinguished from one another. When physically creating the knot, it can be seen that none of the three rings has a privileged position that would distinguish it from the others. The three rings are equivalent, and in order to recognize them, they would need to be given a distinctive feature, such as color.

However, Figure 1a is not the only possible representation of the Borromean knot. Figure 1b is another possibility, different from the first in that it cannot be superimposed on the first by simply sliding it into the plane of the figure.

There are therefore two forms of Borromean knot, which Lacan refers to as the *left-handed* knot and the *right-handed* knot. Figure 1 clearly shows that the two forms of the knot are mirror images of each other. To determine whether a knot is right-handed or left-handed, simply look at its central part, which consists of three curved strands (Figure 2). If these three strands form a clockwise rotation, the knot is said to be dextrorotatory. Otherwise, it is said to be levorotatory. In Figure 1, knot 1a is levorotatory.



Figure 2: Central part of levorotatory (a) and dextrorotatory (b) Borromean knots. (nfigO1.eps)

Another way to distinguish between the two forms of the knot, levorotatory and dextrorotatory, is to consider the superimposition of the rings: In the knot in Figure 1a, the ring at the bottom left is entirely above the ring at the bottom right, which is itself entirely above the ring at the top, which is itself

The order of the layers is counterclockwise. This is called a levorotatory knot. In the knot shown in Figure 1b, the ring at the bottom right is entirely above the ring at the bottom left, which is called a dextrorotatory knot. Following the order of superimposition, we turn clockwise this time. This is called a dextrorotatory knot, even though the knot itself is levorotatory.
that the knot is dextrorotatory.

Like Soury, we will refer to the fact of being dextrorotatory or levorotatory *as the gyration* of a Borromean knot. It should be emphasized that the gyration of a Borromean knot does not depend at all on whether or not the rings have been distinguished from one another. This is indeed a characteristic of the monochrome knot.

2. The polychrome Borromean knot: order.

When we decide to distinguish between the three rings by giving them colors—red (R), green (G), or blue (B) (but it could also be a name or label, such as R, S, or I, for example)—the number of *distinct* possibilities for representing the node increases. In what follows, we will say that two representations of the node are distinct when it is impossible to move from one to the other by simply sliding it across the plane of the sheet of paper. Figure 3 gives an example of two identical representations of the node. Figure 4 gives an example of two distinct representations.

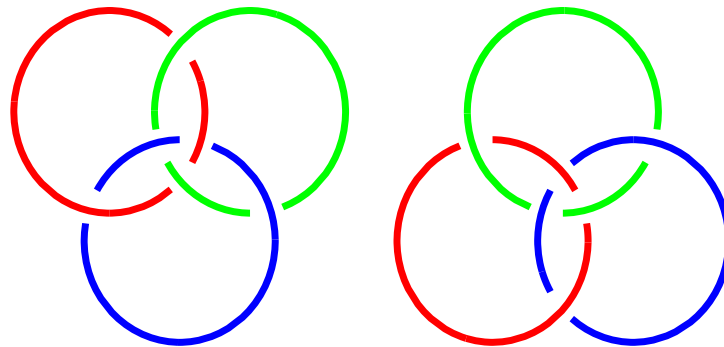


Figure 3: Example of two representations of the node to be considered identical: One can move from one to the other by simple rotation. (nfig4.eps)

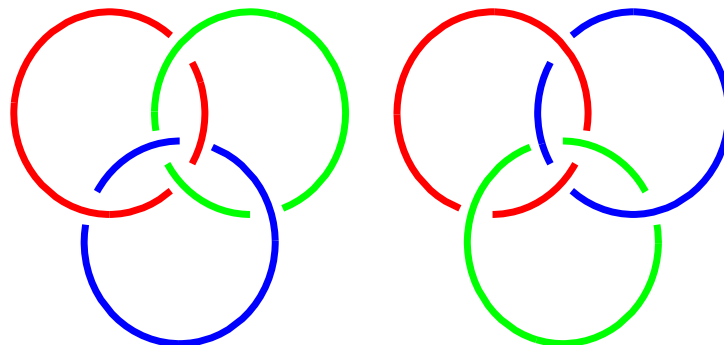


Figure 4: Example of two representations of the node to be considered distinct. (nfig02.eps)

Coloring the rings means that for each monochrome representation of the node, there are two distinct ways of ordering the colors, following the order of superposition:

- The red ring can be above the green ring, which is itself above the blue ring (RGB),
- or
- the red ring can be above the blue ring, which is itself above the green ring (RBV).

This order is independent of the node's gyration. We will refer to *the order* of a Borromean node in the plane as the fact that for this node, the rings follow one of the two possibilities above. The order can be RGB or RBV. (It is clear that RGB, VBR, and BRV are equivalent, as are RBV, BVR, and VRB. There are therefore only two possible orders).

As a result, there are only four distinct representations of the colored Borromean knot in the plane: RGB-L, RGB-D, RBV-L, and RBV-D. These four representations are shown in Figure 5.

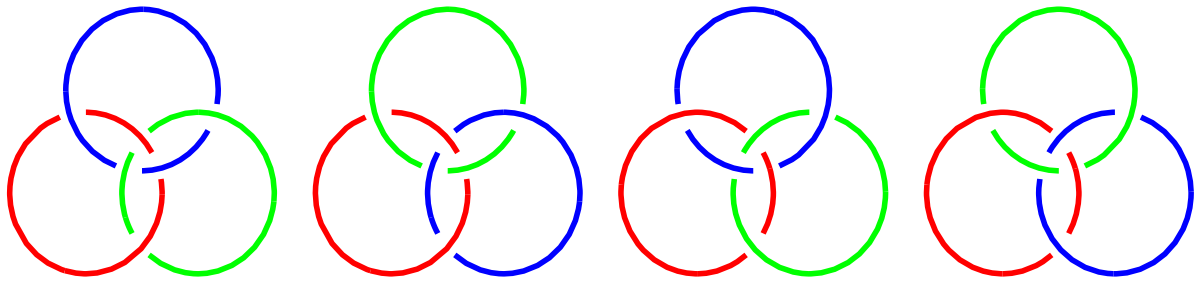


Figure 5: The four distinct forms of the colored planar Borromean knot. From left to right: RGB-L, RBV-L, RBV-D, RGB-D. (nfig7.eps)

3. The Borromean knot in space, flattened

The Borromean knot can also be approached through the three-dimensional figure shown in Figure 6.

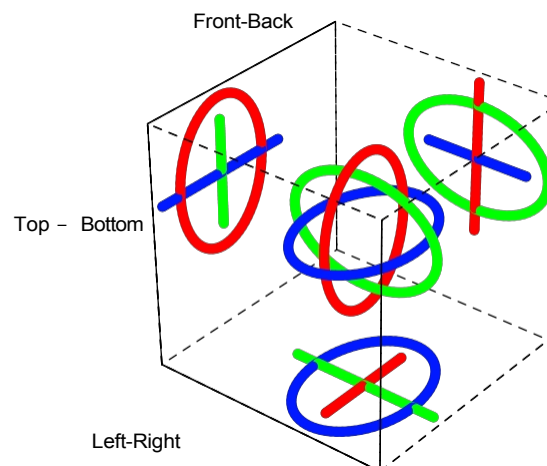


Figure 6: Borromean knot in space. The shadows cast on the three faces of the cube show that the three rings are oval. (nfig1.eps)

In this figure, we can see that there is an order of succession that is not identical to the order defined above: here, the red circle surrounds the green ring, which itself surrounds the blue ring, which itself... It is therefore clear that there is a second representation of the Borromean knot where the red ring surrounds the blue ring, which itself surrounds the green ring, which itself... This second representation is shown in Figure 7.

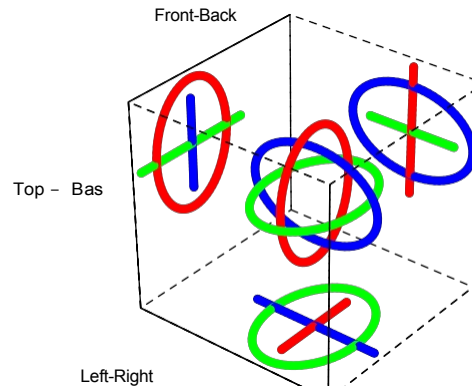


Figure 7: Borromean knot in space. The order of colors is different from that in Figure 6. (nfig6.eps)

A Borromean knot in space can be flattened from different points of view. Below is an example of flattening: flattening the knot in Figure 6 in the Up, Front, Right direction. Flattening takes place in two stages: First, the rings are pushed back, leaving them parallel to each other (for example, in Figure 8, the red ring has been pushed back to the left, the blue ring downwards, and the green ring backwards). Then we rotate them so that they are all in the same plane (Figure 9).

We see that we obtain an RBV-L type knot. However, it should be noted that this result is due to the convention we adopt for the flattening procedure. Readers who familiarize themselves with the actual manipulation of the knot will see that this convention is not unique.

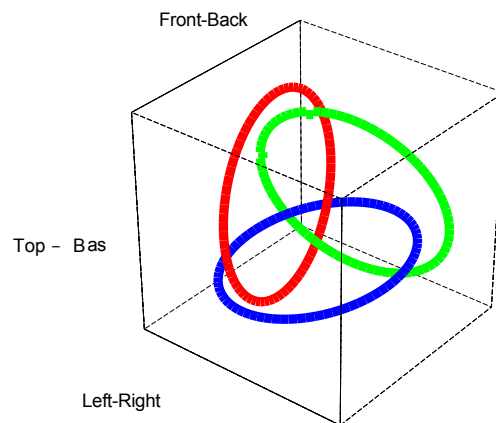


Figure 8: Borromean knot in space. The rings have been pushed apart to prepare for flattening according to the viewpoint: "Up, forward, right (U F R)." (nfig2.eps)

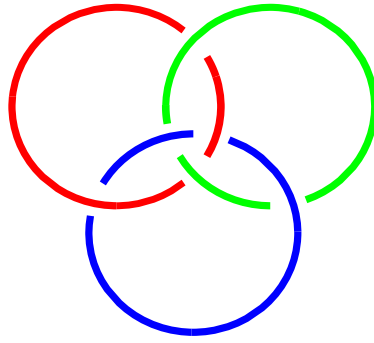


Figure 9: Borromean knot from Figure 6 flattened according to the viewpoint (H Av D). We can see that we obtain an RBV-L type knot. (nfig3.eps)

The same flattening operation can be performed according to the 8 viewpoints: (H Av D), (H Av G), (H Ar D), (H Ar G), (B Av D), (B Av G), (B Ar D), (B Ar G). However, the result of flattening can only give one of the four possible distinct configurations shown in Figure 5.

In fact, we see that the eight flattenings corresponding to the eight possible viewpoints of the node in Figure 6 ultimately provide only two of the four distinct flattenings given in Figure 5.

Thus, in Figure 10, we show the beginning of the flattening of the node in Figure 6, according to the "top right and back" viewpoint. We can see that we obtain a flat node of the RBV-D type.

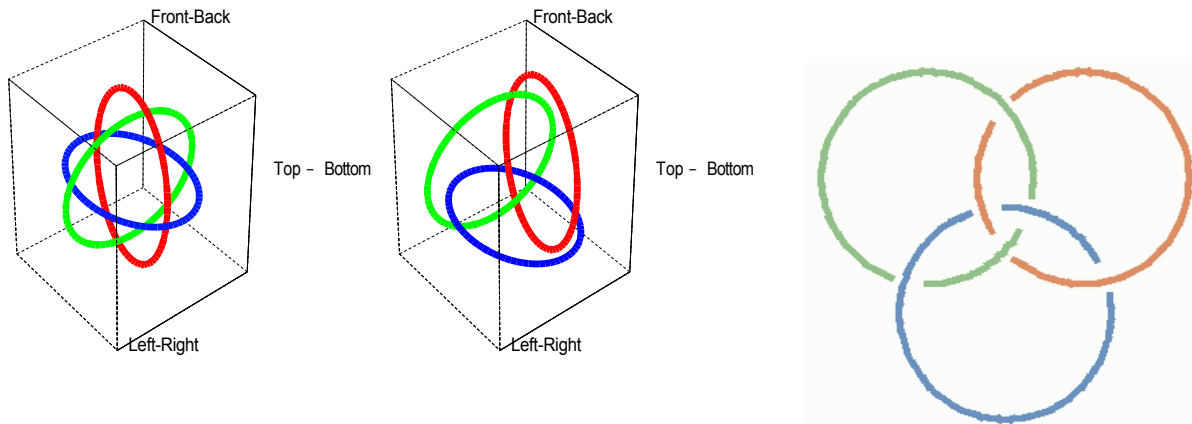


Figure 10: Flattening of the Borromean knot in Figure 6 according to the viewpoint (top, right, rear). We see that we obtain an RBV-D type knot. (HarD.eps and misapla.bmp)

Figure 11 shows how the knot in Figure 6 can be rotated around a horizontal axis to achieve the viewpoint: "bottom, right, front." Figure 12 shows that flattening according to this viewpoint will still result in a flat knot of the RBV-D type.

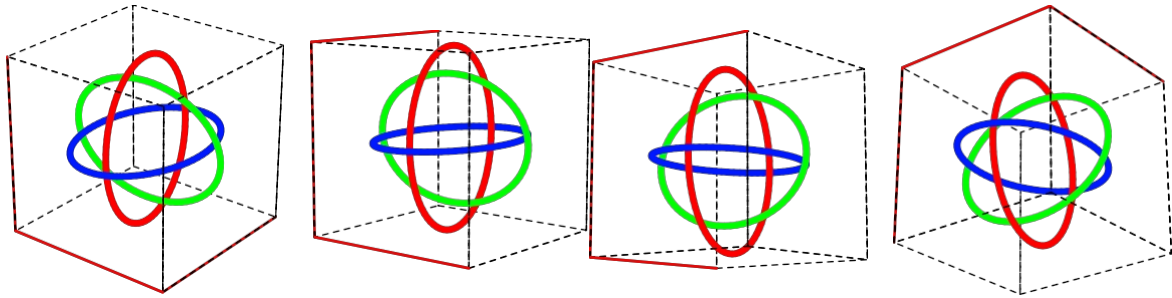


Figure 11: Gradual transition from the "top right forward" viewpoint to the "bottom right forward" viewpoint (rot.eps)

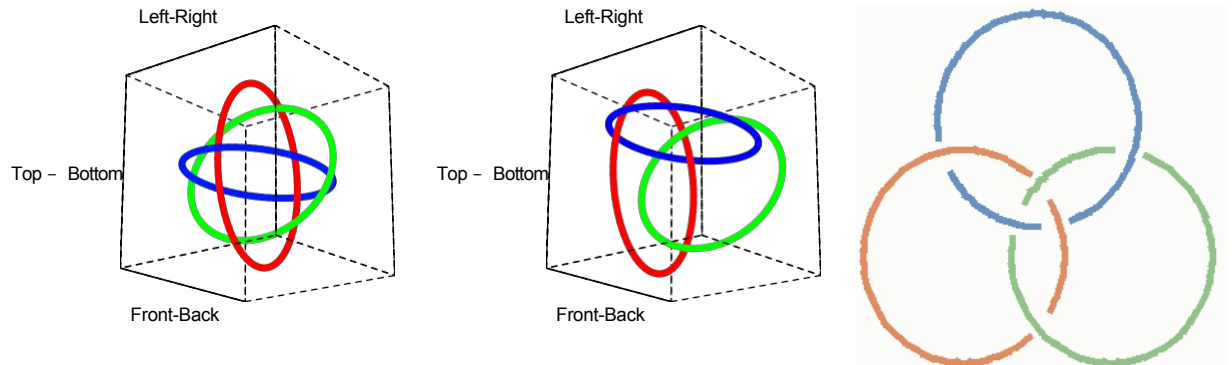


Figure 12: Flat view of the Borromean knot in Figure 6 from the "bottom right front" perspective. We can see that this produces an RBV-D type knot. (BavD.eps and misaplaO.bmp)

We could continue this illustration by reviewing the eight possible viewpoints. However, we already have the information needed to reach the following conclusion: the flattened versions of the knot in Figure 6 are all of the RBV type, and alternately levorotatory or dextrorotatory on the eight vertices of the cube, which give the eight possible viewpoints. To obtain the two other possible flattenings (those with the order RGB, left-handed or right-handed), we must start from Figure 7.

The question therefore arises as to whether the two 6s and 7s are two different representations of the same Borromean knot, or whether there are in fact two different knots. To answer this, we need to know whether it is possible to go from figure 6 to figure 7 without cutting any of the rings, or whether it is necessary to cut a ring to transform the knots in figures 6 and 7 into each other. To do this, we will study the various ways of going from one flattened configuration to another.

4. Transformations of the Borromean knot: from one flattening to another.

The four distinct flattenings possible for a knot whose loops are distinguished (by color or any other means) are shown in Figure 5, and we have chosen to designate them with the four labels: RVB-L, RVB-D, RBV-L, and RBV-D.

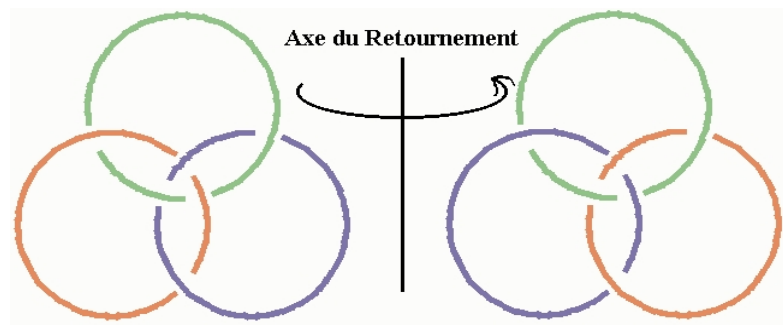


Figure 13: An RVB-D flattening and its reverse: the gyre is preserved and the order is reversed. This results in an RBV-D flattening. (crepe.bmp)

Figure 13 shows what happens to the flattened node in RGB-L form when it is flipped as a whole, "like a pancake." We can see that it becomes RBV-L: the gyration is preserved, while the order is reversed. This is a general rule: when a node is flipped as a whole, "like a pancake," the initial flattening is transformed into a flattening with the same gyration and opposite order.

It should be noted that this transformation is very different from symmetry with respect to a line (a mirror perpendicular to the plane of the figure). This latter transformation, shown in Figure 14, shows that symmetry preserves order and reverses gyration. Block reversal and symmetry are transformations that give the same result when applied to a two-dimensional figure, such as a triangle or a circle. The fact that different results are obtained with the Borromean knot can be considered "proof" that it is a truly three-dimensional figure. This may also be the reason why Lacan describes the flattening of the knot as "real writing."

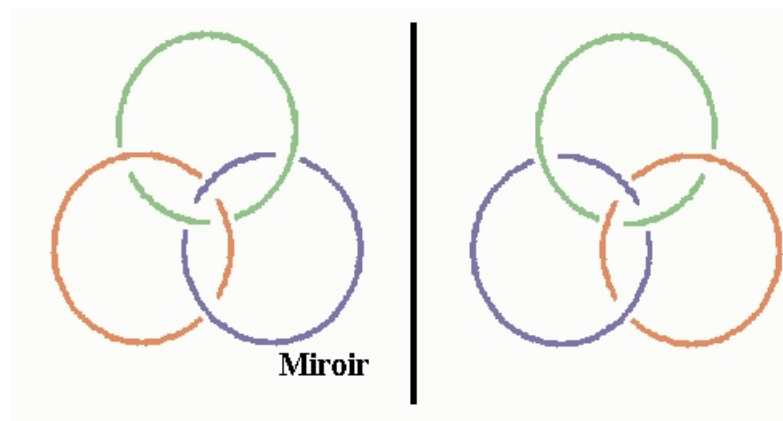


Figure 14: An RGB-D flattening and its image in a mirror perpendicular to the plane of the figure: the order is preserved, the gyri are inverted. The result is RGB-L. (symetr.bmp)

It should also be noted that symmetry is not a "physical" transformation: we do not know a priori whether, in order to transform a given node into its symmetrical counterpart, we need to cut a ring or not.

Figure 15 shows that this is not necessary: it shows what we obtain when we flip one of the rings around the other two: as with symmetry, we obtain a node with an inverted gyri and the same order as the starting node. Flipping a ring is therefore a physical transformation equivalent to symmetry.

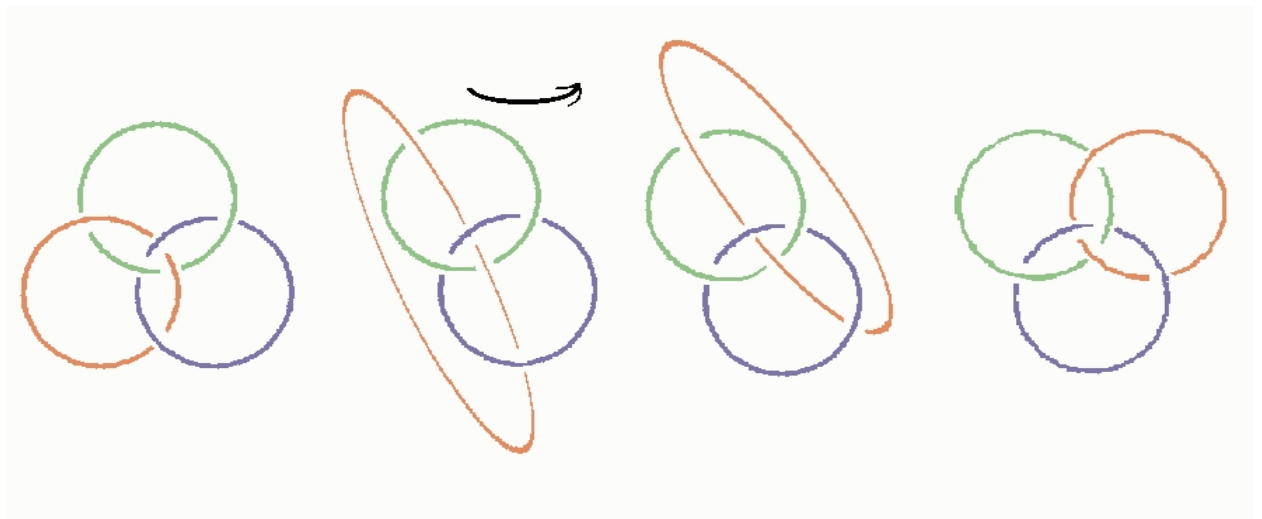


Figure 15: The effect of flipping a ring. The order is preserved, the gyration is reversed, as with symmetry. (flip.bmp)

Regarding this ring reversal operation, it can be noted that it gives the same result regardless of which ring is reversed. Furthermore, we obtain the same result when we flatten a node in space (Figure 6 or 7) following two neighboring points of view on the cube of the eight possible points of view: we change the gyration while maintaining the order.

Thus, using our two transformations, global reversal and ring reversal, we can independently move from an RGB node to an RBV node and vice versa on the one hand, and from a levorotatory node to a dextrorotatory node and vice versa on the other. It is therefore clear that these two transformations are sufficient to transform any of the four possible configurations in Figure 5 into any other.

It also follows that the two knots in space, shown in Figure 6 and Figure 7, can be transformed into each other by means of any flattening operation, followed by a block reversal (as shown in Figure 13), followed by the reverse operation of flattening. The transition from Figure 5 to Figure 6, when performed on a knot made using semi-rigid rings (e.g., electrical wire), appears as a kind of reversal in which the inside becomes the outside and vice versa. This seems to me to explain Lacan's interest in research on the reversal of the sphere.

The conclusion of this section can be summarized as follows:

There is only one colored Borromean knot, and it can only be flattened in four distinct ways.